3

Network Theorems (Application to dc Networks)

3.1 INTRODUCTION

In Chapter 2, we have studied elementary network theorems like Kirchhoff's laws, mesh analysis and node analysis. There are some other methods also to analyse circuits. In this chapter, we will study superposition theorem, Thevenin's theorem, Norton's theorem, maximum power transfer theorem, reciprocity theorem, Millman's theorem, Tellegen's theorem, substitution theorem and compensation theorem. We can find currents and voltages in various parts of the circuits with these methods.

3.2 SUPERPOSITION THEOREM

It states that '*in a linear network containing more than one independent source and dependent source, the resultant current in any element is the algebraic sum of the currents that would be produced by each independent source acting alone, all the other independent sources being represented meanwhile by their respective internal resistances.*'

The independent voltage sources are represented by their internal resistances if given or simply with zero resistances, i.e., short circuits if internal resistances are not mentioned. The independent current sources are represented by infinite resistances, i.e., open circuits.

The dependent sources are not sources but dissipative components—hence they are active at all times. A dependent source has zero value only when its control voltage or current is zero.

A linear network is one whose parameters are constant, i.e., they do not change with voltage and current.

Explanation Consider the network shown in Fig. 3.1. Suppose we have to find current I_4 through resistor R_4.

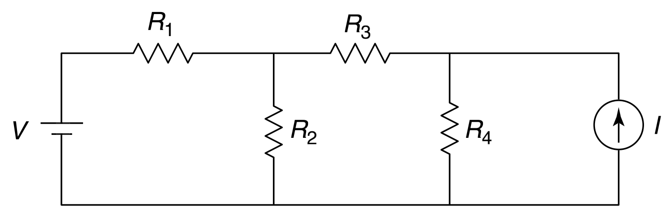


Fig. 3.1 *Network to illustrate superposition theorem*

The current flowing through resistor R_4 due to constant voltage source V is found to be say I_4' (with proper direction), representing constant current source with infinite resistance, i.e., open circuit.

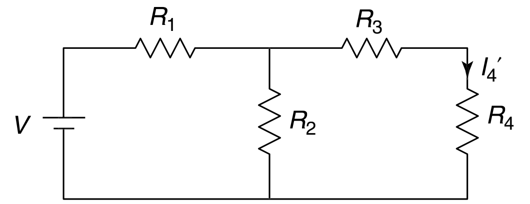


Fig. 3.2 *When voltage source V is acting alone*

The current flowing through resistor R_4 due to constant current source I is found to be say I_4'' (with proper direction), representing the constant voltage source with zero resistance or short circuit.

The resultant current I_4 through resistor R_4 is found by superposition theorem.

$$I_4 = I_4' + I_4''$$

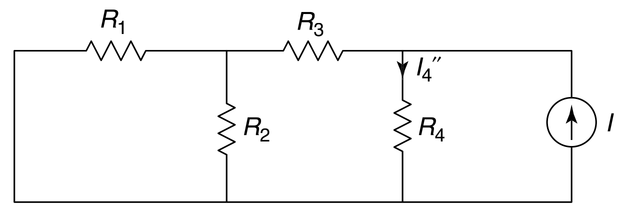


Fig. 3.3 *When current source I is acting alone*

Steps to be followed in Superposition Theorem

1. Find the current through the resistance when only one independent source is acting, replacing all other independent sources by respective internal resistances.
2. Find the current through the resistance for each of the independent sources.
3. Find the resultant current through the resistance by the superposition theorem considering magnitude and direction of each current.

Example 3.1 *Find the current through the 2 Ω resistor in Fig. 3.4.*

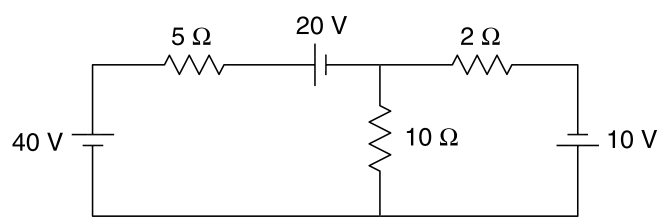


Fig. 3.4

Solution

Step I When the 40 V source is acting alone (Fig. 3.5)

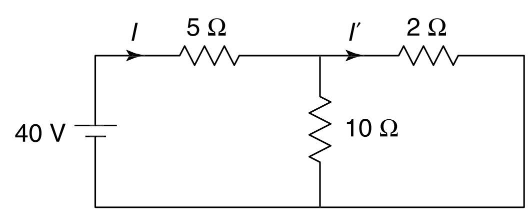


Fig. 3.5

By series parallel reduction technique (Fig. 3.6),

$$I = \frac{40}{5+1.67} = 6 \text{ A}$$

From Fig. 3.5, by current-division rule,

$$I' = 6 \times \frac{10}{10+2} = 5 \text{ A } (\rightarrow)$$

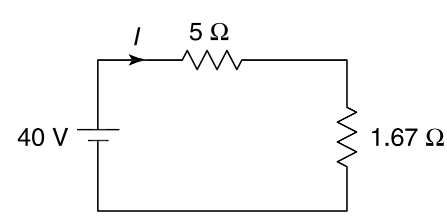


Fig. 3.6

Step II When the 20 V source is acting alone (Fig. 3.7)

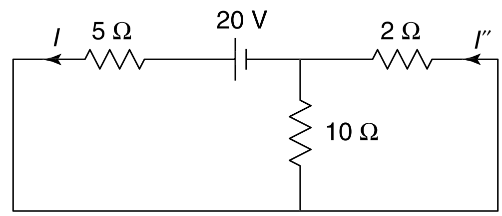


Fig. 3.7

By series–parallel reduction technique (Fig. 3.8),

$$I = \frac{20}{5+1.67} = 3\text{ A}$$

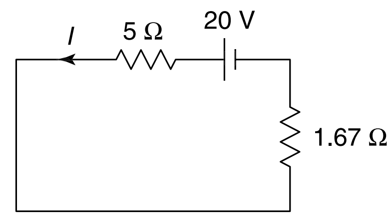


Fig. 3.8

From Fig. 3.7, by current-division rule,

$$I'' = 3 \times \frac{10}{10+2} = 2.5\text{ A}(\leftarrow) = -2.5\text{ A} \quad (\rightarrow)$$

Step III When the 10 V source is acting alone (Fig. 3.9)

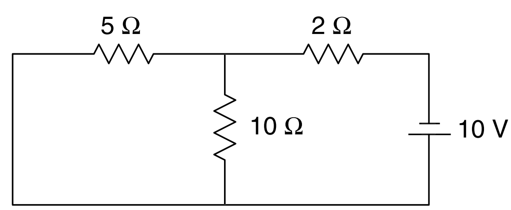


Fig. 3.9

By series–parallel reduction technique (Fig. 3.10),

$$I''' = \frac{10}{3.33+2} = 1.88\text{ A}(\rightarrow)$$

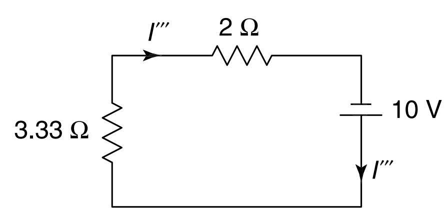


Fig. 3.10

Step IV By superposition theorem,

$$I = I' + I'' + I''' = 5 - 2.5 + 1.88 = 4.38\text{ A} \quad (\rightarrow)$$

Example 3.2 *Find the current through the 1 Ω resistor in Fig. 3.11.*

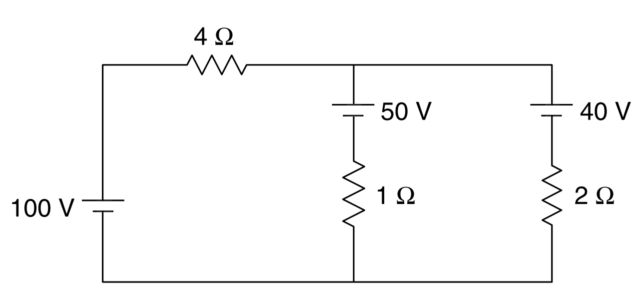


Fig. 3.11

Solution

Step I When the 100 V source is acting alone (Fig. 3.12)

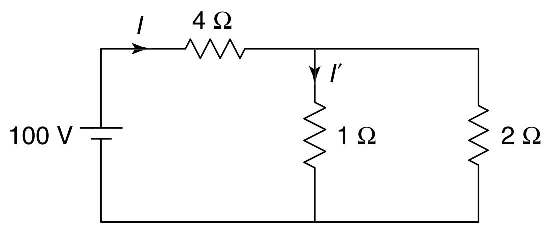


Fig. 3.12

By series–parallel reduction technique (Fig. 3.13),

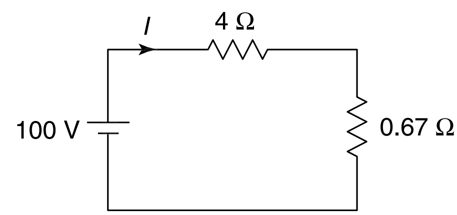


Fig. 3.13

$$I = \frac{100}{4+0.67} = 21.41 \text{ A}$$

From Fig. 3.12, by current-division rule,

$$I' = 21.41 \times \frac{2}{2+1} = 14.27 \text{ A}(\downarrow)$$

Step II When the 50 V source is acting alone (Fig. 3.14)

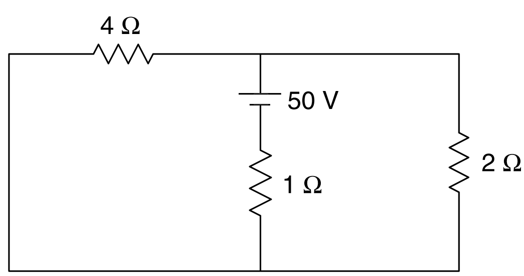


Fig. 3.14

By series–parallel reduction technique (Fig. 3.15),

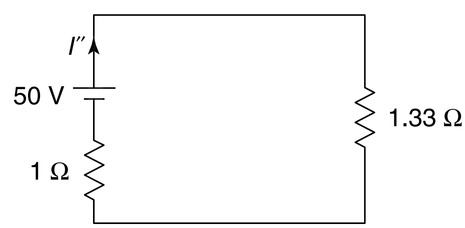


Fig. 3.15

$$I'' = \frac{50}{1+1.33} = 21.46 \text{ A}(\uparrow) = -21.46 \text{ A}(\downarrow)$$

Step III When the 40 V source is acting alone (Fig. 3.16)

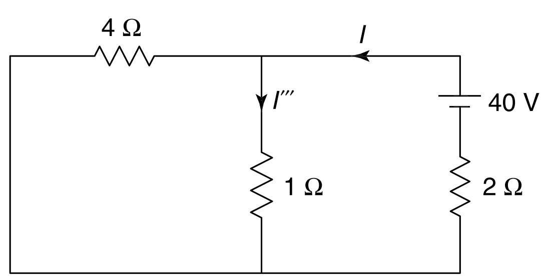


Fig. 3.16

By series–parallel reduction technique (Fig. 3.17),

$$I = \frac{40}{0.8+2} = 14.29 \text{ A}$$

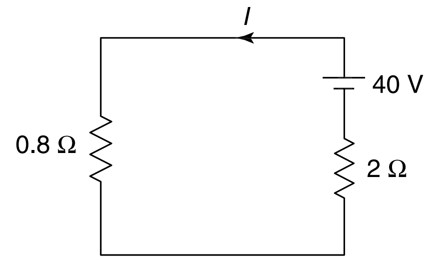


Fig. 3.17

From Fig. 3.16, by current-division rule,

$$I''' = 14.29 \times \frac{4}{4+1} = 11.43 \text{ A}(\downarrow)$$

Step IV By superposition theorem,

$$I = I' + I'' + I''' = 14.27 - 21.46 + 11.43 = 4.24 \text{ A}(\downarrow)$$

Example 3.3 *Find the current through the 8 Ω resistor in Fig. 3.18.*

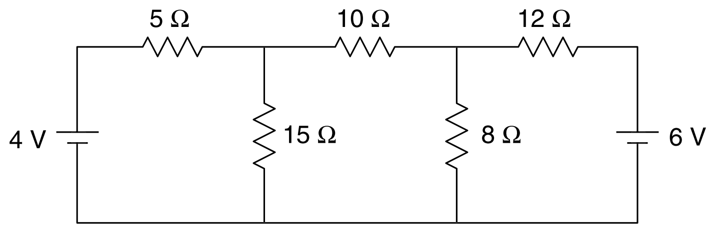


Fig. 3.18

Solution

Step I When the 4 V source is acting alone (Fig. 3.19)

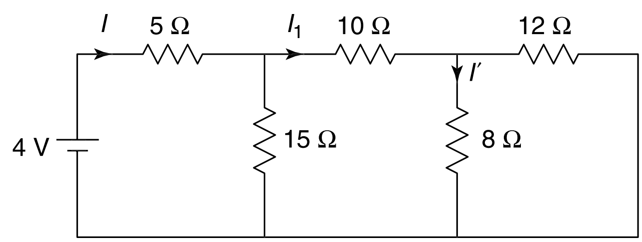
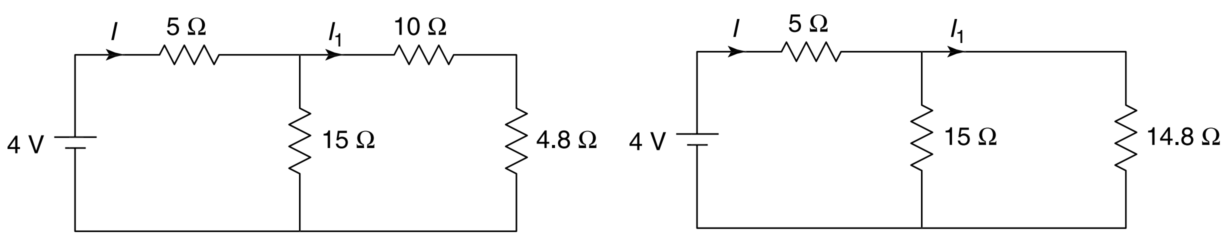


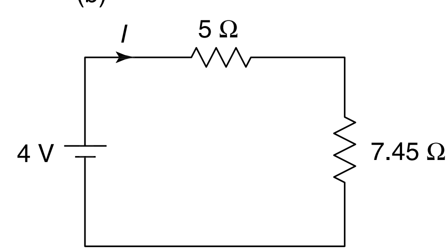
Fig. 3.19

By series–parallel reduction technique (Fig. 3.20),



(a) (b)

$$I = \frac{4}{5+7.45} = 0.32 \text{ A}$$



(c)

Fig. 3.20

From Fig. 3.20(b), by current-division rule,

$$I_1 = 0.32 \times \frac{15}{15+14.8} = 0.16 \text{ A}$$

From Fig. 3.19, by current-division rule,

$$I' = 0.16 \times \frac{12}{12+8} = 0.096 \text{ A}(\downarrow)$$

Step II When the 6 V source is acting alone (Fig. 3.21)

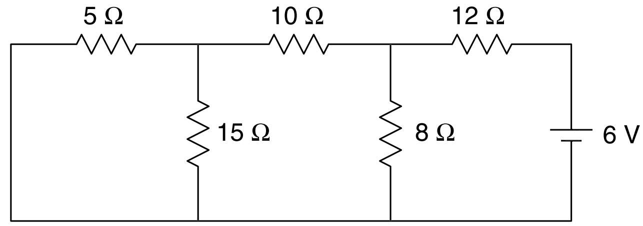
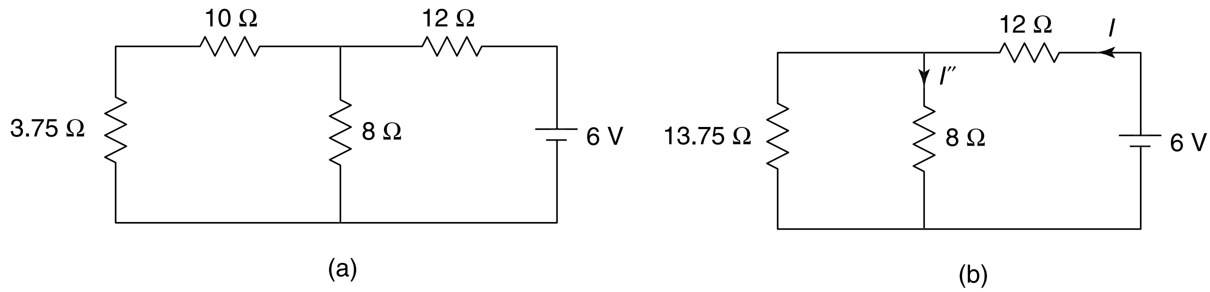


Fig. 3.21

By series–parallel reduction technique (Fig. 3.22),



$$I = \frac{6}{12+5.06} = 0.35 \text{ A}$$

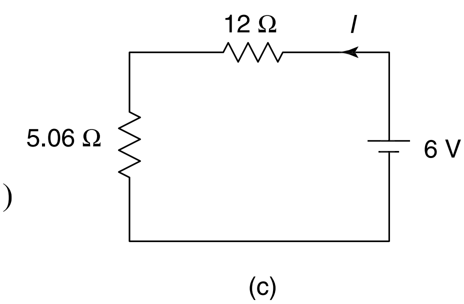


Fig. 3.22

From Fig. 3.22(b), by current division rule,

$$I'' = 0.35 \times \frac{13.75}{13.75+8} = 0.22 \text{ A}(\downarrow)$$

Step III By superposition theorem,

$$I = I' + I'' = 0.096 + 0.22 = 0.316 \text{ A}(\downarrow)$$

Example 3.4 *Find the current through the 4 Ω resistor in Fig. 3.23.*

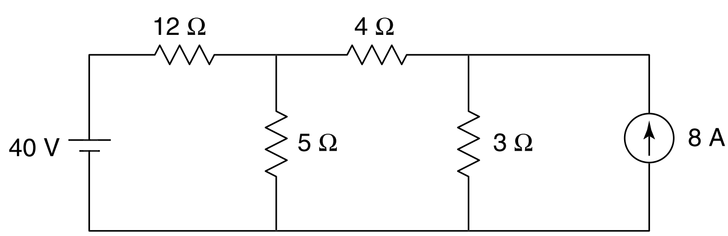


Fig. 3.23

Solution

Step I When the 40 V source is acting alone (Fig. 3.24)

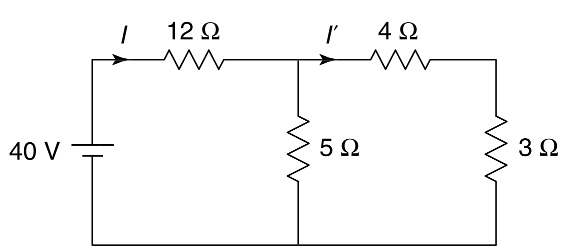


Fig. 3.24

By series–parallel reduction technique (Fig. 3.25),

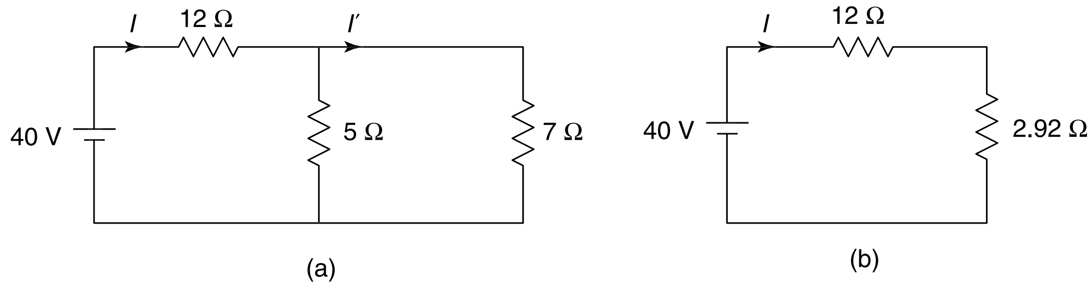


Fig. 3.25

$$I = \frac{40}{12+2.92} = 2.68 \text{ A}$$

From Fig. 3.25(a), by current-division rule,

$$I' = 2.68 \times \frac{5}{5+7} = 1.12 \text{ A}(\rightarrow) = -1.12 \text{ A}(\leftarrow)$$

Step II When the 8 A source is acting alone (Fig. 3.26)

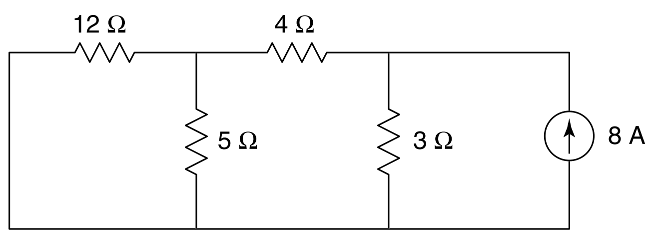


Fig. 3.26

By series–parallel reduction technique (Fig. 3.27),

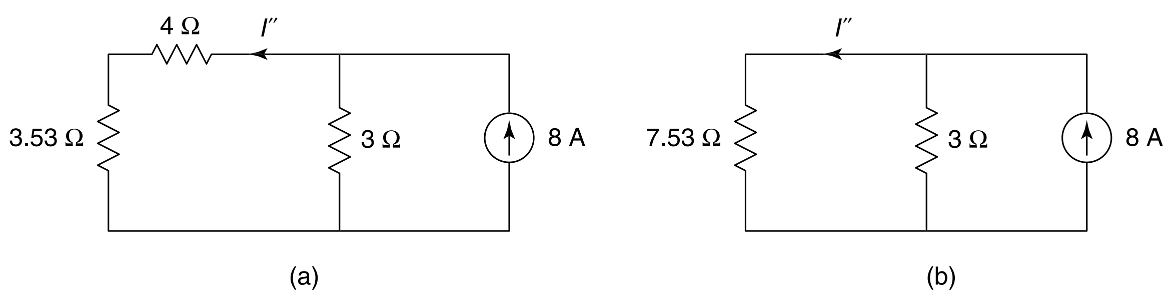


Fig. 3.27

$$I'' = 8 \times \frac{3}{7.53+3} = 2.28 \text{ A}(\leftarrow)$$

Step III By superposition theorem,

$$I = I' + I'' = -1.12 + 2.28 = 1.16 \text{ A}(\leftarrow)$$

Example 3.5 *Find the current in the 10 Ω resistor in Fig. 3.28.*

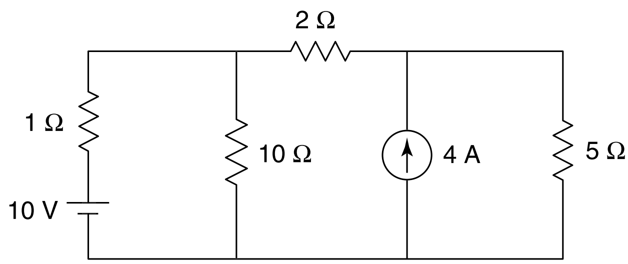


Fig. 3.28

Solution

Step I When the 10 V source is acting alone (Fig. 3.29)

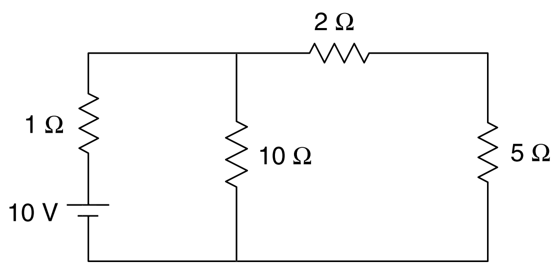


Fig. 3.29

By series–parallel reduction technique (Fig. 3.30),

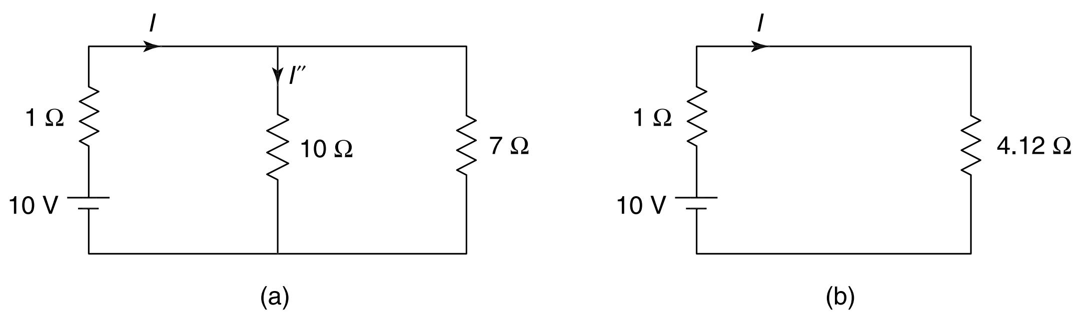


Fig. 3.30

$$I = \frac{10}{1+4.12} = 1.95 \text{ A}$$

From Fig. 3.30 (a), by current-division rule,

$$I' = 1.95 \times \frac{7}{7+10} = 0.8 \text{ A}(\downarrow)$$

Step II When the 4 A source is acting alone (Fig. 3.31)

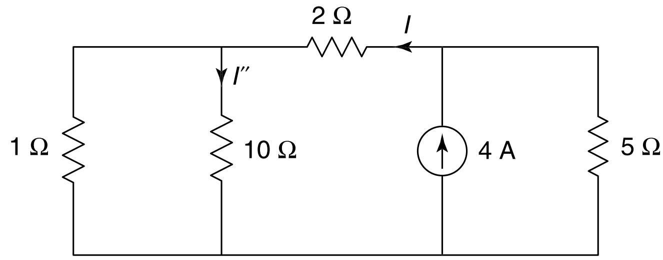


Fig. 3.31

By series–parallel reduction technique (Fig. 3.32),

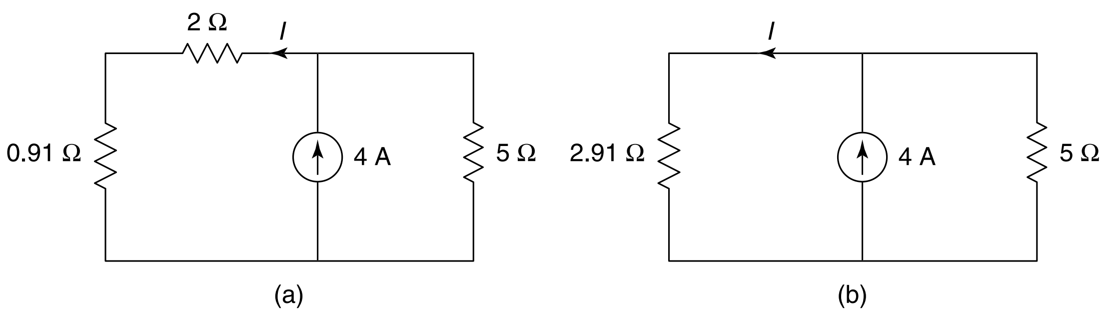


Fig. 3.32

$$I = 4 \times \frac{5}{2.91+5} = 2.53 \text{ A}$$

From Fig. 3.31, by current-division rule,

$$I'' = 2.53 \times \frac{1}{1+10} - 0.23 \text{ A}(\downarrow)$$

Step III By superposition theorem,

$$I = I' + I'' = 0.8 + 0.23 = 1.03 \text{ A}(\downarrow)$$

Example 3.6 *Find the current through the 8 Ω resistor in Fig. 3.33.*

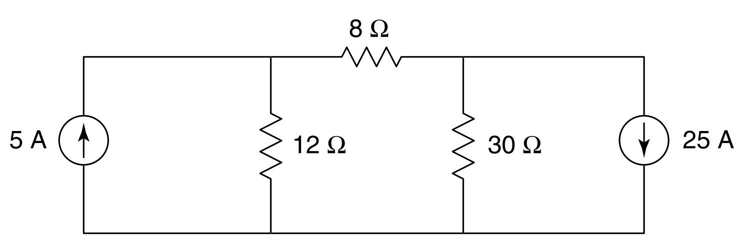


Fig. 3.33

Solution

Step I When the 5 A source is acting alone (Fig. 3.34)

$$I' = 5 \times \frac{12}{12+8+30} = 1.2 \text{ A } (\rightarrow)$$

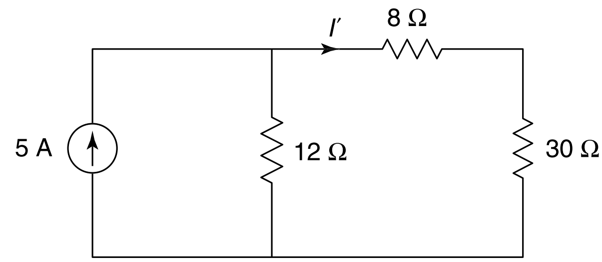


Fig. 3.34

Step II When the 25 A source is acting alone (Fig. 3.35)

$$I'' = 25 \times \frac{30}{30+12+8} = 15 \text{ A } (\rightarrow)$$

Step III By superposition theorem,

$$I = I' + I'' = 1.2 + 15 = 16.2 \text{ A}(\rightarrow)$$

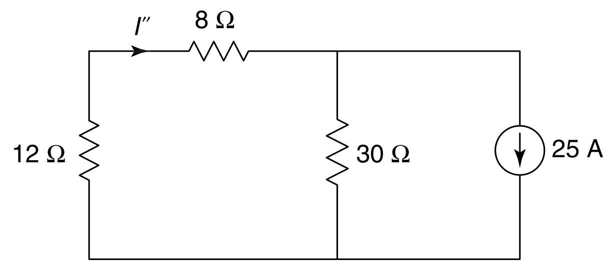


Fig. 3.35

Example 3.7 *Find the current through the 4 Ω resistor in Fig. 3.36.*

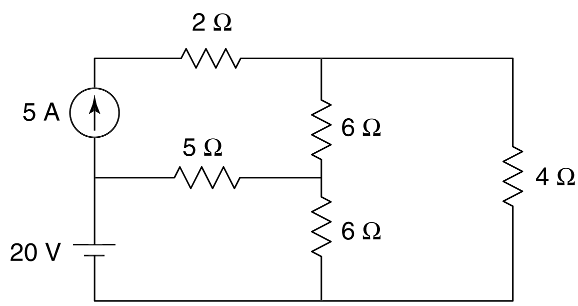


Fig. 3.36

Solution

Step I When the 5 A source is acting alone (Fig. 3.37)

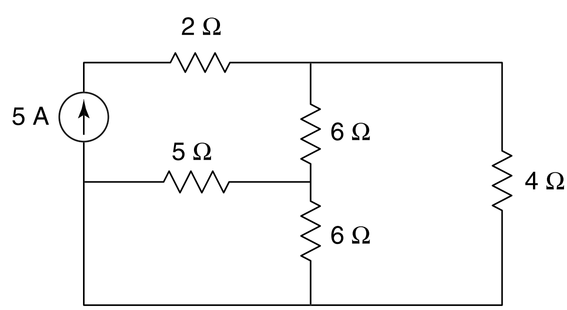


Fig. 3.37

By series–parallel reduction technique (Fig. 3.38),

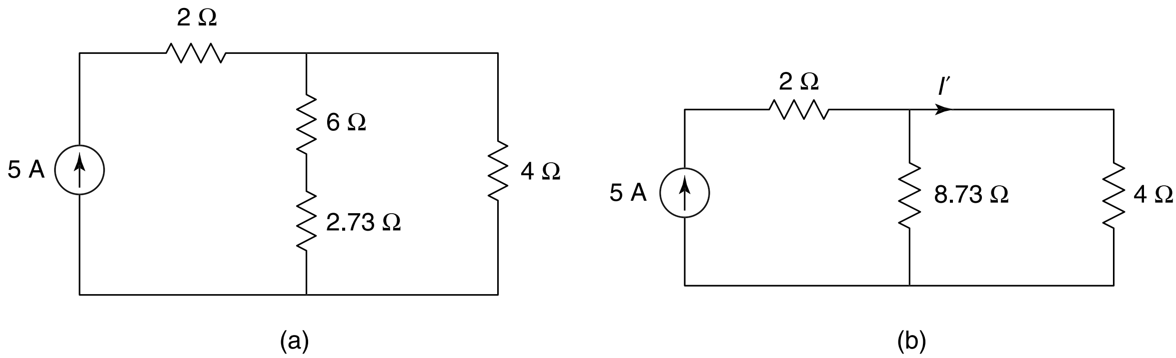


Fig. 3.38

$$I' = 5 \times \frac{8.73}{8.73+4} = 3.43\ \text{A}(\downarrow)$$

Step II When the 20 V source is acting alone (Fig. 3.39)

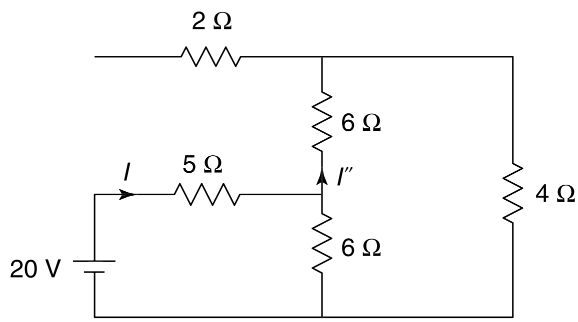


Fig. 3.39

By series–parallel reduction technique (Fig. 3.40),

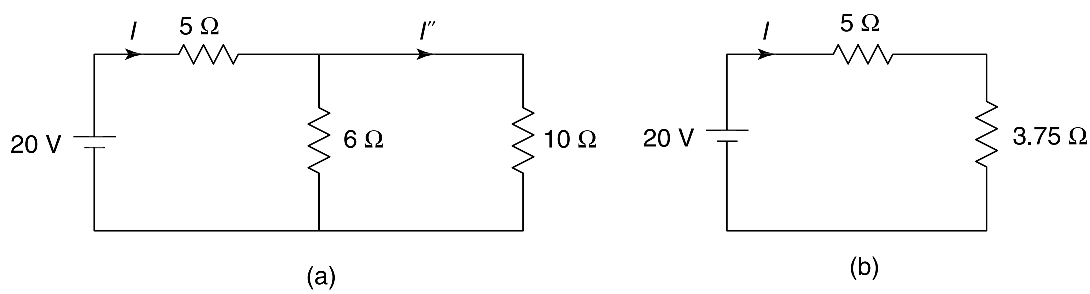


Fig. 3.40

$$I = \frac{20}{5+3.75} = 2.29\ \text{A}$$

From Fig. 3.40 (a), by current-division rule,

$$I'' = 2.29 \times \frac{6}{6+10} = 0.86\ \text{A}(\downarrow)$$

Step III By superposition theorem

$$I = I' + I'' = 3.43 + 0.86 = 4.29\ \text{A}\ (\downarrow)$$

Example 3.8 *Find the current through the 3 Ω resistor in Fig. 3.41.*

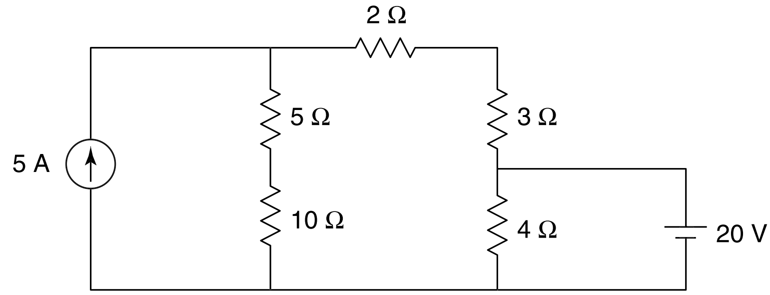


Fig. 3.41

Solution

Step I When the 5 A source is acting alone (Fig. 3.42)

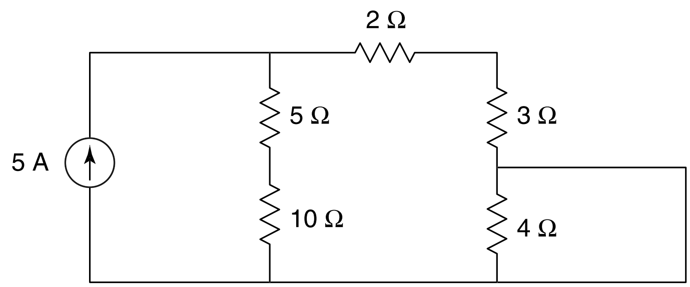


Fig. 3.42

By series–parallel reduction technique (Fig. 3.43),

$$I' = 5 \times \frac{15}{15+2+3} = 3.75\ \text{A}(\downarrow)$$

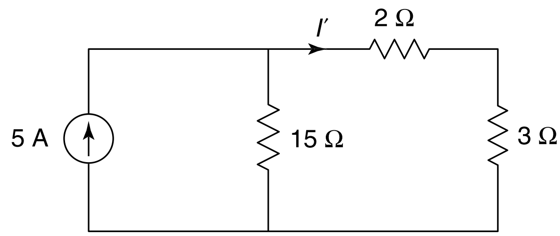


Fig. 3.43

Step II When the 20 V source is acting alone (Fig. 3.44)

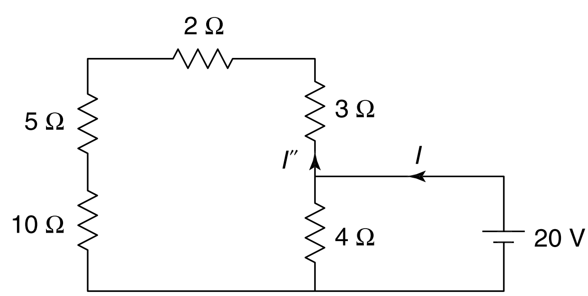
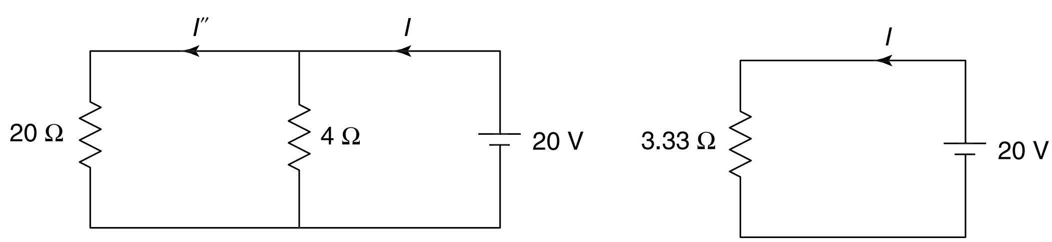


Fig. 3.44

By series–parallel reduction technique (Fig. 3.45),



(a) (b)

Fig. 3.45

$$I = \frac{20}{3.33} = 6 \text{ A}$$

From Fig. 3.45(a), by current-division rule,

$$I'' = 6 \times \frac{4}{20+4} = 1 \text{ A}(\uparrow) = -1 \text{ A}(\downarrow)$$

Step III By superposition theorem,

$$I = I' + I'' = 3.75 - 1 = 2.75 \text{ A } (\downarrow)$$

Example 3.9 *Find the current in the 1 Ω resistors in Fig. 3.46.*

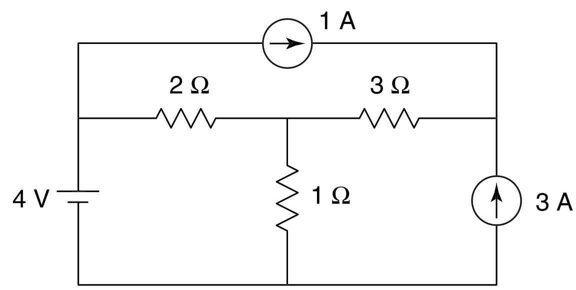


Fig. 3.46

Solution

Step I When the 4 V source is acting alone (Fig. 3.47)

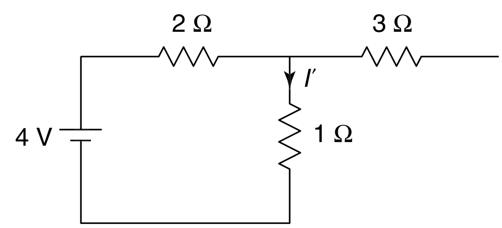


Fig. 3.47

$$I' = \frac{4}{2+1} = 1.33 \text{ A}(\downarrow)$$

Step II When the 3 A source is acting alone (Fig. 3.48)
By current-division rule,

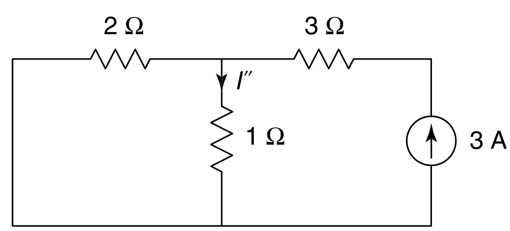


Fig. 3.48

$$I'' = 3 \times \frac{2}{1+2} = 2 \text{ A } (\downarrow)$$

Step III When the 1 A source is acting alone (Fig. 3.49)

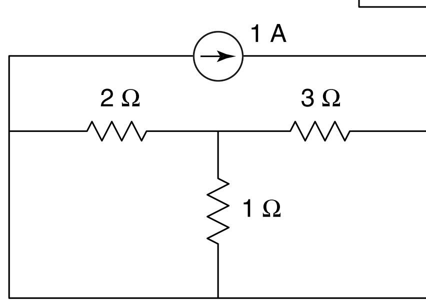


Fig. 3.49

Redrawing the network (Fig. 3.50),
By current-division rule,

$$I''' = 1 \times \frac{2}{2+1} = 0.66 \text{ A}(\downarrow)$$

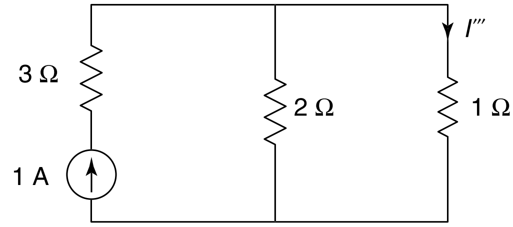


Fig. 3.50

Step IV By superposition theorem,

$$I = I' + I'' + I''' = 1.33 + 2 + 0.66 = 4 \text{ A } (\downarrow)$$

Example 3.10 *Find the voltage V_{AB} in Fig. 3.51.*

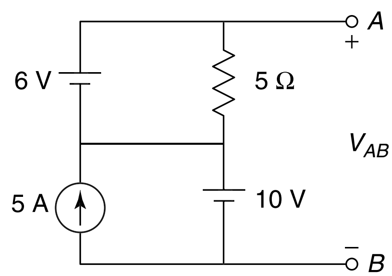


Fig. 3.51

Solution

Step I When the 6 V source is acting alone (Fig. 3.52)

$$V'_{AB} = 6 \text{ V}$$

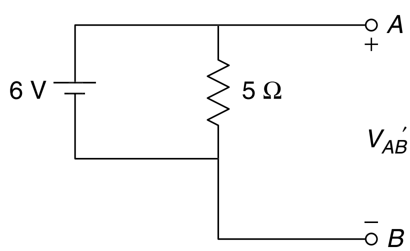


Fig. 3.52

Step II When the 10 V source is acting alone (Fig. 3.53)
Since the resistor of 5 Ω is shorted, the voltage across it is zero.

$$V''_{AB} = 10 \text{ V}$$

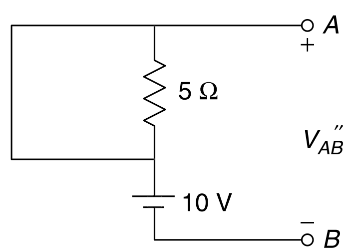


Fig. 3.53

Step III When the 5 A source is acting alone (Fig. 3.54)
Due to short circuit in both the parts,

$$V'''_{AB} = 0$$

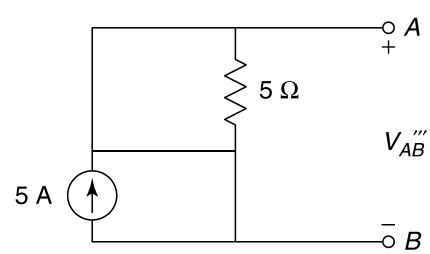


Fig. 3.54

Step IV By superposition theorem,

$$V_{AB} = V'_{AB} + V''_{AB} + V'''_{AB} = 6 + 10 + 0 = 16 \text{ V}$$

Example 3.11 *Find the current through the 5 Ω resistor in Fig. 3.55.*

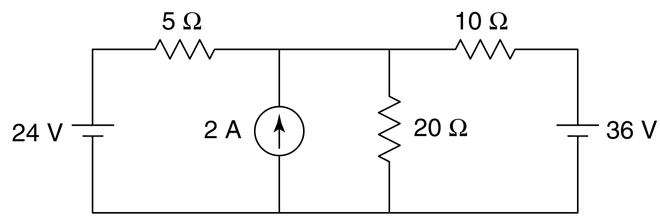


Fig. 3.55

Solution

Step I When the 24 V source is acting alone (Fig. 3.56)

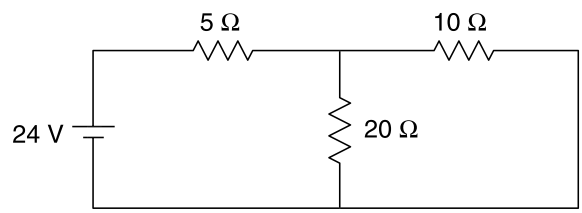


Fig. 3.56

By series–parallel reduction technique (Fig. 3.57),

$$I' = \frac{24}{5+6.67} = 2.06\ \text{A}(\rightarrow) = -2.06\ \text{A}(\leftarrow)$$

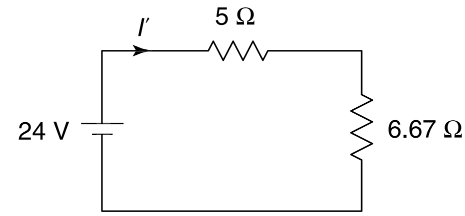


Fig. 3.57

Step II When the 2 A source is acting alone
By series–parallel reduction technique (Fig. 3.58),

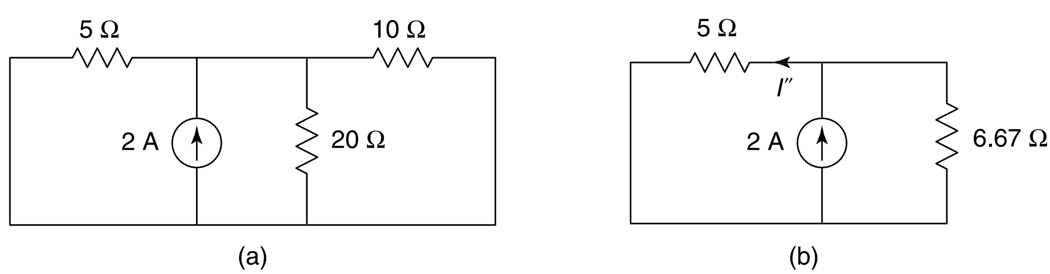


Fig. 3.58

By current-division rule,

$$I'' = 2 \times \frac{6.67}{5+6.67} = 1.14\ \text{A}(\leftarrow)$$

Step III When the 36 V source is acting alone (Fig. 3.59)
By series–parallel reduction technique,

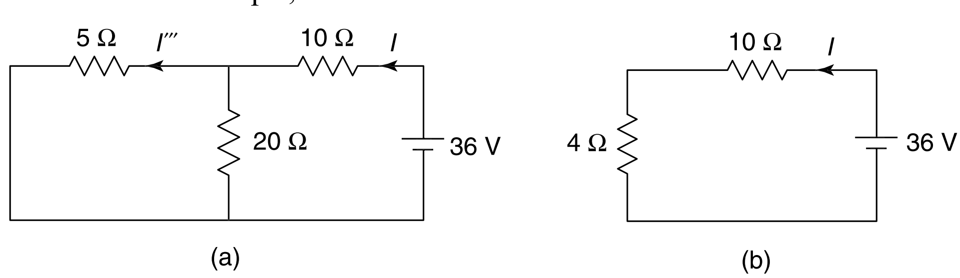


Fig. 3.59

$$I = \frac{36}{10+4} = 2.57 \text{ A}$$

By current-division rule,

$$I''' = 2.57 \times \frac{20}{20+5} = 2.06 \text{ A}(\leftarrow)$$

Step IV By superposition theorem,

$$I = I' + I'' + I''' = -2.06 + 1.14 + 2.06 = 1.14 \text{ A } (\leftarrow)$$

Example 13.12 *Find the current through the 4 Ω resistor in Fig. 3.60.*

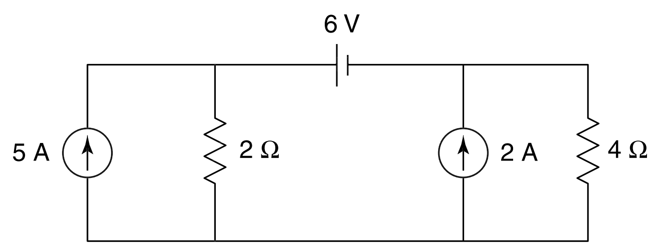


Fig. 3.60

Solution

Step I When the 5 A source is acting alone (Fig. 3.61)
By current-division rule,

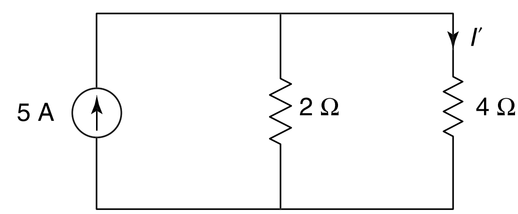


Fig. 3.61

$$I' = 5 \times \frac{2}{2+4} = 1.67 \text{ A}(\downarrow)$$

Step II When the 2 A source is acting alone (Fig. 3.62)
By current-division rule,

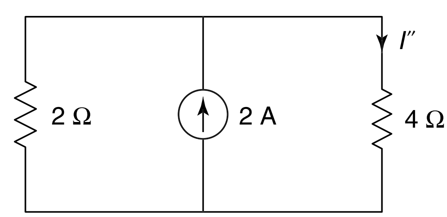


Fig. 3.62

$$I'' = 2 \times \frac{2}{2+4} = 0.67 \text{ A}(\downarrow)$$

Step III When the 6 V source is acting alone (Fig. 3.63)
Applying KVL to the mesh,

$$-2I''' - 6 - 4I''' = 0$$

$$I''' = -1 \text{ A } (\downarrow)$$

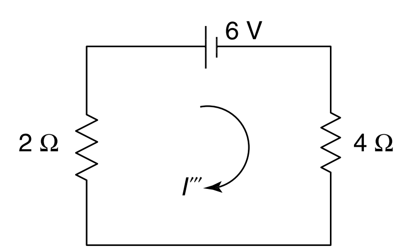


Fig. 3.63

Step IV By superposition theorem,

$$I = I' + I'' + I''' = 1.67 + 0.67 - 1 = 1.34 \text{ A } (\downarrow)$$

EXAMPLES WITH DEPENDENT SOURCES

Example 3.13 *Find the current through the 6 Ω resistor in Fig. 3.64.*

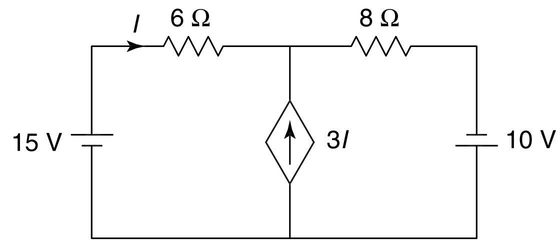


Fig. 3.64

Solution

Step I When the 15 V source is acting alone (Fig 3.65)

From Fig. 3.65,

$$I' = I_1 \qquad \ldots\text{(i)}$$

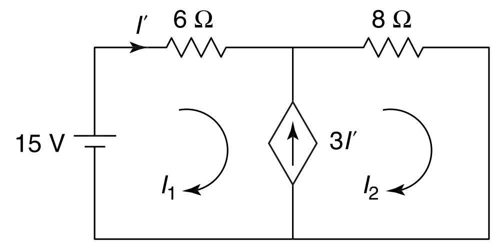


Fig. 3.65

Meshes 1 and 2 will form a supermesh.

Writing current equation for the supermesh,

$$I_2 - I_1 = 3I' = 3I_1$$

$$4I_1 - I_2 = 0 \qquad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$15 - 6I_1 - 8I_2 = 0$$

$$6I_1 + 8I_2 = 15 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 0.39\text{A}$$

$$I_2 = 1.59\text{A}$$

$$I' = I_1 = 0.39 \text{ A } (\rightarrow)$$

Step II When the 10 V source is acting alone (Fig 3.66)

From Fig. 3.66,

$$I'' = I_1 \qquad \ldots\text{(i)}$$

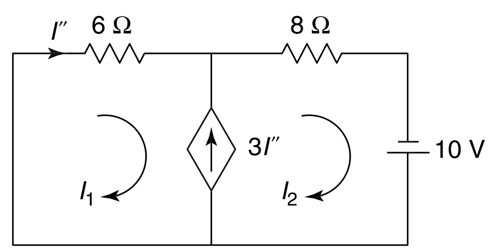


Fig. 3.66

Meshes 1 and 2 will form a supermesh.

Writing current equation for the supermesh,

$$I_2 - I_1 = 3I'' = 3I_1$$

$$4I_1 - I_2 = 0 \qquad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$-6I_1 - 8I_2 + 10 = 0$$

$$6I_1 + 8I_2 = 10 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 0.26 \text{ A}$$
$$I_2 = 1.05 \text{ A}$$
$$I'' = I_1 = 0.26 \text{ A} \quad (\rightarrow)$$

Step III By superposition theorem,

$$I = I' + I'' = 0.39 + 0.26 = 0.65 \text{ A} (\rightarrow)$$

Example 3.14 *Find the current I_x in Fig. 3.67.*

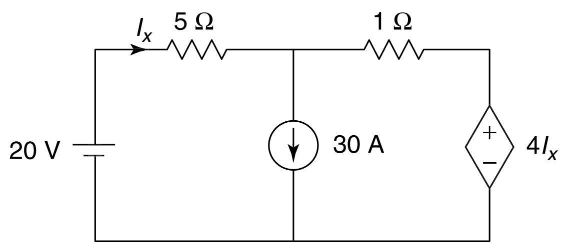


Fig. 3.67

Solution

Step I When the 30 A source is acting alone (Fig. 3.68)
From Fig. 3.68,

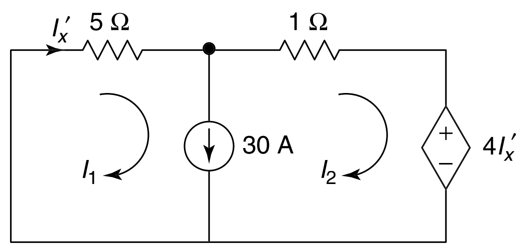


Fig. 3.68

$$I_x' = I_1 \qquad \ldots\text{(i)}$$

Meshes 1 and 2 will form a supermesh.
Writing current equation for the supermesh,

$$I_1 - I_2 = 30 \qquad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$-5I_1 - 1I_2 - 4I_x' = 0$$
$$-5I_1 - I_2 - 4I_1 = 0$$
$$9I_1 + I_2 = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 3 \text{ A}$$
$$I_2 = -27 \text{ A}$$
$$I_x' = I_1 = 3 \text{ A} (\rightarrow)$$

Step II When the 20 V source is acting alone (Fig. 3.69)
Applying KVL to the mesh,

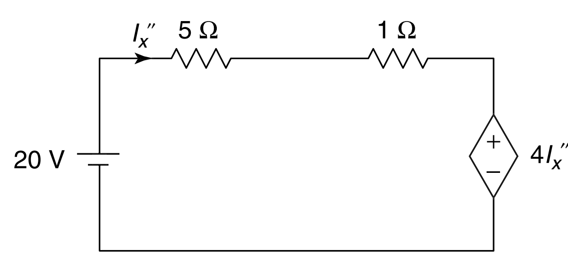


Fig. 3.69

$$20 - 5I_x'' - 1I_x'' - 4I_x'' = 0$$
$$I_x'' = 2 \text{ A}(\rightarrow)$$

Step III By superposition theorem,

$$I_x = I_x' + I_x'' = 3 + 2 = 5 \text{ A}(\rightarrow)$$

Example 3.15 *Find the current I_1 in Fig. 3.70.*

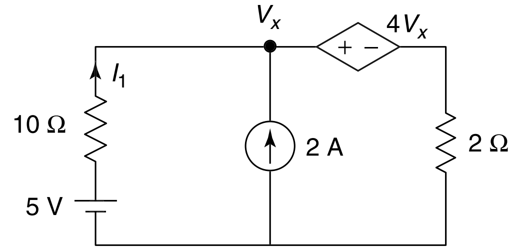


Fig. 3.70

Solution

Step I When the 5 V source is acting alone (Fig 3.71)

From the figure,

$$V_x = 5 - 10I_1'$$

Applying KVL to the mesh,

$$5 - 10I_1' - 4V_x - 2I_1' = 0$$

$$5 - 10I_1' - 4(5 - 10I_1') - 2I_1' = 0$$

$$5 - 10I_1' - 20 + 40I_1' - 2I_1' = 0$$

$$I_1' = \frac{15}{28} = 0.54 \text{ A}(\uparrow)$$

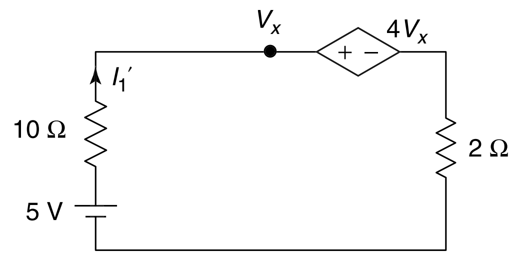


Fig. 3.71

Step II When the 2 A source is acting alone (Fig 3.72)

From Fig . 3.72,

$$V_x = -10I_1' \qquad \ldots\text{(i)}$$

Meshes 1 and 2 will form a supermesh.

Writing current equation for the supermesh,

$$I_2 - I_1' = 2 \qquad \ldots\text{(ii)}$$

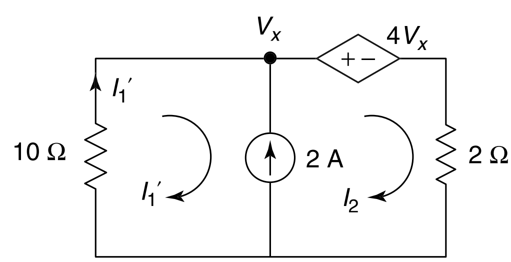


Fig. 3.72

Applying KVL to the outer path of the supermesh,

$$-10I_1' - 4V_x - 2I_2 = 0$$

$$-10I_1' - 4(-10I_1') - 2I_2 = 0$$

$$30I_1' - 2I_2 = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 0.14 \text{ A } (\uparrow)$$

$$I_2 = 2.14 \text{ A}$$

Step III By superposition theorem,

$$I_1 = I_1' + I_1'' = 0.54 + 0.14 = 0.68 \text{ A } (\uparrow)$$

Example 3.16 *Determine the current through the 10 Ω resistor in Fig. 3.73.*

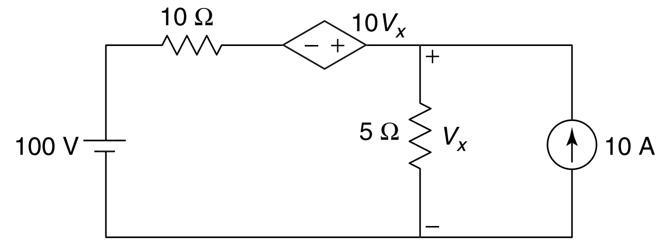


Fig. 3.73

Solution

Step I When the 100 V source is acting alone (Fig. 3.74)

From the figure,

$$V_x = 5I'$$

Applying KVL to the mesh,

$$100 - 10I' + 10V_x - 5I' = 0$$
$$100 - 10I' + 10(5I') - 5I' = 0$$
$$I' = -2.86 \text{ A } (\rightarrow)$$

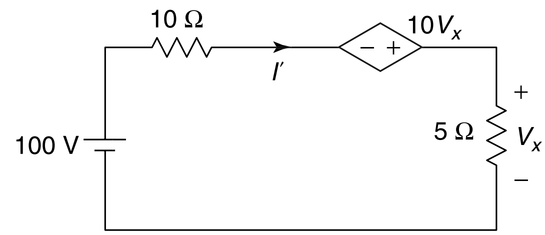


Fig. 3.74

Step II When the 10 A source is acting alone (Fig. 3.75)

From Fig. 3.75,

$$V_x = 5(I_1 - I_2) \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 1,

$$-10I_1 + 10V_x - 5(I_1 - I_2) = 0$$
$$-10I_1 + 10\{5(I_1 - I_2)\} - 5(I_1 - I_2) = 0$$
$$35I_1 - 45I_2 = 0 \qquad \ldots\text{(ii)}$$

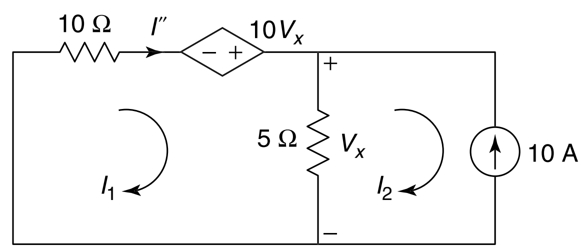


Fig. 3.75

For Mesh 2,

$$I_2 = -10 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = -12.86 \text{ A}$$
$$I_2 = -10 \text{ A}$$
$$I'' = I_1 = -12.86 \text{ A } (\rightarrow)$$

Step III By superposition theorem,

$$I = I' + I'' = -2.86 - 12.86 = -15.72 \text{ A } (\rightarrow)$$

Example 3.17 *Find the current I in the network of Fig. 3.76.*

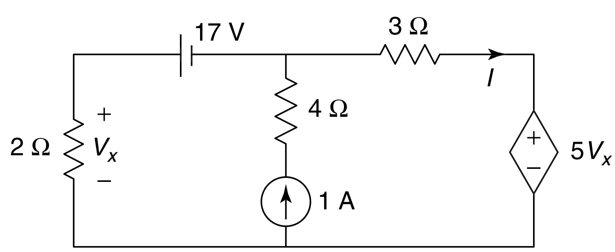


Fig. 3.76

Solution

Step I When the 17 V source is acting alone (Fig. 3.77)

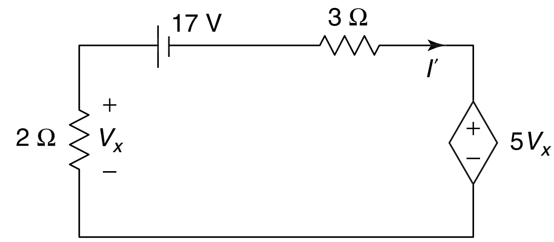


Fig. 3.77

From the figure,

$$V_x = -2I'$$

Applying KVL to the mesh,

$$-2I' - 17 - 3I' - 5V_x = 0$$
$$-2I' - 17 - 3I' - 5(-2I') = 0$$
$$I' = 3.4 \text{ A } (\rightarrow)$$

Step II When the 1 A source is acting alone (Fig. 3.78)

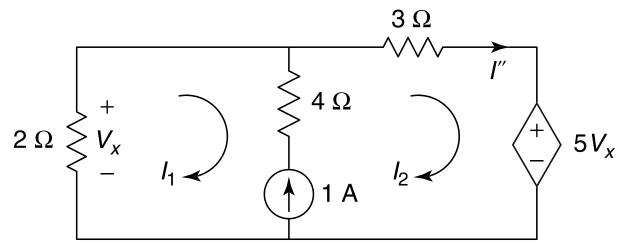


Fig. 3.78

From Fig. 3.78,

$$V_x = -2I_1 \qquad \text{...(i)}$$

Meshes 1 and 2 will form a supermesh.
Writing current equation for the supermesh,

$$I_2 - I_1 = 1 \qquad \text{...(ii)}$$

Applying KVL to the outer path of the supermesh,

$$-2I_1 - 3I_2 - 5V_x = 0$$
$$-2I_1 - 3I_2 - 5(-2I_1) = 0$$
$$8I_1 - 3I_2 = 0 \qquad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 0.6 \text{ A}$$
$$I_2 = 1.6 \text{ A}$$
$$I'' = I_2 = 1.6 \text{ A } (\rightarrow)$$

Step III By superposition theorem,

$$I = I' + I'' = 3.4 + 1.6 = 5 \text{ A } (\rightarrow)$$

Example 3.18 *Find the voltage V_1 in Fig. 3.79.*

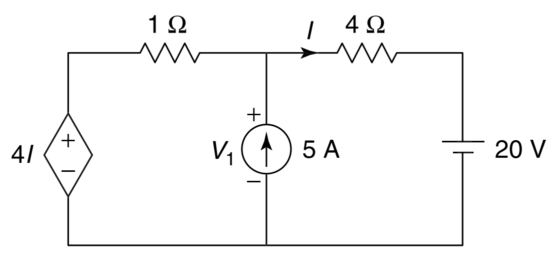


Fig. 3.79

Solution

Step I When the 5 A source is acting alone (Fig. 3.80)

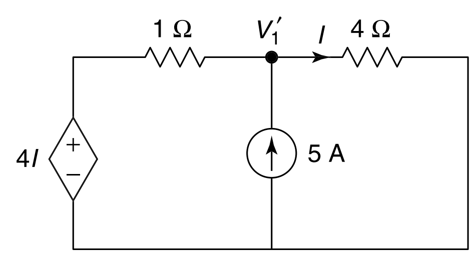


Fig. 3.80

From Fig. 3.80,

$$I = \frac{V_1'}{4}$$

Applying KCL at Node 1,

$$\frac{V_1' - 4I}{1} + \frac{V_1'}{4} = 5$$

$$V_1' - 4\left(\frac{V_1'}{4}\right) + \frac{V_1'}{4} = 5$$

$$V_1' = 20 \text{ V}$$

Step II When the 20 V source is acting alone (Fig. 3.81)
Applying KVL to the mesh,

$$4I - I - 4I - 20 = 0$$

$$I = -20 \text{ A}$$

$$V_1'' = 4I - 1(I) = 3I = 3\,(-20) = -60 \text{ V}$$

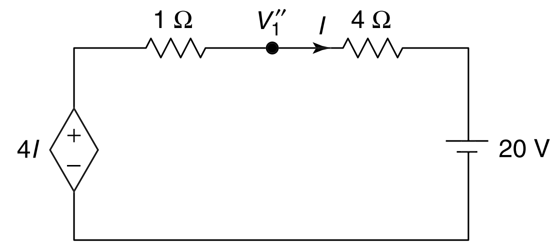


Fig. 3.81

Step III By superposition theorem,

$$V_1 = V_1' + V_1'' = 20 - 60 = -40 \text{ V}$$

Example 3.19 *Find the current in the 6 Ω resistor in Fig. 3.82.*

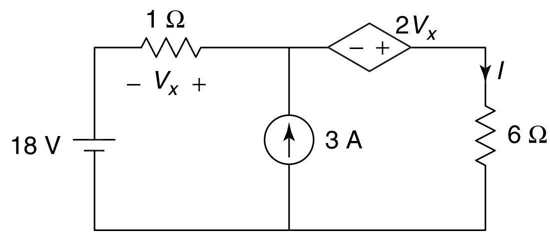


Fig. 3.82

Solution

Step I When the 18 V source is acting alone (Fig. 3.83)
From Fig. 3.83,

$$V_x = -I'$$

Applying KVL to the mesh,

$$18 - I' + 2V_x - 6I' = 0$$

$$18 - I' - 2I' - 6I' = 0$$

$$I' = 2 \text{ A } (\downarrow)$$

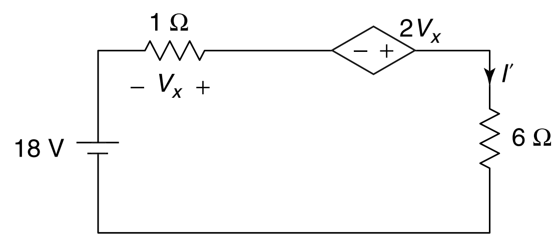


Fig. 3.83

Step II When the 3 A source is acting alone (Fig. 3.84)
From Fig. 3.84,

$$V_x = -1\,I_1 = -I_1 \qquad \text{...(i)}$$

Meshes 1 and 2 will form a supermesh.
Writing current equation for the supermesh,

$$I_2 - I_1 = 3 \qquad \text{...(ii)}$$

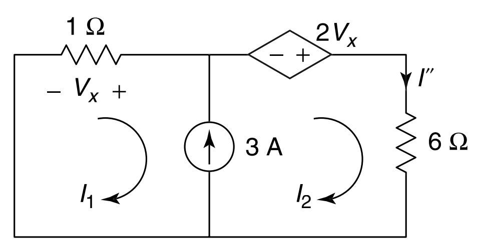


Fig. 3.84

Applying KVL to the outerpath of the supermesh,

$$-1I_1 + 2V_x - 6I_2 = 0$$

$$-I_1 + 2(-I_1) - 6I_2 = 0$$

$$3I_1 + 6I_2 = 0 \qquad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = -2 \text{ A}$$
$$I_2 = 1 \text{ A}$$
$$I'' = I_2 = 1 \text{ A } (\downarrow)$$

Step III By superposition theorem,

$$I_{6\,\Omega} = I' + I'' = 2 + 1 = 3 \text{ A } (\downarrow)$$

Example 3.20 *Find the current I_y in Fig. 3.85.*

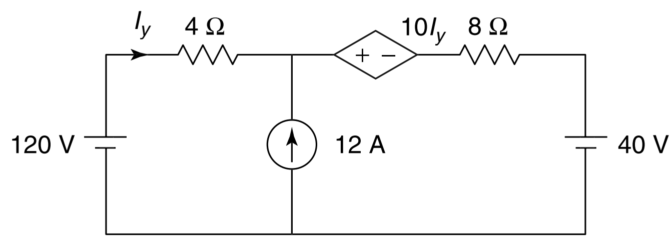


Fig. 3.85

Solution

Step I When the 120 V source is acting alone (Fig. 3.86)
Applying KVL to the mesh,

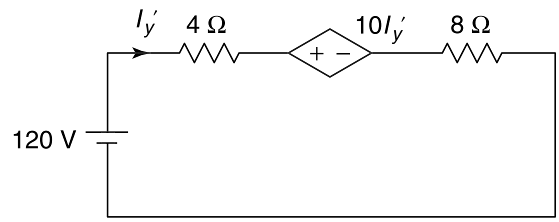


Fig. 3.86

$$120 - 4I_y' - 10I_y' - 8I_y' = 0$$

$$I_y' = 5.45 \text{ A } (\rightarrow)$$

Step II When the 12 A source is acting alone (Fig. 3.87)
From Fig. 3.87,

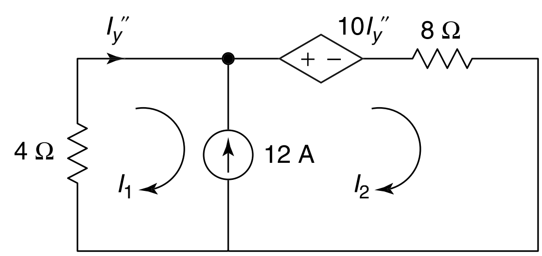


Fig. 3.87

$$I_y'' = I_1 \qquad \text{...(i)}$$

Meshes 1 and 2 will form a supermesh.
Writing current equation for the supermesh,

$$I_2 - I_1 = 12 \qquad \text{...(ii)}$$

Applying KVL to the outer path of the supermesh,

$$-4I_1 - 10I_y'' - 8I_2 = 0$$
$$-4I_1 - 10I_1 - 8I_2 = 0$$
$$14I_1 + 8I_2 = 0 \qquad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = -4.36 \text{ A}$$
$$I_2 = 7.64 \text{ A}$$
$$Iy'' = I_1 = -4.36 \text{ A } (\rightarrow)$$

Step III When the 40 V source is acting alone (Fig. 3.88)
Applying KVL to the mesh,

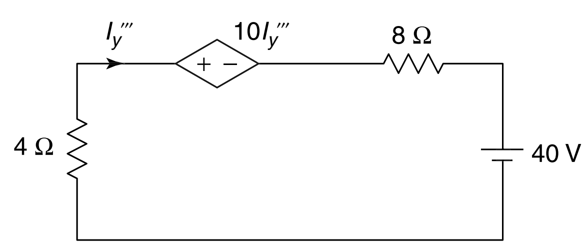


Fig. 3.88

$$-4\,I_y''' - 10I_y''' - 8I_y''' - 40 = 0$$

$$I_y''' = -\frac{40}{22} = -1.82 \text{ A } (\rightarrow)$$

Step IV By superposition theorem,

$$I_y = I_y' + I_y'' + I_y''' = 5.45 - 4.36 - 1.82 = -0.73 \text{ A } (\rightarrow)$$

Example 3.21 *Find the voltage V_x in Fig. 3.89.*

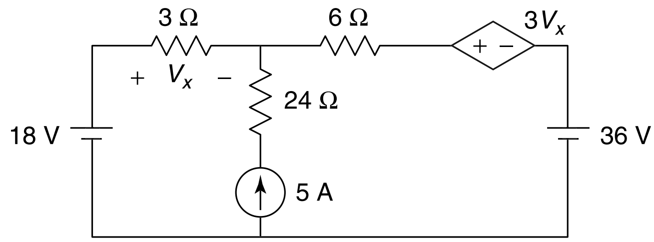


Fig. 3.89

Solution

Step I When the 18 V source is acting alone (Fig. 3.90)

From Fig. 3.90,

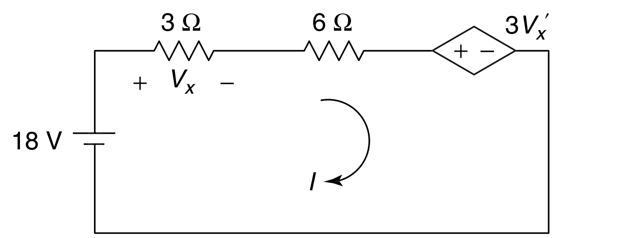


Fig. 3.90

$$V_x' = 3I$$

Applying KVL to the mesh,

$$18 - 3I - 6I - 3V_x' = 0$$
$$18 - 3I - 6I - 3\,(3I) = 0$$
$$I = 1 \text{ A}$$
$$V_x' = 3 \text{ V}$$

Step II When the 5 A source is acting alone (Fig. 3.91)

From Fig. 3.91,

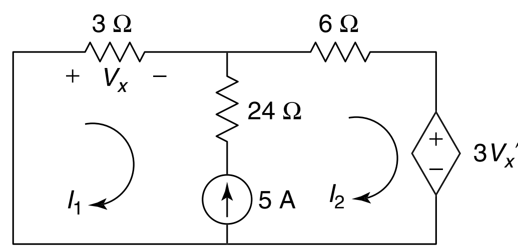


Fig. 3.91

$$V_x'' = -3I_1 \qquad \ldots\text{(i)}$$

Meshes 1 and 2 will form a supermesh.

Writing current equation for the supermesh,

$$I_2 - I_1 = 5 \qquad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$-3I_1 - 6I_2 - 3V_x'' = 0$$
$$-3I_1 - 6I_2 - 3(3I_1) = 0$$
$$12I_1 + 6I_2 = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = -1.67\text{A}$$
$$I_2 = 3.33 \text{ A}$$
$$V_x'' = 3I_1 = 3(-1.67) = -5 \text{ V}$$

Step III When the 36 V source is acting alone (Fig. 3.92)

From the figure,

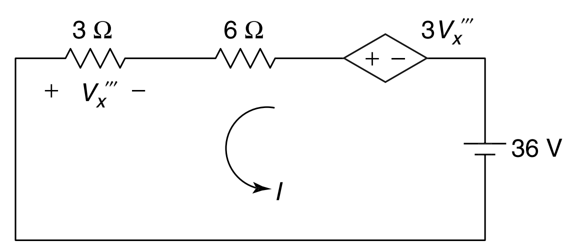


Fig. 3.92

$$V_x''' = -3I$$

Applying KVL to the mesh,

$$36 + 3V_x''' - 6I - 3I = 0$$

$$36 + 3V_x''' - 6\left(\frac{-V_x'''}{3}\right) - 3\left(\frac{-V_x'''}{3}\right) = 0$$

$$36 + 3V_x''' + 2V_x''' + V_x''' = 0$$

$$V_x''' = -6 \text{ V}$$

Step IV By superposition theorem,

$$V_x = V_x' + V_x'' + V_x''' = 3 - 5 - 6 = -8 \text{ V}$$

Example 3.22 *Find the voltage V in the network of Fig. 3.93.*

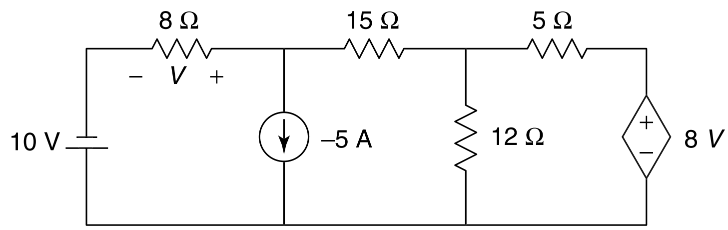


Fig. 3.93

Solution

Step I When the 10 V source is acting alone (Fig. 3.94)

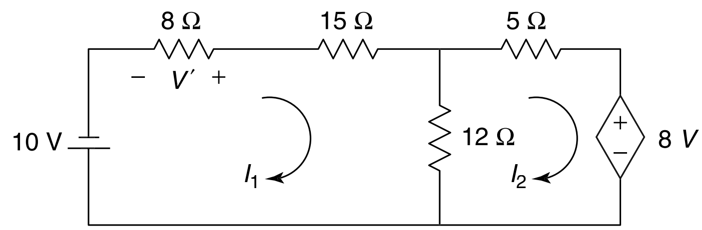


Fig. 3.94

From Fig. 3.94,

$$V' = -8I_1 \qquad \text{...(i)}$$

Applying KVL to Mesh 1,

$$-10 - 8I_1 - 15\,I_1 - 12\,(I_1 - I_2) = 0$$

$$35I_1 - 12I_2 = -10 \qquad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$-12(I_2 - I_1) - 5I_2 - 8V' = 0$$

$$-12I_2 + 12I_1 - 5I_2 - 8(-8I_1) = 0$$

$$76I_1 - 17I_2 = 0 \qquad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 0.54 \text{ A}$$

$$I_2 = 2.4 \text{ A}$$

$$V' = -8I_1 = -8(0.54) = -4.32 \text{ V}$$

Step II When the –5 A source is acting alone (Fig. 3.95)

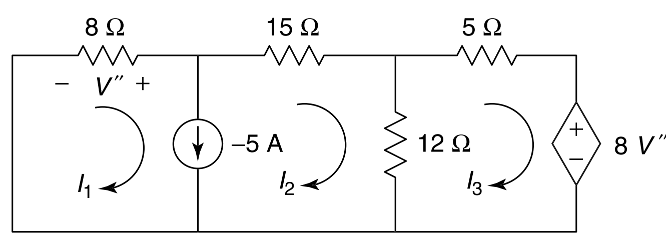


Fig. 3.95

From Fig. 3.95,

$$V'' = -8I_1 \qquad \text{...(i)}$$

Meshes 1 and 2 will form a supermesh.

Writing current equation for the supermesh,

$$I_1 - I_2 = -5 \qquad \text{...(ii)}$$

Applying KVL to the outer path of the supermesh,

$$-8I_1 - 15I_2 - 12(I_2 - I_3) = 0$$

$$-8I_1 - 27I_2 + 12I_3 = 0 \qquad \text{...(iii)}$$

Applying KVL to Mesh 3,

$$-12(I_3 - I_2) - 5I_3 - 8V'' = 0$$

$$-12I_3 + 12I_2 - 5I_3 - 8(-8I_1) = 0$$

$$64I_1 + 12I_2 - 17I_3 = 0 \qquad \text{...(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = 4.97 \text{ A}$$

$$I_2 = 9.97 \text{ A}$$

$$I_3 = 25.74 \text{ A}$$

$$V'' = -8I_1 = -8(-4.97) = -39.76 \text{ V}$$

Step III By superposition theorem,

$$V = V' + V'' = -4.32 - 39.76 = -44.08 \text{ V}$$

Example 3.23 *For the network shown in Fig. 3.96, find the voltage V_0.*

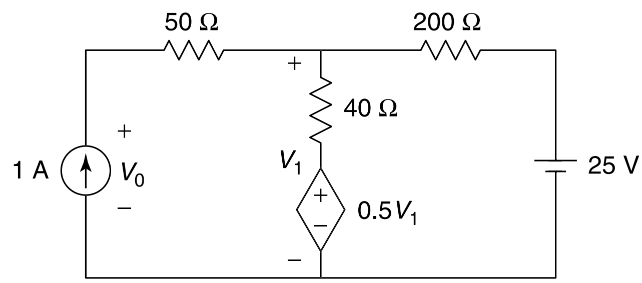


Fig. 3.96

Solution

Step I When the 1 A source is acting alone (Fig. 3.97)

From Fig. 3.97,

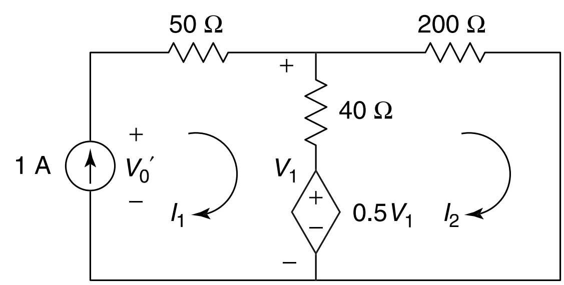


Fig. 3.97

$$V_1 = 200\,I_2 \qquad \text{...(i)}$$

For Mesh 1,

$$I_1 = 1 \qquad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$0.5V_1 - 40(I_2 - I_1) - 200\,I_2 = 0$$

$$0.5(200I_2) - 40I_2 + 40I_1 - 200I_2 = 0$$

$$40I_1 - 140\,I_2 = 0 \qquad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 1 \text{ A}$$

$$I_2 = 0.29 \text{ A}$$

$$V_0' - 50\,I_1 - 200I_2 = 0$$

$$V_0' - 50(1) - 200(0.29) = 0$$

$$V_0' = 108 \text{ V}$$

Step II When the 25 V source is acting alone (Fig. 3.98)
From Fig. 3.98,

$$V_1 - 200\,I - 25 = 0$$

$$V_1 = 200I + 25 \quad \text{...(i)}$$

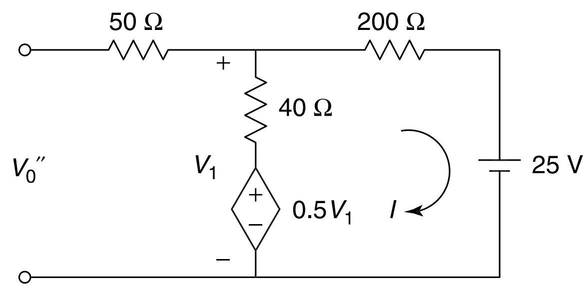


Fig. 3.98

Applying KVL to Mesh 1,

$$0.5V_1 - 40\,I - 200\,I - 25 = 0$$

$$0.5(200\,I + 25) - 40\,I - 200\,I - 25 = 0$$

$$I = -0.09 \text{ A}$$

$$V_0'' = V_1 = 200\,I + 25 = 200(-0.09) + 25 = 7 \text{ V}$$

Step III By superposition theorem,

$$V_0 = V_0' + V_0'' = 108 + 7 = 115\,\text{V}$$

Example 3.24 *For the network shown in Fig. 3.99, find the voltage V_x.*

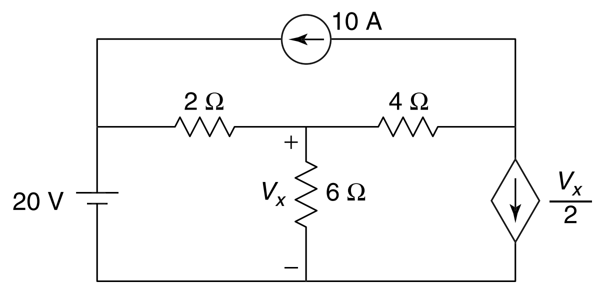


Fig. 3.99

Solution

Step I When the 20 V source is acting alone (Fig. 3.100)
From Fig. 3.100,

$$V_x' = 6(I_1 - I_2) \qquad \text{...(i)}$$

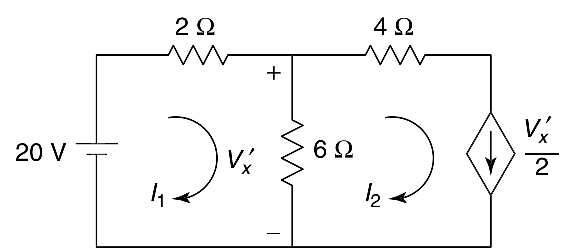


Fig. 3.100

Applying KVL to Mesh 1,

$$20 - 2I_1 - 6(I_1 - I_2) = 0$$

$$8I_1 - 6I_2 = 20 \qquad \text{...(ii)}$$

For Mesh 2,

$$I_2 = \frac{V_x'}{2} = \frac{6(I_1 - I_2)}{2} = 3I_1 - 3I_2$$

$$3I_1 - 4I_2 = 0 \qquad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 5.71 \text{ A}$$

$$I_2 = 4.29 \text{ A}$$

$$V_x' = 6(I_1 - I_2) = 6(5.71 - 4.29) = 8.52 \text{ V}$$

Step II When the 10 A source is acting alone (Fig. 3.101)
From Fig. 3.101,

$$V_x'' = 6(I_1 - I_2) \qquad \text{...(i)}$$

Applying KVL to Mesh 1,

$$-2(I_1 - I_3) - 6(I_1 - I_2) = 0$$

$$8I_1 - 6I_2 - 2I_3 = 0 \qquad \text{...(ii)}$$

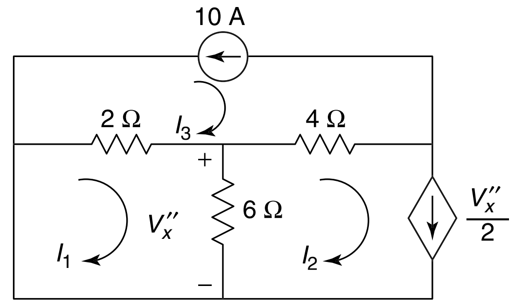


Fig. 3.101

For Mesh 2,

$$I_2 = \frac{V_x''}{2} = \frac{6(I_1 - I_2)}{2} = 3I_1 - 3I_2$$

$$3I_1 - 4I_2 = 0 \qquad \text{...(iii)}$$

For Mesh 3,

$$I_3 = -10 \qquad \text{...(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = -5.71 \text{ A}$$

$$I_2 = -4.29 \text{ A}$$

$$I_3 = -10 \text{ A}$$

$$V_x'' = 6(I_1 - I_2) = 6(-5.71 + 4.29) = -8.52 \text{ V}$$

Step III By superposition theorem,

$$V_x = V_x' + V_x'' = 8.52 - 8.52 = 0$$

Example 3.25 *Calculate the current I in the network shown in Fig. 3.102.*

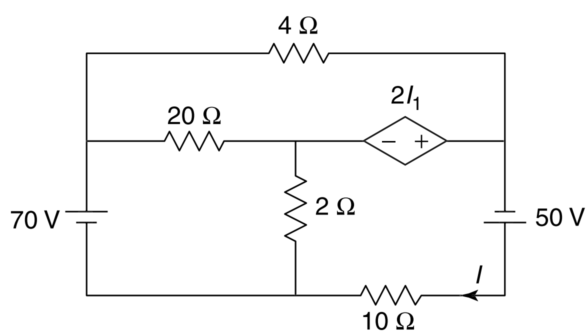


Fig. 3.102

Solution

Step I When the 70 V source is acting alone (Fig. 3.103)
From Fig. 3.103,

$$I' = I_3 \qquad \text{...(i)}$$

Applying KVL to Mesh 1,

$$-4I_1 - 2I_1 - 20(I_1 - I_2) = 0$$

$$26I_1 - 20I_2 = 0 \qquad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$70 - 20(I_2 - I_1) - 2(I_2 - I_3) = 0$$

$$-20I_1 + 22I_2 - 2I_3 = 70 \qquad \text{...(iii)}$$

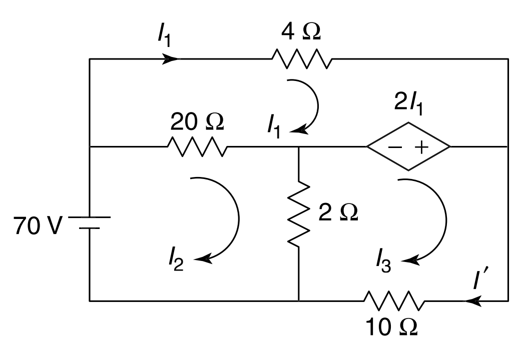


Fig. 3.103

Applying KVL to Mesh 3,

$$-2(I_3 - I_2) + 2I_1 - 10I_3 = 0$$

$$2I_1 + 2I_2 - 12I_3 = 0 \qquad \text{...(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = 8.94 \text{ A}$$
$$I_2 = 11.62 \text{ A}$$
$$I_3 = 3.43 \text{ A}$$
$$I' = I_3 = 3.43 \text{ A} (\leftarrow)$$

Step II When the 50 V source is acting alone (Fig. 3.104)
From Fig. 3.104,

$$I'' = I_3 \quad \text{...(i)}$$

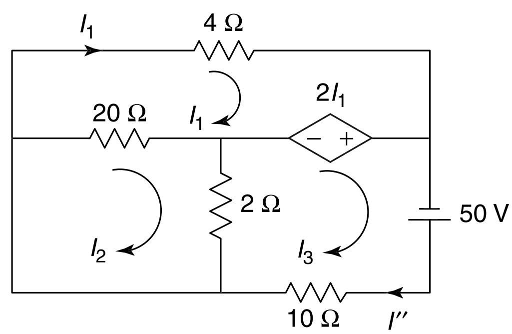


Fig. 3.104

Applying KVL to Mesh 1,

$$-4I_1 - 2I_1 - 20(I_1 - I_2) = 0$$

$$26I_1 - 20I_2 = 0 \quad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$-20(I_2 - I_1) - 2(I_2 - I_3) = 0$$

$$-20I_1 + 22I_2 - 2I_3 = 0 \qquad \text{...(iii)}$$

Applying KVL to Mesh 3,

$$-2(I_3 - I_2) + 2I_1 + 50 - 10I_3 = 0$$

$$2I_1 + 2I_2 - 12I_3 = -50 \qquad \text{...(iv)}$$

Solving Eqs (ii), (iii), and (iv),

$$I_1 = 1.06 \text{ A}$$
$$I_2 = 1.38 \text{ A}$$
$$I_3 = 4.57 \text{ A}$$
$$I'' = I_3 = 4.57 \text{ A} (\leftarrow)$$

Step III By superposition theorem,

$$I = I' + I'' = 3.43 + 4.57 = 8 \text{ A} (\leftarrow)$$

Example 3.26 *Find the voltage V_0 in the network of Fig. 3.105.*

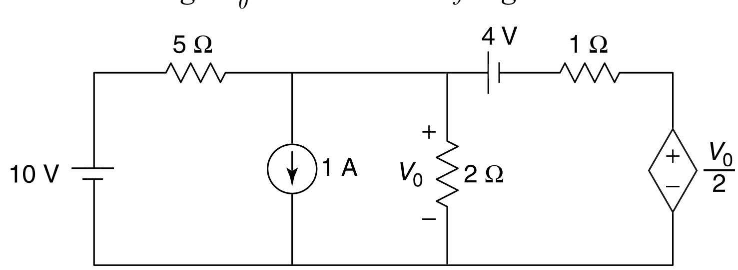


Fig. 3.105

Solution

Step I When the 10 V source is acting alone (Fig. 3.106)

Applying KCL at the node,

$$\frac{V_0'-10}{5}+\frac{V_0'}{2}+\frac{V_0'-\dfrac{V_0'}{2}}{1}=0$$

$$\left(\frac{1}{5}+\frac{1}{2}+\frac{1}{2}\right)V_0'=2$$

$$V_0'=1.67\text{ V}$$

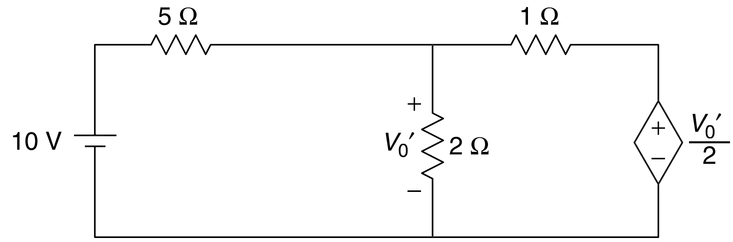


Fig. 3.106

Step II When the 1A current source is acting alone (Fig. 3.107)

Applying KCL at the node,

$$\frac{V_0''}{5}+1+\frac{V_0''}{2}+\frac{V_0''-\dfrac{V_0''}{2}}{1}$$

$$\left(\frac{1}{5}+\frac{1}{2}+\frac{1}{2}\right)V_0''=-1$$

$$V_0''=-0.83\text{ V}$$

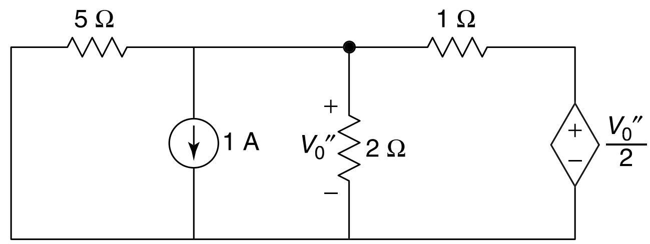


Fig. 3.107

Step III When the 4 V source is acting alone (Fig. 3.108)

Applying KCL at the node,

$$\frac{V_0'''}{5}+\frac{V_0'''}{2}+\frac{V_0'''-4-\dfrac{V_0'''}{2}}{1}=0$$

$$\left(\frac{1}{5}+\frac{1}{2}+\frac{1}{2}\right)V_0'''=4$$

$$V_0'''=3.33\text{ V}$$

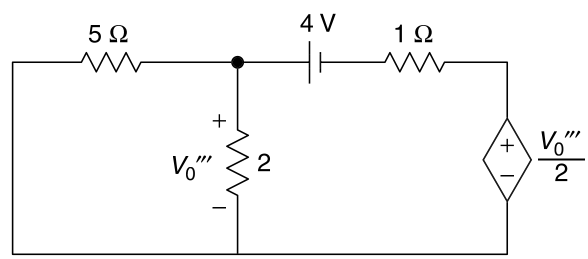


Fig. 3.108

Step IV By superposition theorem,

$$V_0=V_0'+V_0''+V_0'''=1.67-0.83+3.33=4.17\text{ V}$$

3.3 THEVENIN'S THEOREM

It states that '*any two terminals of a network can be replaced by an equivalent voltage source and an equivalent series resistance. The voltage source is the voltage across the two terminals with load, if any, removed. The series resistance is the resistance of the network measured between two terminals with load removed and constant voltage source being replaced by its internal resistance (or if it is not given with zero resistance, i.e., short circuit) and constant current source replaced by infinite resistance, i.e., open circuit.*'

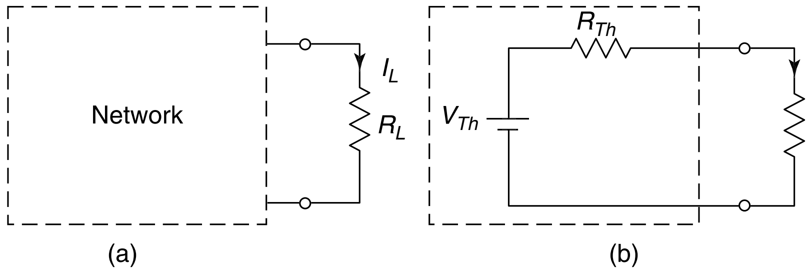


Fig. 3.109 *Network illustrating Thevenin's theorem*

Explanation Consider a simple network as shown in Fig. 3.110.

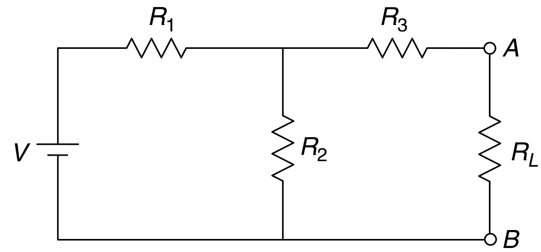


Fig. 3.110 *Network*

For finding load current through R_L, first remove the load resistor R_L from the network and calculate open circuit voltage V_{Th} across points A and B as shown in Fig. 3.111.

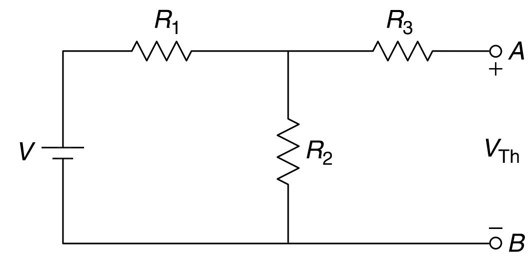


Fig. 3.111 *Calculation of V_{Th}*

$$V_{\text{Th}} = \frac{R_2}{R_1 + R_2} \text{V}$$

For finding series resistance R_{Th}, replace the voltage source by a short circuit and calculate resistance between points A and B as shown in Fig. 3.112.

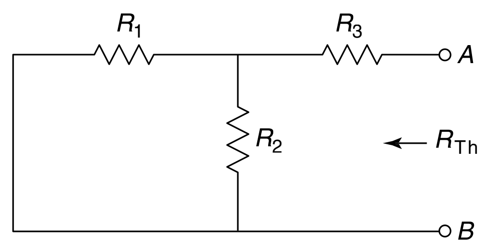


Fig. 3.112 *Calculation of R_{Th}*

$$R_{\text{Th}} = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

Thevenin's equivalent network is shown in Fig. 3.113.

$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L}$$

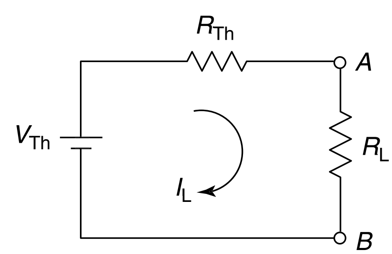


Fig 3.113 *Thevenin's equivalent network*

If the network contains both independent and dependent sources, Thevenin's resistance R_{Th} is calculated as,

$$R_{\text{Th}} = \frac{V_{\text{Th}}}{I_N}$$

where I_N is the short-circuit current which would flow in a short circuit placed across the terminals A and B. Dependent sources are active at all times. They have zero values only when the control voltage or current is zero. R_{Th} may be negative in some cases which indicates negative resistance region of the device, i.e., as voltage increases, current decreases in the region and vice-versa.

If the network contains only dependent sources then

$$V_{\text{Th}} = 0$$

$$I_N = 0$$

For finding R_{Th} in such a network, a known voltage V is applied across the terminals A and B and current is calculated through the path AB.

$$R_{\text{Th}} = \frac{V}{I}$$

or a known current source I is connected across the terminals A and B and voltage is calculated across the terminals A and B.

$$R_{\text{Th}} = \frac{V}{I}$$

Thevenin's equivalent network for such a network is shown in Fig. 3.114.

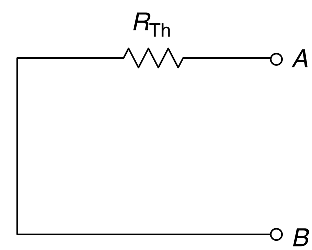


Fig. 3.114 *Thevenin's equivalent network*

Steps to be Followed in Thevenin's Theorem

1. Remove the load resistance R_L.
2. Find the open circuit voltage V_{Th} across points A and B.
3. Find the resistance R_{Th} as seen from points A and B.
4. Replace the network by a voltage source V_{Th} in series with resistance R_{Th}.
5. Find the current through R_L using Ohm's law.

$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L}$$

Example 3.27 *Find the current through the 2 Ω resistor in Fig. 3.115.*

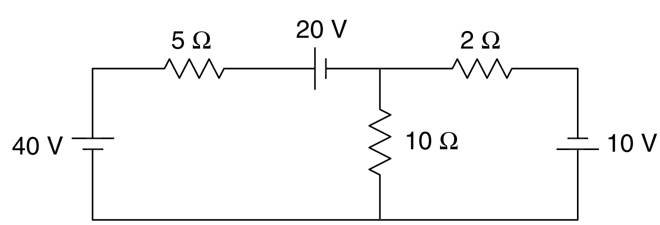


Fig. 3.115

Solution

Step I Calculation of V_{Th} (Fig. 3.116)

Applying KVL to the mesh,

$$40 - 5I - 20 - 10I = 0$$

$$15I = 20$$

$$I = 1.33 \text{ A}$$

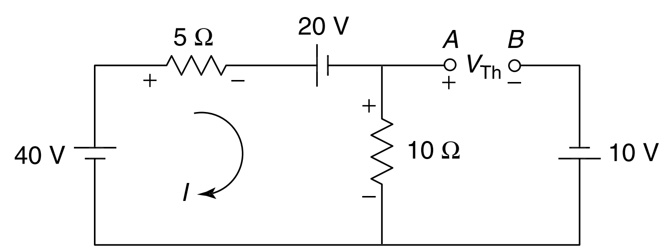


Fig. 3.116

Writing the V_{Th} equation,

$$10I - V_{Th} + 10 = 0$$

$$V_{Th} = 10I + 10 = 10(1.33) + 10 = 23.33 \text{ V}$$

Step II Calculation of R_{Th} (Fig. 3.117)

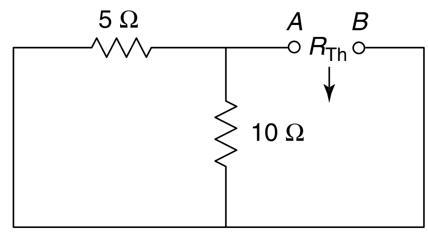


Fig. 3.117

$$R_{Th} = 5 \,||\, 10 = 3.33\ \Omega$$

Step III Calculation of I_L (Fig. 3.118)

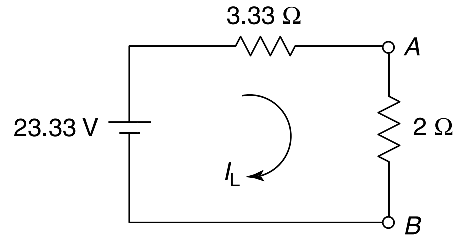


Fig. 3.118

$$I_L = \frac{23.33}{3.33 + 2} = 4.38 \text{ A}$$

Example 3.28 *Find the current through the 8 Ω resistor in Fig. 3.119.*

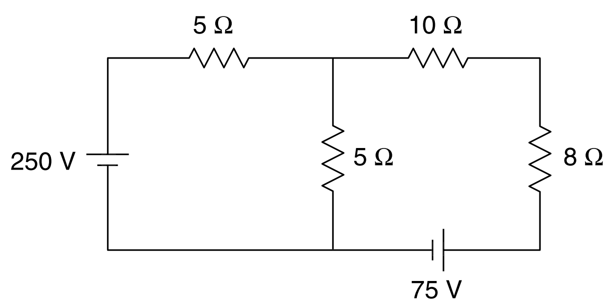


Fig. 3.119

Solution

Step I Calculation of V_{Th} (Fig. 3.120)

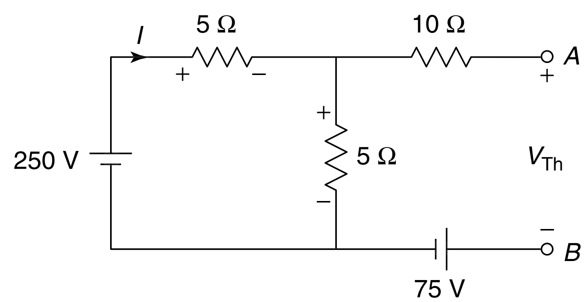


Fig. 3.120

$$I = \frac{250}{5 + 5} = 25 \text{ A}$$

Writing the V_{Th} equation,

$$250 - 5I - V_{Th} - 75 = 0$$

$$V_{Th} = 175 - 5I = 175 - 5(25) = 50 \text{ V}$$

Step II Calculation of R_{Th} (Fig. 3.121)

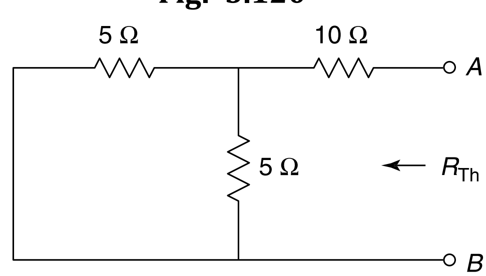


Fig. 3.121

$$R_{Th} = (5 \,||\, 5) + 10 = 12.5\ \Omega$$

Step III Calculation of I_L (Fig. 3.122)

$$I_L = \frac{50}{12.5+8} = 2.44 \text{ A}$$

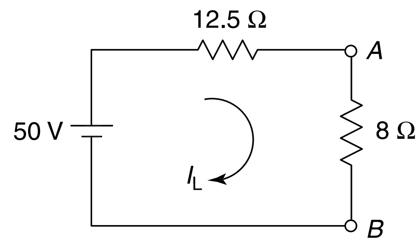


Fig. 3.122

Example 3.29 *Find the current through the 2 Ω resistor connected between terminals A and B in the Fig. 3.123.*

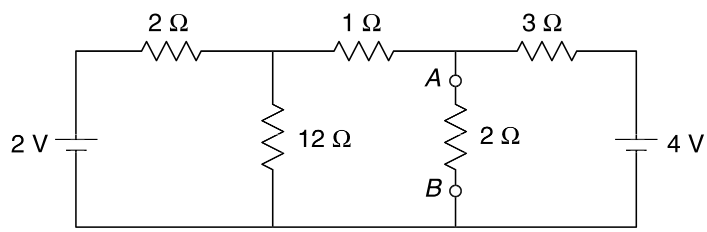


Fig. 3.123

Solution

Step I Calculation of V_{Th} (Fig. 3.124)

Applying KVL to Mesh 1,

$$2 - 2I_1 - 12(I_1 - I_2) = 0$$

$$14I_1 - 12I_2 = 2 \quad \text{...(i)}$$

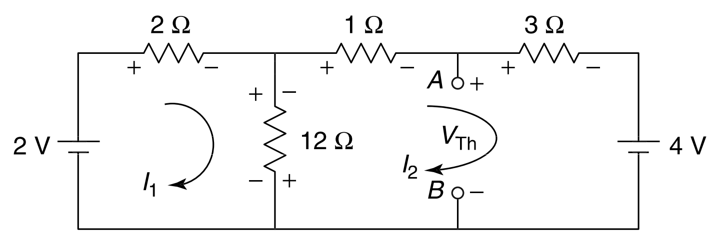


Fig. 3.124

Applying KVL to Mesh 2,

$$-12(I_2 - I_1) - 1I_2 - 3I_2 - 4 = 0$$

$$-12I_1 + 16I_2 = -4 \quad \text{...(ii)}$$

Solving Eqs (i) and (ii),

$$I_2 = -0.4 \text{ A}$$

Writing the V_{Th} equation,

$$V_{Th} - 3I_2 - 4 = 0$$

$$V_{Th} = 4 + 3I_2 = 4 + 3(-0.4) = 2.8 \text{ V}$$

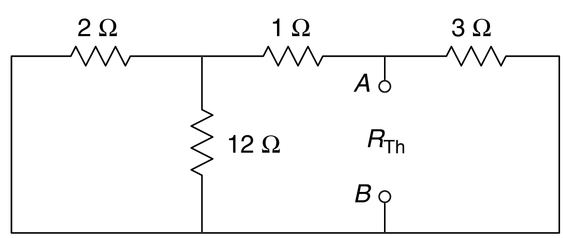


Fig. 3.125

Step II Calculation of R_{Th} (Fig. 3.125)

$$R_{Th} = [(2 \,||\, 12) + 1] \,||\, 3 = 1.43 \, \Omega$$

Step III Calculation of I_L (Fig. 3.126)

$$I_L = \frac{2.8}{1.43+2} = 0.82 \text{ A}$$

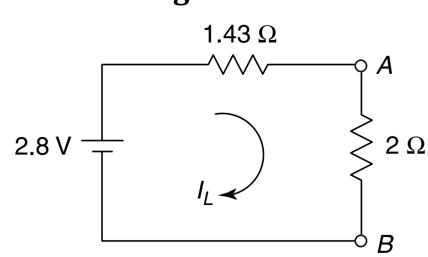


Fig. 3.126

Example 3.30 *Find the current through the 10 Ω resistor in Fig. 3.127.*

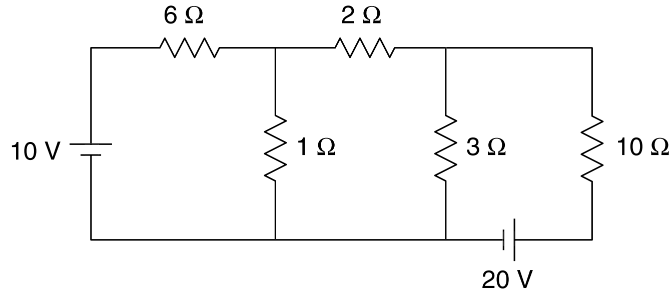


Fig. 3.127

Solution

Step I Calculation of V_{Th} (Fig. 3.128)

Applying KVL to Mesh 1,

$$10-6I_1-1(I_1-I_2)=0$$
$$7I_1-I_2=10 \quad \text{.....(i)}$$

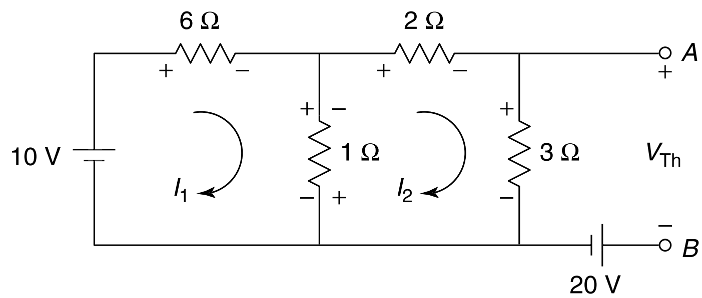


Fig. 3.128

Applying KVL to Mesh 2,

$$-1(I_2-I_1)-2I_2-3I_2=0$$
$$-I_1+6I_2=0 \quad \text{...(ii)}$$

Solving Eqs (i) and (ii),

$$I_2=0.24\text{ A} \quad \text{...(ii)}$$

Writing the V_{Th} equation,

$$3I_2-V_{\text{Th}}-20=0$$
$$V_{\text{Th}}=3I_2-20=3(0.24)-20=-19.28\text{ V}$$

=19.28 V (terminal *B* is positive w.r.t *A*)

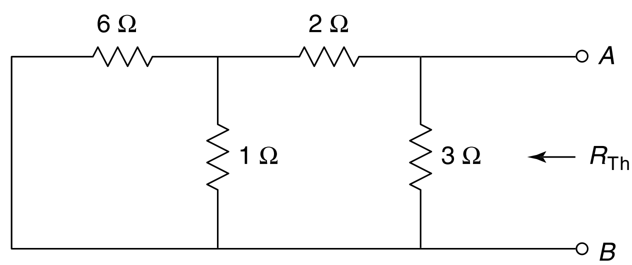


Fig. 3.129

Step II Calculation of R_{Th} (Fig. 3.129)

$$R_{\text{Th}}=[(6\,||\,1)+2]\,||\,3=1.47\ \Omega$$

Step III Calculation of I_L (Fig. 3.130)

$$I_L=\frac{19.28}{1.47+10}=1.68\text{ A}(\uparrow)$$

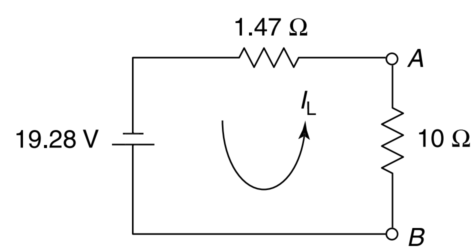


Fig. 3.130

Example 3.31 *Find the current through the 10 Ω resistor in Fig. 3.131.*

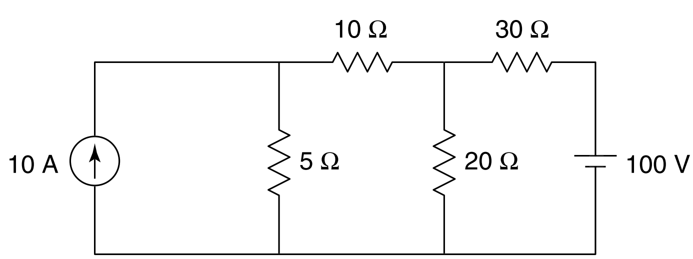


Fig. 3.131

Solution

Step I Calculation of V_{Th} (Fig. 3.132)

For Mesh 1,

$$I_1 = 10$$

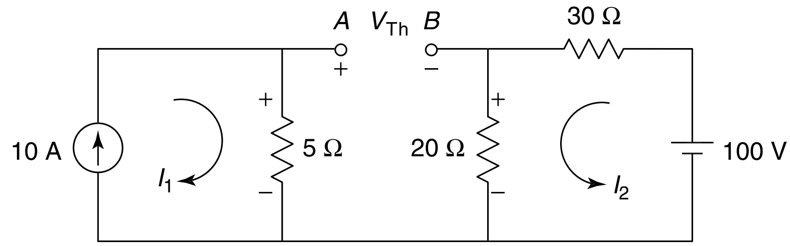


Fig. 3.132

Applying KVL to Mesh 2,

$$100 - 30I_2 - 20I_2 = 0$$

$$I_2 = 2 \text{ A}$$

Writing the V_{Th} equation,

$$5I_1 - V_{Th} - 20I_2 = 0$$

$$V_{Th} = 5I_1 - 20I_2 = 5(10) - 20(2) = 10 \text{ V}$$

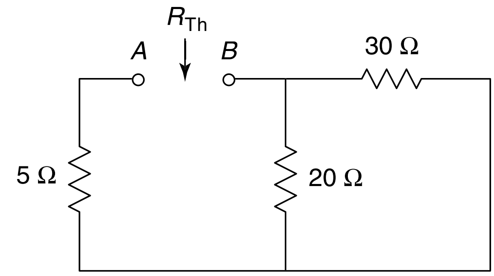


Fig. 3.133

Step II Calculation of R_{Th} (Fig. 3.133)

$$R_{Th} = 5 + (20 \parallel 30) = 17\ \Omega$$

Step III Calculation of I_L (Fig. 3.134)

$$I_L = \frac{10}{17+10} = 0.37 \text{ A}$$

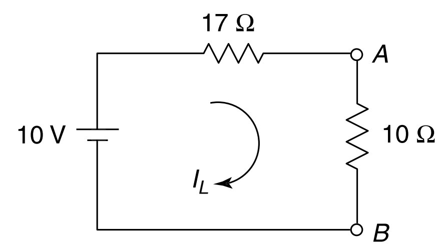


Fig. 3.134

Example 3.32 *Find the current through the 40 Ω resistor in Fig. 3.135.*

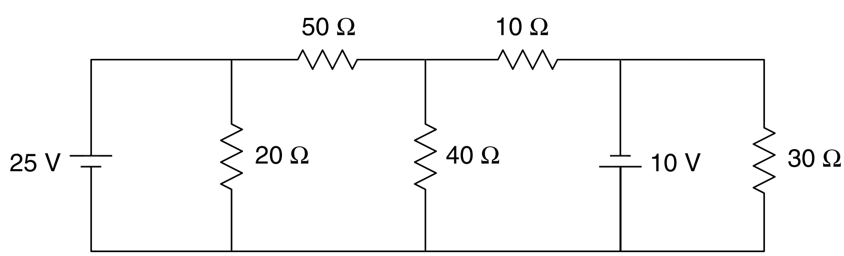


Fig. 3.135

Solution

Step I Calculation of V_{Th} (Fig. 3.136)

Since the 20 Ω resistor is connected across the 25 V source, the resistor becomes redundant.

$$V_{20\,\Omega} = 25 \text{ V}$$

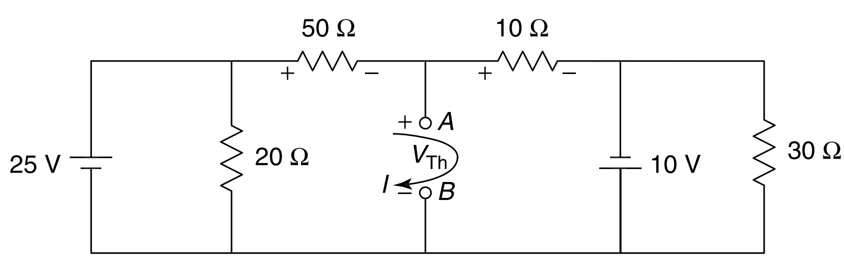


Fig. 3.136

Applying KVL to the mesh,

$$25-50I-10I+10=0$$
$$I=0.58\text{ A}$$

Writing the V_{Th} equation,

$$V_{\text{Th}}-10\text{I}+10=0$$
$$V_{\text{Th}}=10(I)-10=10(0.58)-10=-4.2\text{ V}$$
$$=4.2\text{ V(terminal } B \text{ is positive w.r.t } A)$$

Step II Calculation of R_{Th} (Fig. 3.137)

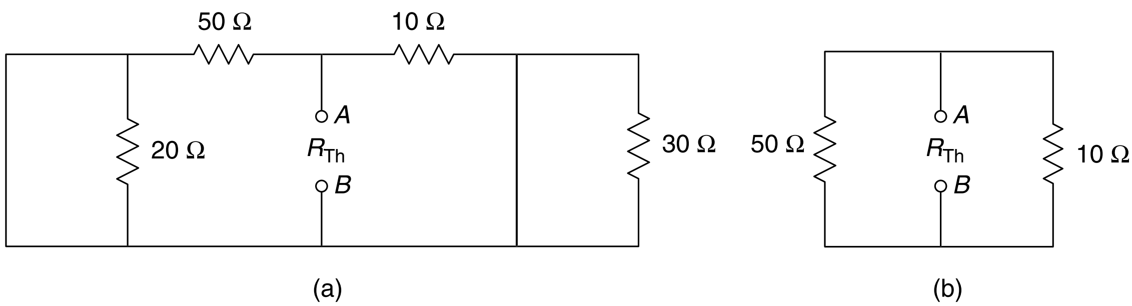


Fig. 3.137

$$R_{\text{Th}}=50\,\|\,10=8.33\ \Omega$$

Step III Calculation of I_L (Fig. 3.138)

$$I_L=\frac{4.2}{8.33+40}=0.09\text{ A}(\uparrow)$$

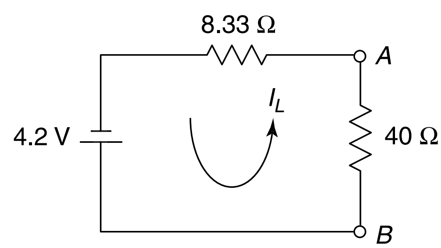


Fig. 3.138

Example 3.33 *Find the current through the 10 Ω resistor in Fig. 3.139.*

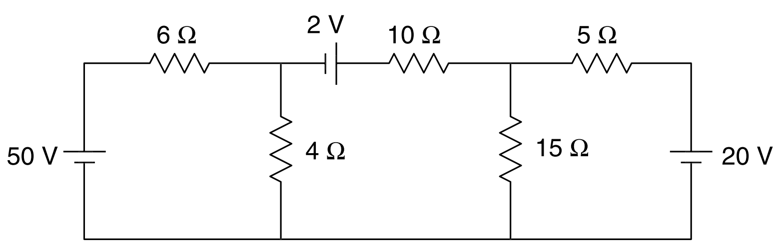


Fig. 3.139

Solution

Step I Calculation of V_{Th} (Fig. 3.140)

$$I_1=\frac{50}{6+4}=5\text{ A}$$

$$I_2=\frac{20}{5+15}=1\text{ A}$$

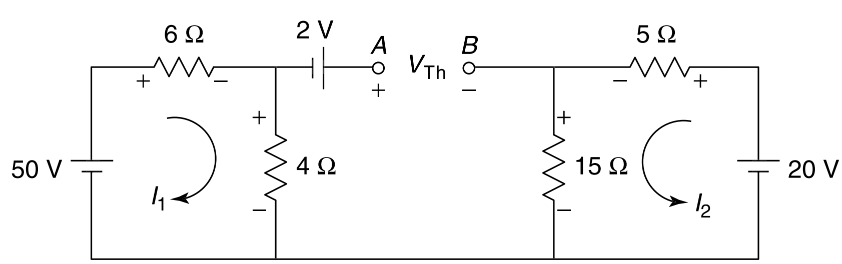


Fig. 3.140

Writing the V_{Th} equation,

$$4I_1 + 2 - V_{\text{Th}} - 15I_2 = 0$$

$$V_{\text{Th}} = 4I_1 + 2 - 15I_2 = 4(5) + 2 - 15(1) = 7 \text{ V}$$

Step II Calculation of R_{Th} (Fig. 3.141)

$$R_{\text{Th}} = (6 \,||\, 4) + (5 \,||\, 15) = 6.15\ \Omega$$

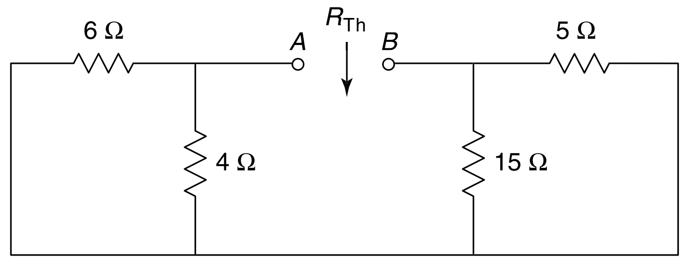


Fig. 3.141

Step III Calculation of I_L (Fig. 3.142)

$$I_L = \frac{7}{6.15 + 10} = 0.43 \text{ A}$$

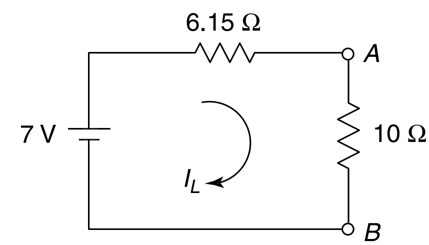


Fig. 3.142

Example 3.34 *Determine the current through the 24 Ω resistor in Fig. 3.143.*

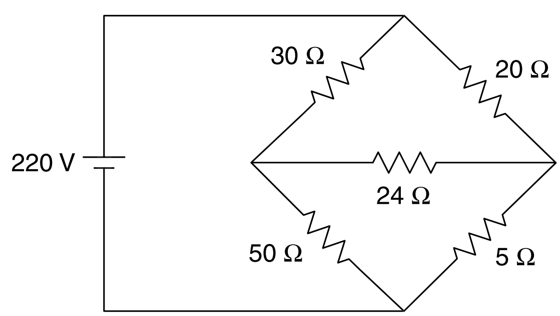


Fig. 3.143

Solution

Step I Calculation of V_{Th} (Fig. 3.144)

$$I_1 = \frac{220}{30 + 50} = 2.75 \text{ A}$$

$$I_2 = \frac{220}{20 + 5} = 8.8 \text{ A}$$

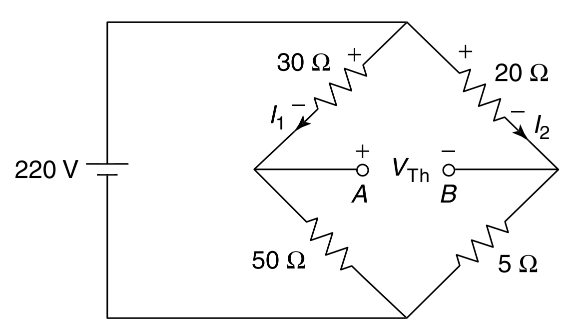


Fig. 3.144

Writing the V_{Th} equation,

$$V_{\text{Th}} + 30I_1 - 20I_2 = 0$$

$$V_{\text{Th}} = 20I_2 - 30I_1 = 20(8.8) - 30(2.75) = 93.5 \text{ V}$$

Step II Calculation of R_{Th} (Fig. 3.145)

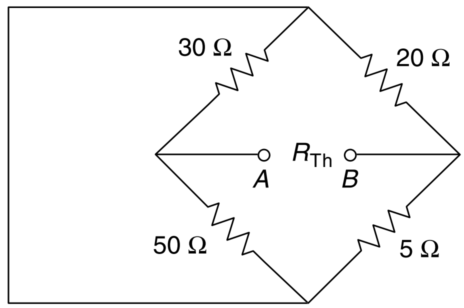


Fig. 3.145

Redrawing the circuit (Fig. 3.146),

$$R_{Th} = (30 \,||\, 50) + (20 \,||\, 5) = 22.75\ \Omega$$

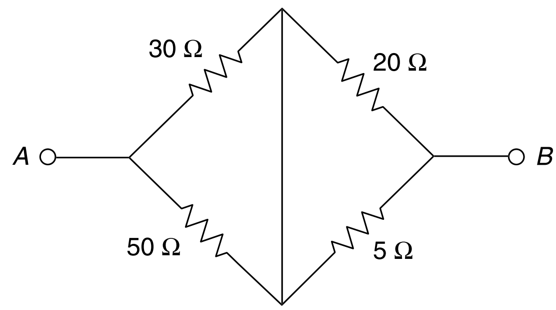


Fig. 3.146

Step III Calculation of I_L (Fig. 3.147)

$$I_L = \frac{93.5}{22.75 + 24} = 2\text{ A}$$

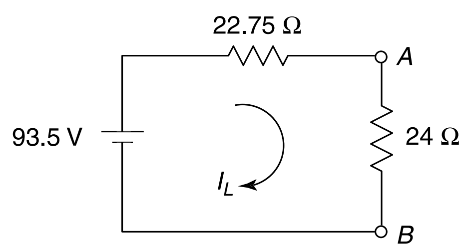


Fig. 3.147

Example 3.35 *Find the current through the 3 Ω resistor in Fig. 3.148.*

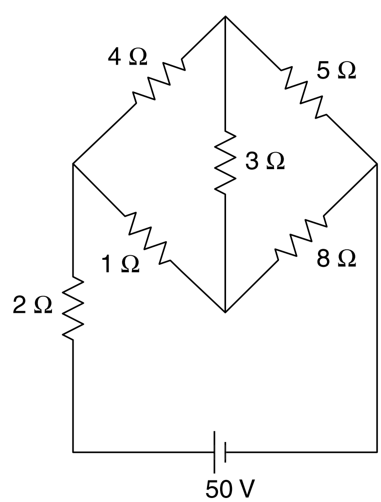


Fig. 3.148

Solution

Step I Calculation of V_{Th} (Fig. 3.149)

Applying KVL to Mesh 1,

$$50 - 2I_1 - 1(I_1 - I_2) - 8(I_1 - I_2) = 0$$

$$11I_1 - 9I_2 = 50 \quad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-4I_2 - 5I_2 - 8(I_2 - I_1) - 1(I_2 - I_1) = 0$$

$$-9I_1 + 18I_2 = 0 \quad \ldots\text{(ii)}$$

Solving Eqs (i) and (ii),

$$I_1 = 7.69 \text{ A}$$

$$I_2 = 3.85 \text{ A}$$

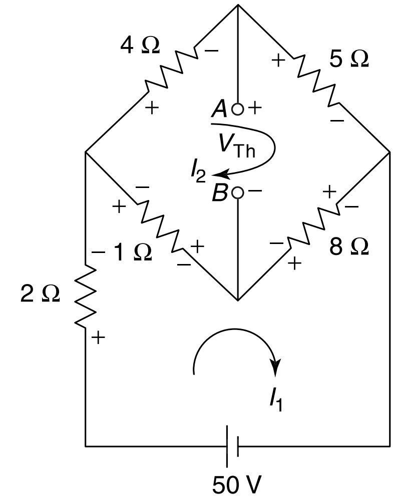


Fig. 3.149

Writing the V_{Th} equation,

$$V_{Th} - 5I_2 - 8(I_2 - I_1) = 0$$

$$V_{Th} = 5I_2 + 8(I_2 - I_1) = 5(3.85) + 8(3.85 - 7.69) = -11.47 \text{ V}$$

$$= 11.47 \text{ V(the terminal } B \text{ is positive w.r.t. } A)$$

Step II Calculation of R_{Th} (Fig. 3.150)

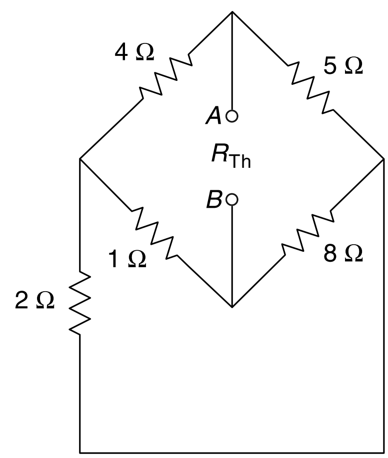


Fig. 3.150

Redrawing the network (Fig. 3.151),

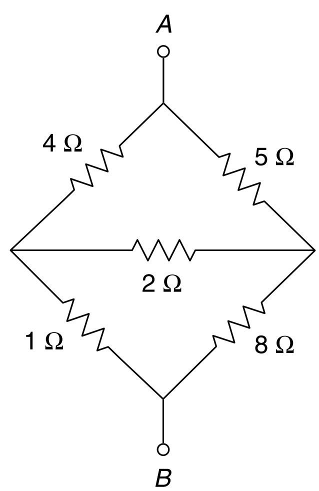


Fig. 3.151

Converting the upper delta into equivalent star network (Fig. 3.152),

$$R_1 = \frac{4\times 2}{4+2+5} = 0.73\ \Omega$$

$$R_2 = \frac{4\times 5}{4+2+5} = 1.82\ \Omega$$

$$R_3 = \frac{5\times 2}{4+2+5} = 0.91\ \Omega$$

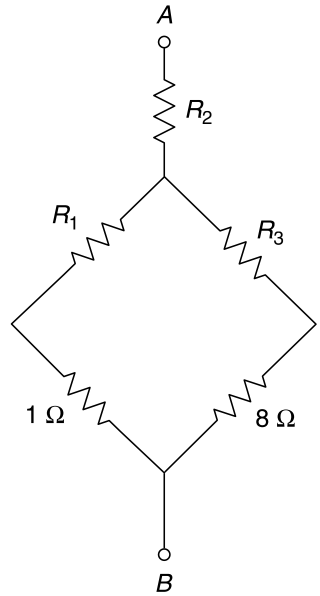


Fig. 3.152

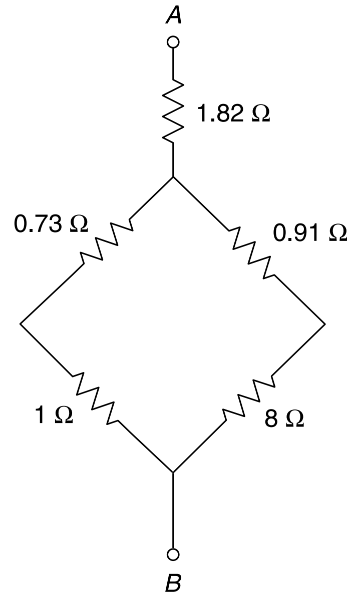


Fig. 3.153

Simplifying the network (Fig. 3.153),

$$R_{\text{Th}} = 1.82 + (1.73 \,||\, 8.91) = 3.27\ \Omega$$

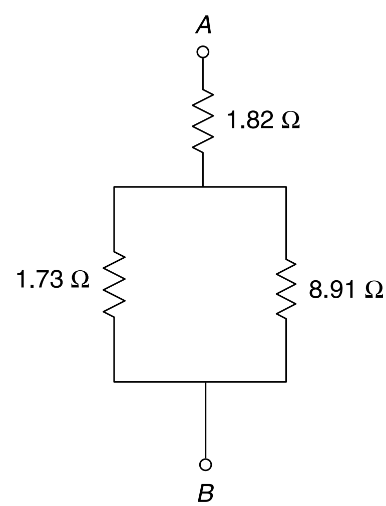


Fig. 3.154

Step III Calculation of I_L (Fig. 3.155)

$$I_L = \frac{11.47}{3.27+3} = 1.83\ \text{A}(\uparrow)$$

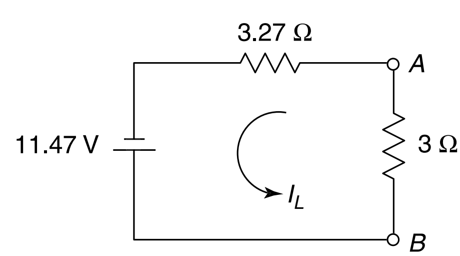


Fig. 3.155

Example 3.36 *Find the current through the 20 Ω resistor in Fig. 3.156.*

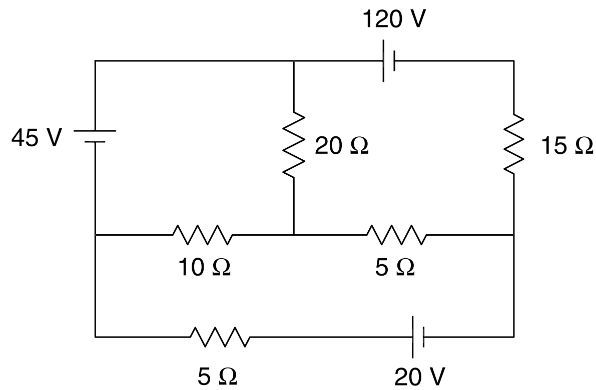


Fig. 3.156

Solution

Step I Calculation of V_{Th} (Fig. 3.157)

Applying KVL to Mesh 1,

$$45-120-15I_1-5(I_1-I_2)-10(I_1-I_2)=0$$

$$30I_1-15I_2=-75 \qquad \text{...(i)}$$

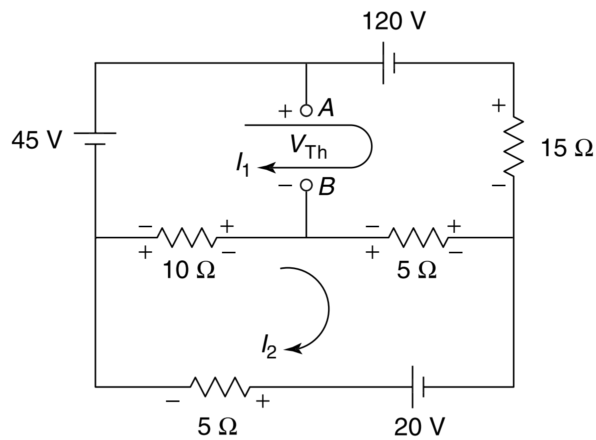


Fig. 3.157

Applying KVL to Mesh 2,

$$20-5I_2-10(I_2-I_1)-5(I_2-I_1)=0$$

$$-15I_1+20I_2=20 \qquad \text{...(ii)}$$

Solving Eqs (i) and (ii),

$$I_1=-3.2\text{ A}$$

$$I_2=-1.4\text{ A}$$

Writing the V_{Th} equation,

$$45-V_{Th}-10(I_1-I_2)=0$$

$$V_{Th}=45-10(I_1-I_2)=45-10[-3.2-(-1.4)]=63\text{ V}$$

Step II Calculation of R_{Th} (Fig. 3.158)

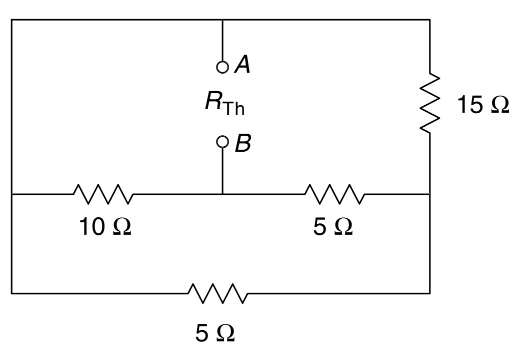


Fig. 3.158

Converting the delta formed by resistors of 10 Ω, 5 Ω and 5 Ω into an equivalent star network (Fig. 3.159),

$$R_1 = \frac{10 \times 5}{20} = 2.5\ \Omega$$

$$R_2 = \frac{10 \times 5}{20} = 2.5\ \Omega$$

$$R_3 = \frac{5 \times 5}{20} = 1.25\ \Omega$$

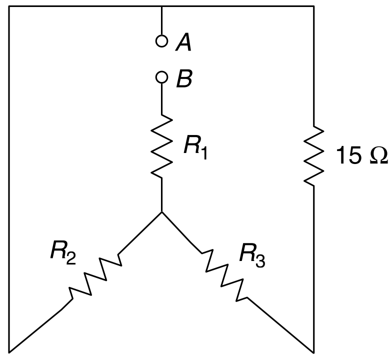


Fig. 3.159

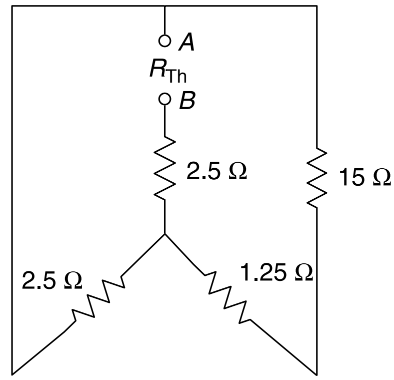


Fig. 3.160

Simplifying the network (Fig. 3.160),

$$R_{Th} = (16.25 \,||\, 2.5) + 2.5 = 4.67\ \Omega$$

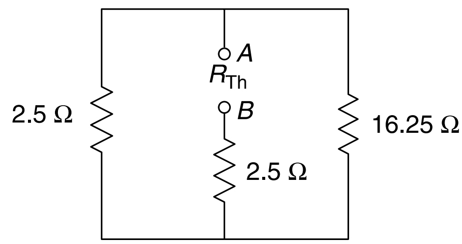


Fig. 3.161

Step III Calculation of I_L

$$I_L = \frac{63}{4.67 + 20} = 2.55\ \text{A}$$

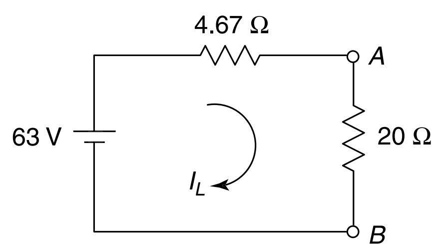


Fig. 3.162

Example 3.37 *Find the current through the 3 Ω resistor in Fig. 3.163.*

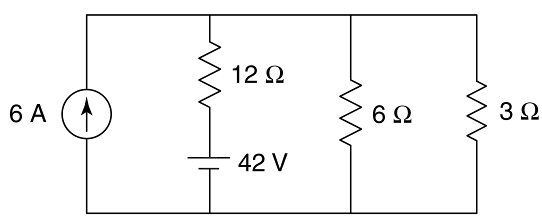


Fig. 3.163

Solution

Step I Calculation of V_{Th} (Fig. 3.164)

For Mesh 1,

$$I_1 = 6 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$42 - 12(I_2 - I_1) - 6I_2 = 0$$

$$-12I_1 + 18I_2 = 42 \qquad \text{...(ii)}$$

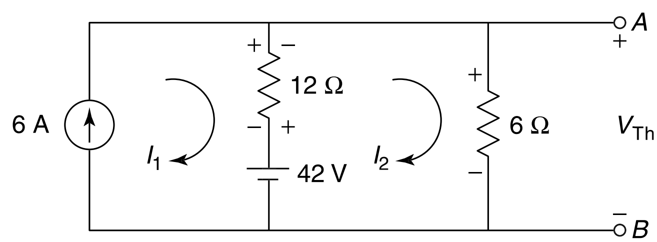


Fig. 3.164

Solving Eqs (i) and (ii),

$$I_2 = 6.33 \text{ A}$$

Writing the V_{Th} equation,

$$V_{\text{Th}} = 6I_2 = 38 \text{ V}$$

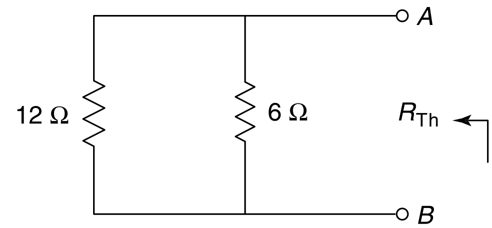


Fig. 3.165

Step II Calculation of R_{Th} (Fig. 3.165)

$$R_{\text{Th}} = 6 \,||\, 12 = 4\,\Omega$$

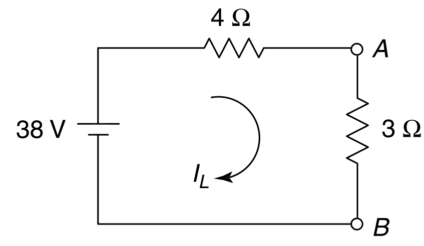


Fig. 3.166

Step III Calculation of I_L (Fig. 3.166)

$$I_L = \frac{38}{4+3} = 5.43 \text{ A}$$

Example 3.38

Find the current through the 30 Ω resistor in Fig. 3.167.

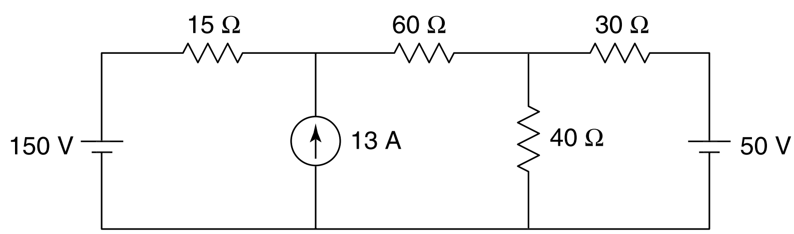


Fig. 3.167

Solution

Step I Calculation of V_{Th} (Fig. 3.168)

Meshes 1 and 2 form a supermesh.

Writing current equation for supermesh,

$$I_2 - I_1 = 13 \qquad \text{...(i)}$$

Writing voltage equation for supermesh,

$$150 - 15I_1 - 60I_2 - 40I_2 = 0$$

$$15I_1 + 100I_2 = 150 \quad \text{...(ii)}$$

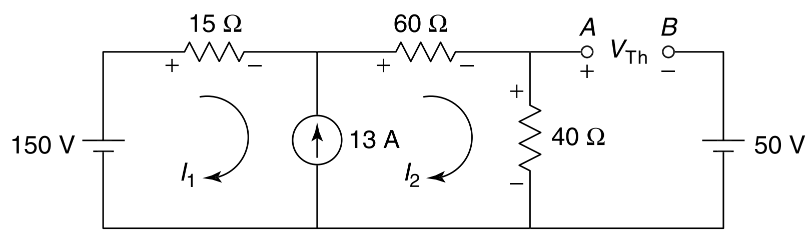


Fig. 3.168

Solving Eqs (i) and (ii),

$$I_1 = -10 \text{ A}$$

$$I_2 = 3 \text{ A}$$

Writing the V_{Th} equation,

$$40I_2 - V_{\text{Th}} - 50 = 0$$

$$V_{\text{Th}} = 40I_2 - 50 = 40(3) - 50 = 70 \text{ V}$$

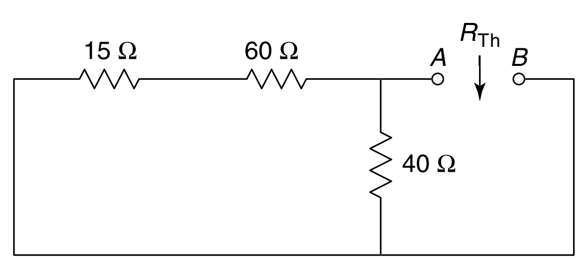


Fig. 3.169

Step II Calculation of R_{Th} (Fig. 3.169)

$$R_{\text{Th}} = 75 \,||\, 40 = 26.09\,\Omega$$

Step III Calculation of I_L (Fig. 3.170)

$$I_L = \frac{70}{26.09+30} = 1.25\text{ A}$$

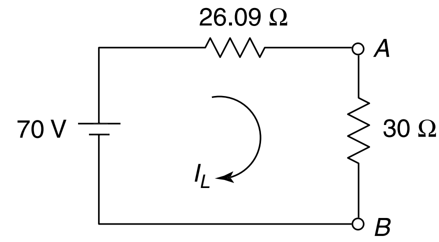


Fig. 3.170

Example 3.39 *Find the current through the 20 Ω resistor in Fig. 3.171.*

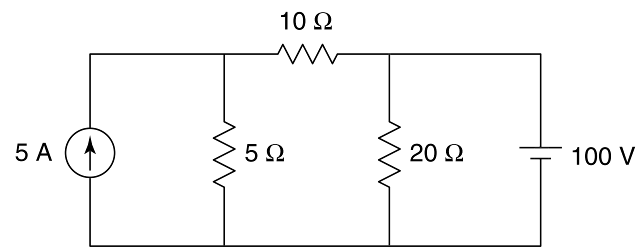


Fig. 3.171

Solution

Step I Calculation of V_{Th} (Fig. 3.172)

$$V_{\text{Th}} = 100\text{ V}$$

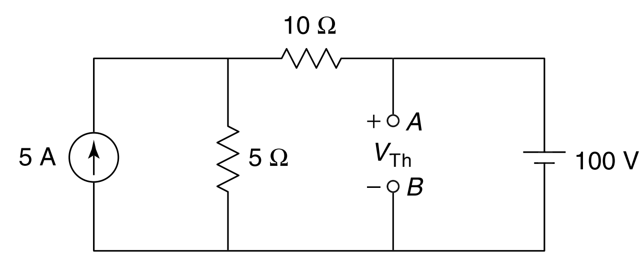


Fig. 3.172

Step II Calculation of R_{Th} (Fig.3.173)

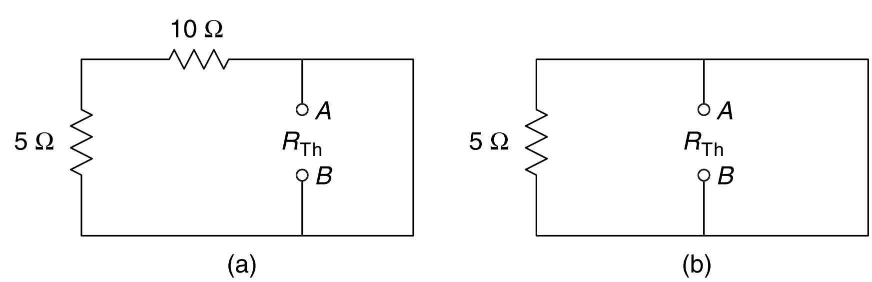


Fig. 3.173

$$R_{\text{Th}} = 0$$

Step III Calculation of I_L (Fig. 3.174)

$$I_L = \frac{100}{20} = 5\text{ A}$$

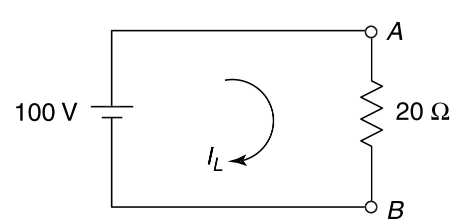


Fig. 3.174

Example 3.40 *Find the current through the 20 Ω resistor in Fig. 3.175.*

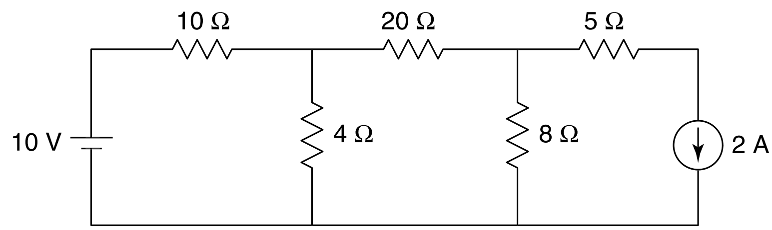


Fig. 3.175

Solution

Step I Calculation of V_{Th} (Fig. 3.176)

$$I_1 = \frac{10}{10+4} = 0.71 \text{ A}$$

$$I_2 = 2 \text{ A}$$

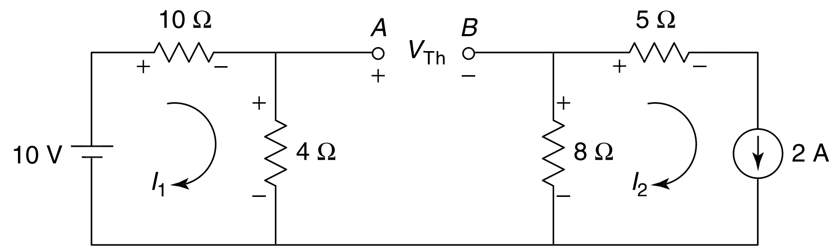


Fig. 3.176

Writing the V_{Th} equation,

$$4I_1 - V_{Th} + 8I_2 = 0$$

$$V_{Th} = 4(0.71) + 8(2) = 18.84 \text{ V}$$

Step II Calculation of R_{Th} (Fig. 3.177).

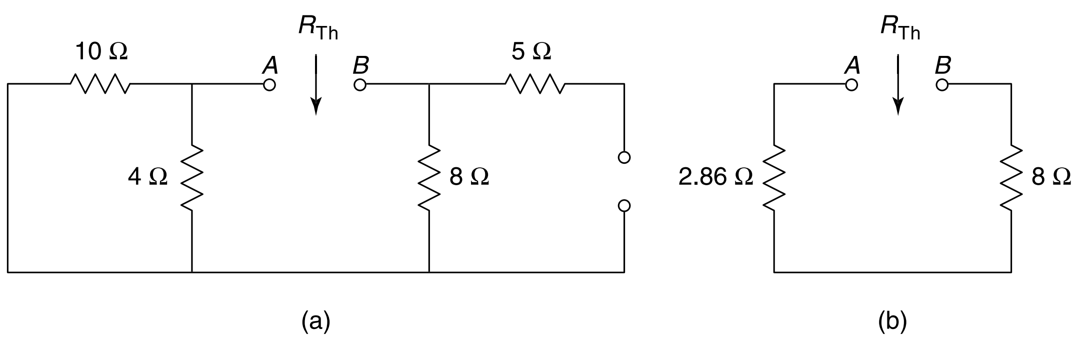


Fig. 3.177

$$R_{Th} = 10.86 \ \Omega$$

Step III Calculation of I_L (Fig. 3.178)

$$I_L = \frac{18.84}{10.86+20} = 0.61 \text{A}$$

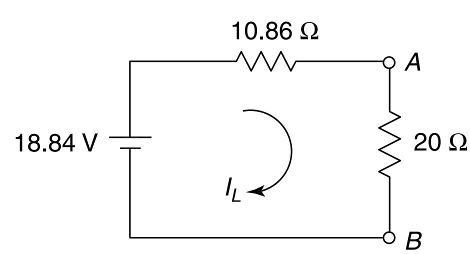


Fig. 3.178

Example 3.41 *Find the current through the 5 Ω resistor in Fig. 3.179.*

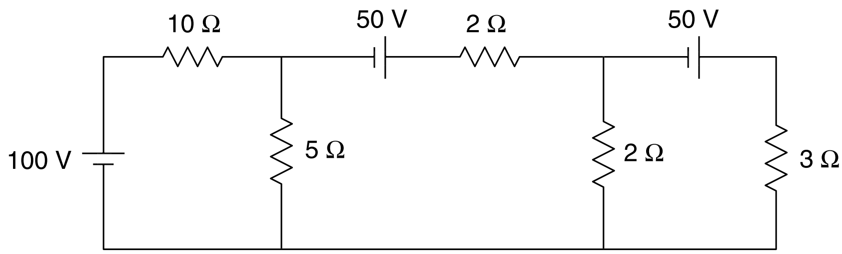


Fig. 3.179

Solution

Step I Calculation of V_{Th} (Fig. 3.180)

Applying KVL to Mesh 1,

$$100-10I_1+50-2I_1-2(I_1-I_2)=0$$

$$14I_2-2I_2=150 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-2(I_2-I_1)+50-3I_2=0$$

$$-2I_1+5I_2=50 \qquad \text{...(ii)}$$

Solving Eqs (i) and (ii),

$$I_1=12.88\text{ A}$$

$$I_2=15.15\text{ A}$$

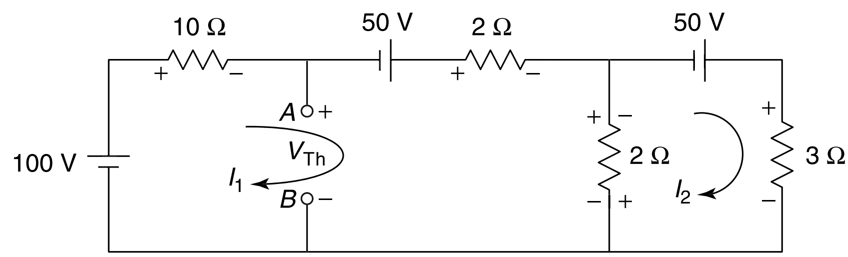


Fig. 3.180

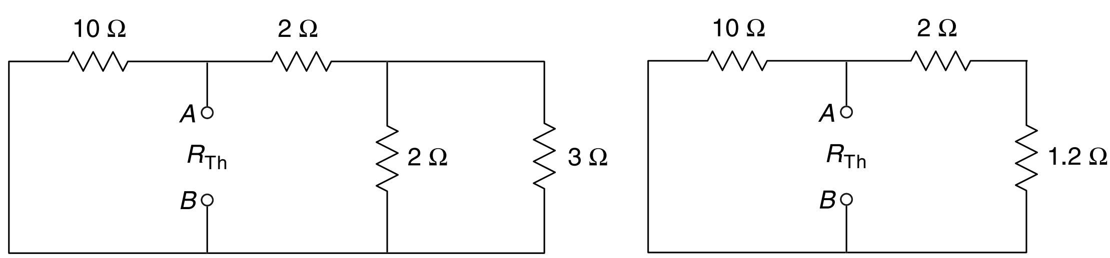
Writing the V_{Th} equation,

$$100-10I_1-V_{Th}=0$$

$$V_{Th}=100-10(12.88)=-28.8\text{ V}$$

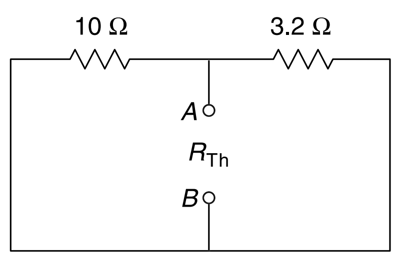
$$=28.8\text{ V(terminal } B \text{ is positive w.r.t. } A)$$

Step II Calculation of R_{Th} (Fig. 3.181)



(a) (b)

$$R_{Th}=10\,||\,3.2=2.42\ \Omega$$



(c)

Fig. 3.181

Step III Calculation of I_L (Fig. 3.182)

$$I_L = \frac{28.8}{2.42+5} = 3.88\text{ A }(\uparrow)$$

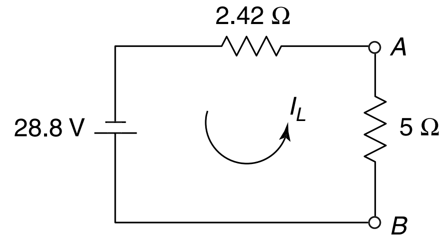


Fig. 3.182

Example 3.42 *Find the current through the 10 Ω resister in Fig. 3.183.*

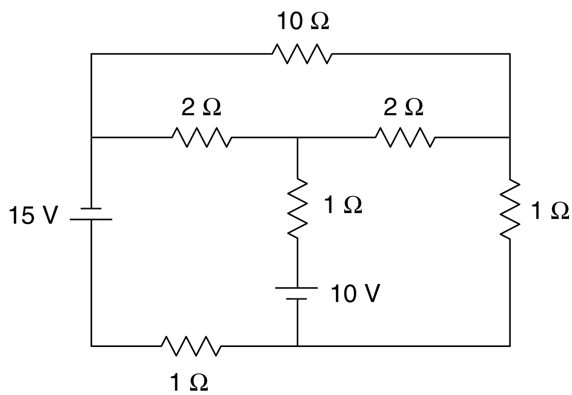


Fig. 3.183

Solution

Step I Calculation of V_{Th} (Fig. 3.183)

Applying KVL to Mesh 1,

$$-15 - 2I_1 - 1(I_1 - I_2) - 10 - 1I_1 = 0 \quad \text{...(i)}$$
$$4I_1 - I_2 = -25$$

Applying KVL to Mesh 2,

$$10 - 1(I_2 - I_1) - 2I_2 - 1I_2 = 0 \quad \text{...(ii)}$$
$$-I_1 + 4I_2 = 10$$

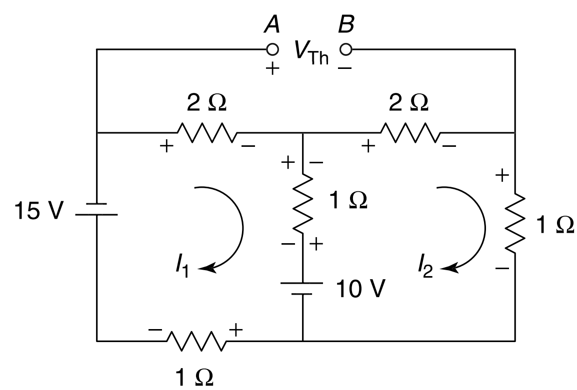


Fig. 3.184

Solving Eqs (i) and (ii),

$$I_1 = -6\text{ A}$$
$$I_2 = 1\text{ A}$$

Writing the V_{Th} equation,

$$-V_{Th} + 2I_2 + 2I_1 = 0$$
$$V_{Th} = 2I_1 + 2I_2 = 2(-6) + 2(1) = -10\text{ V}$$
$$= 10\text{ V} (\text{the terminal } B \text{ is positive w.r.t. } A)$$

Step II Calculation of R_{Th} (Fig. 3.185)

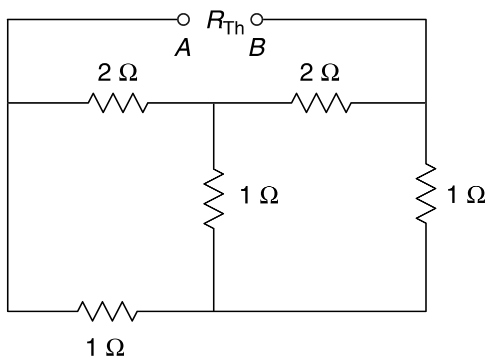


Fig. 3.185

Converting the star network formed by resistors of 2 Ω, 2 Ω and 1 Ω into an equivalent delta network (Fig. 3.186),

$$R_1 = 2+2+\frac{2\times2}{1} = 8\ \Omega$$
$$R_2 = 2+1+\frac{2\times1}{2} = 4\ \Omega$$
$$R_3 = 2+1+\frac{2\times1}{2} = 4\ \Omega$$

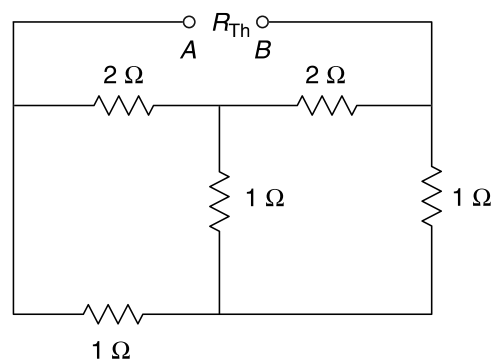


Fig. 3.186

Simplifying the network (Fig. 3.187),

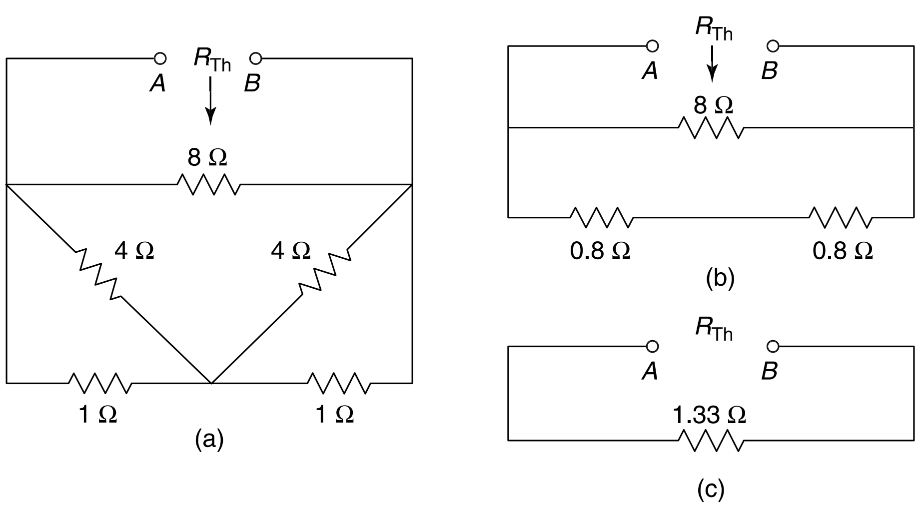


Fig. 3.187

$$R_{Th} = 1.33\ \Omega$$

Step III Calculation of I_L (Fig. 3.188)

$$I_L = \frac{10}{1.33+10} = 0.88\text{ A }(\uparrow)$$

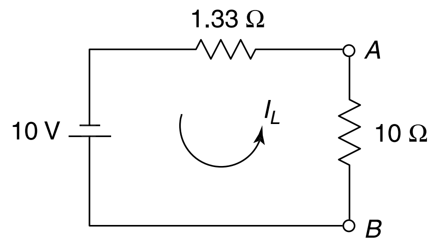


Fig. 3.188

Example 3.43 *Find the current through the 1 Ω resistor in Fig. 3.189.*

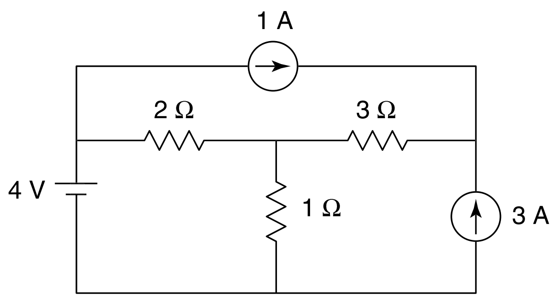


Fig. 3.189

Solution

Step I Calculation of V_{Th} (Fig. 3.190)

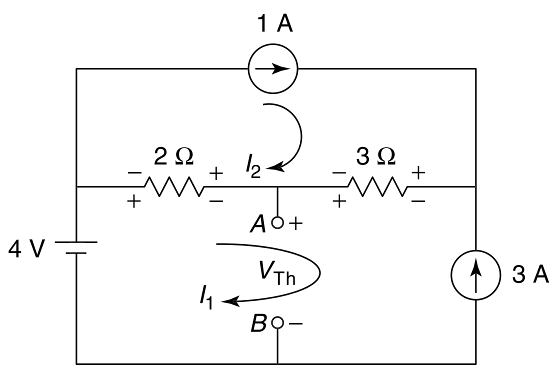


Fig. 3.190

Writing the current equations for Meshes 1 and 2,

$$I_1 = -3$$
$$I_2 = 1$$

Writing the V_{Th} equation,

$$4 - 2(I_1 - I_2) - V_{\text{Th}} = 0$$
$$V_{\text{Th}} = 4 - 2(I_1 - I_2) = 4 - 2(-4) = 12\text{ V}$$

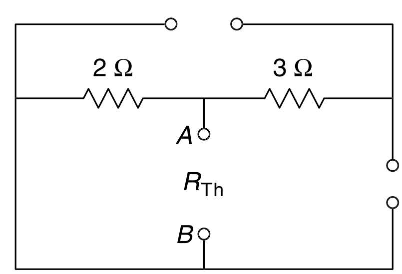


Fig. 3.191

Step II Calculation of R_{Th} (Fig. 3.191)

$$R_{\text{Th}} = 2\ \Omega$$

Step III Calculation of I_L (Fig. 3.192)

$$I_L = \frac{12}{2+1} = 4\text{ A}$$

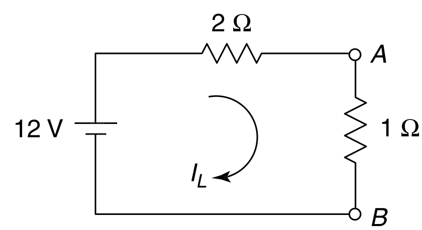


Fig. 3.192

Example 3.44 *Find the current through the 3 Ω resistor in Fig. 3.193.*

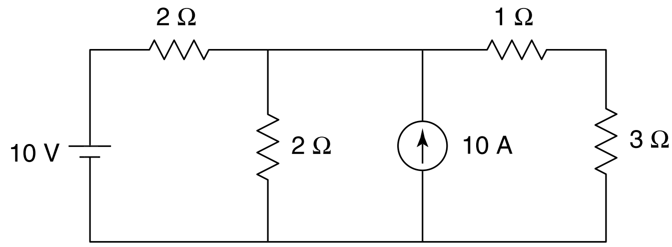


Fig. 3.193

Solution

Step I Calculation of V_{Th} (Fig. 3.194)

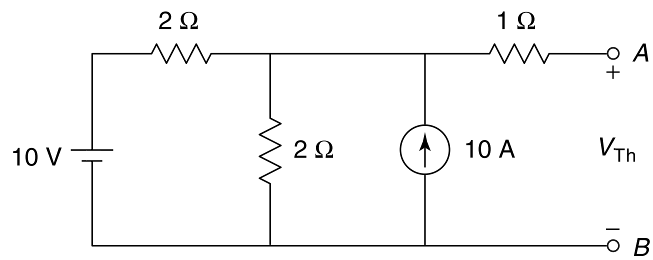


Fig. 3.194

By source transformation (Fig. 3.195),

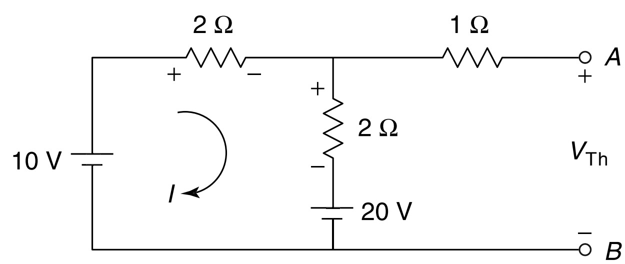


Fig. 3.195

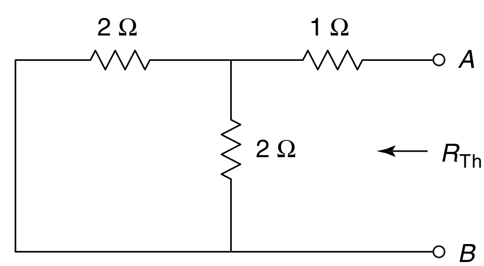


Fig. 3.196

Applying KVL to the mesh,

$$10 - 2I - 2I - 20 = 0$$
$$4I = -10$$
$$I = -2.5\text{ A}$$

Writing the V_{Th} equation,

$$10 - 2I - V_{\text{Th}} = 0$$
$$V_{\text{Th}} = 10 - 2I = 10 - 2(-2.5) = 15\text{ V}$$

Step II Calculation of R_{Th} (Fig. 3.196)

$$R_{\text{Th}} = (2 \parallel 2) + 1 = 1 + 1 = 2\ \Omega$$

Step III Calculation of I_L (Fig. 3.197)

$$I_L = \frac{15}{2+3} = 3\text{ A}$$

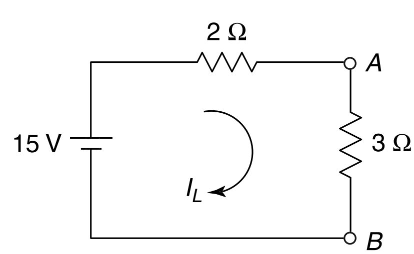


Fig. 3.197

EXAMPLES WITH DEPENDENT SOURCES

Example 3.45 *Obtain the Thevenin equivalent network for the given network of Fig. 3.198 at terminals A and B.*

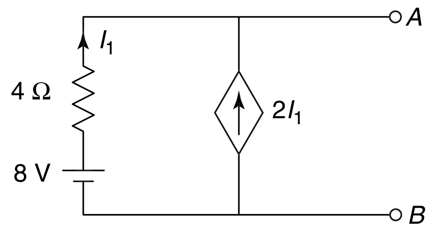


Fig. 3.198

Solution

Step I Calculation of V_{Th} (Fig. 3.199)

From Fig. 3.199,

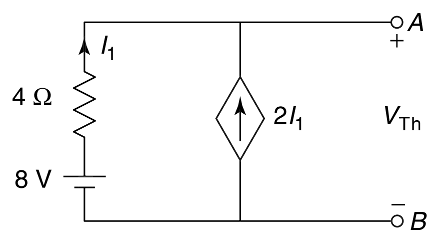


Fig. 3.199

$$I_1 = -2I_1$$
$$3I_1 = 0$$
$$I_1 = 0$$

Writing the V_{Th} equation,

$$8 - 0 - V_{Th} = 0$$
$$V_{Th} = 8\text{ V}$$

Step II Calculation of I_N (Fig. 3.200),

Meshes 1 and 2 will form a supermesh.

Writing current equation for the supermesh,

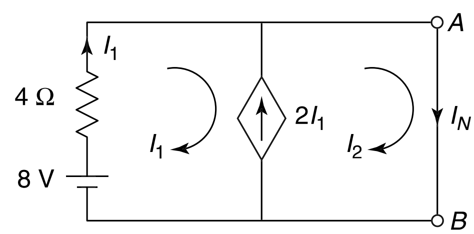


Fig. 3.200

$$I_2 - I_1 = 2I_1$$
$$3I_1 - I_2 = 0 \qquad \text{...(i)}$$

Applying KVL to the outer path of the supermesh,

$$8 - 4I_1 = 0$$
$$I_1 = 2 \qquad \text{...(ii)}$$

Solving Eqs (i) and (ii),

$$I_2 = 6\text{ A}$$
$$I_N = I_2 = 6\text{ A}$$

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{8}{6} = 1.33\ \Omega$$

Step IV Thevenin's Equivalent Network (Fig. 3.201)

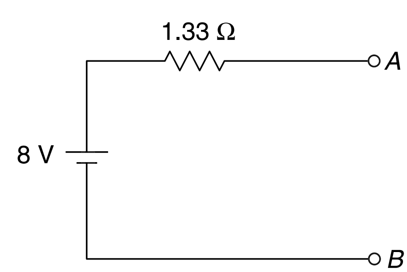


Fig. 3.201

Example 3.46 *Find Thevenin's equivalent network of Fig. 3.202.*

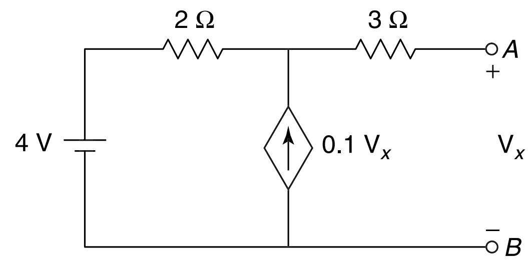


Fig. 3.202

Solution

Step I Calculation of V_{Th} (Fig. 3.203)

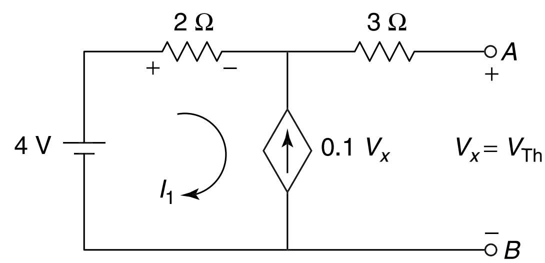


Fig. 3.203

$$V_x = V_{\text{Th}}$$

$$I_1 = -0.1\,V_x$$

Writing the V_{Th} equation,

$$4 - 2I_1 - V_x = 0$$

$$4 - 2(-0.1V_x) - V_x = 0$$

$$4 - 0.8V_x = 0$$

$$V_x = 5\text{ V}$$

$$V_x = V_{\text{Th}} = 5\text{ V}$$

Step II Calculation of I_N (Fig. 3.204)

From Fig. 3.204,

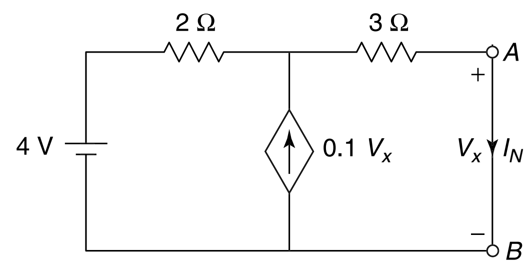


Fig. 3.204

$$V_x = 0$$

The dependent source 0.1 V_x depends on the controlling variable V_x. When $V_x = 0$, the dependent source vanishes, i.e., $0.1V_x = 0$ as shown in Fig. 3.205.

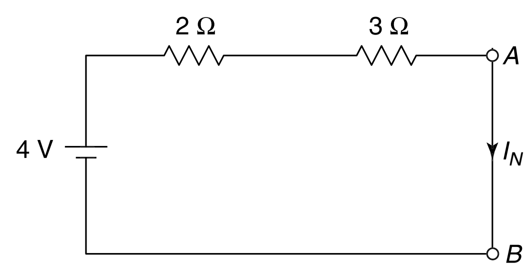


Fig. 3.205

$$I_N = \frac{4}{2+3} = 0.8\text{ A}$$

Step III Calculation of R_{Th}

$$R_{\text{Th}} = \frac{V_{\text{Th}}}{I_N} = \frac{5}{0.8} = 6.25\ \Omega$$

Step IV Thevenin's Equivalent Network (Fig. 3.206)

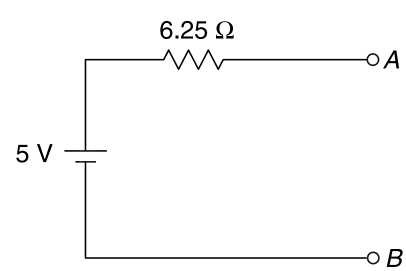


Fig. 3.206

Example 3.47 *Obtain the Thevenin equivalent network of Fig. 3.207 for the terminals A and B.*

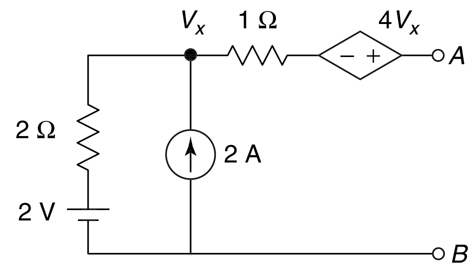


Fig. 3.207

Solution

Step I Calculation of V_{Th} (Fig. 3.208)

From Fig. 3.208,

$$2 - 2I_1 - V_x = 0$$
$$V_x = 2 - 2I_1$$

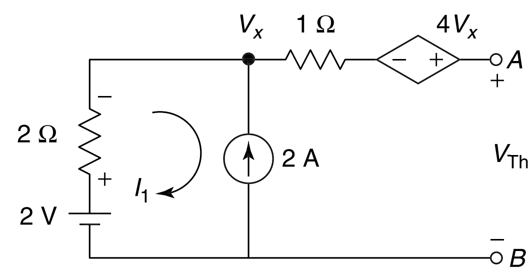


Fig. 3.208

For Mesh 1,

$$I_1 = -2 \text{ A}$$
$$V_x = 2 - 2(-2) = 6 \text{ V}$$

Writing the V_{Th} equation,

$$2 - 2I_1 - 0 + 4V_x - V_{Th} = 0$$
$$2 - 2(-2) - 0 + 4(6) - V_{Th} = 0$$
$$V_{Th} = 30 \text{ V}$$

Step II Calculation of I_N (Fig. 3.209)

From Fig. 3.209,

$$V_x = 2 - 2I_1 \qquad \ldots\text{(i)}$$

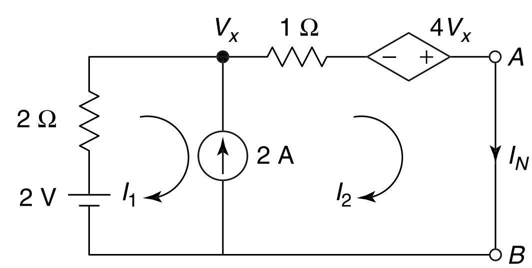


Fig. 3.209

Meshes 1 and 2 will form a supermesh,

Writing current equation for the supermesh

$$I_2 - I_1 = 2 \qquad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$2 - 2I_1 - 1I_2 + 4V_x = 0$$
$$2 - 2I_1 - I_2 + 4(2 - 2I_1) = 0$$
$$10I_1 + I_2 = 10 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 0.73 \text{ A}$$
$$I_2 = 2.73 \text{ A}$$
$$I_N = I_2 = 2.73 \text{ A}$$

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{30}{2.73} = 10.98\ \Omega$$

Step IV Thevenin's Equivalent Network (Fig. 3.210)

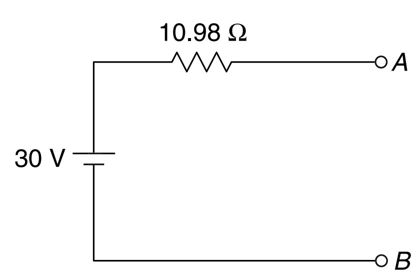


Fig. 3.210

Example 3.48 *Find the Thevenin equivalent network of Fig. 3.211 for the terminals A and B.*

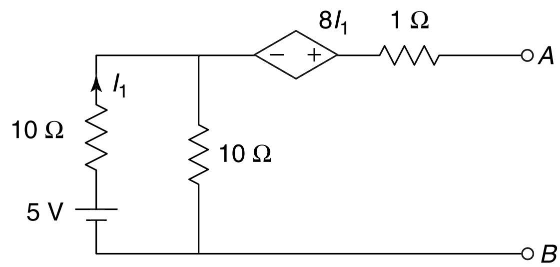


Fig. 3.211

Solution

Step I Calculation of V_{Th} (Fig. 3.212)
Applying KVL to the mesh,

$$5 - 10I_1 - 10I_1 = 0$$

$$I_1 = \frac{5}{20} = 0.25 \text{ A}$$

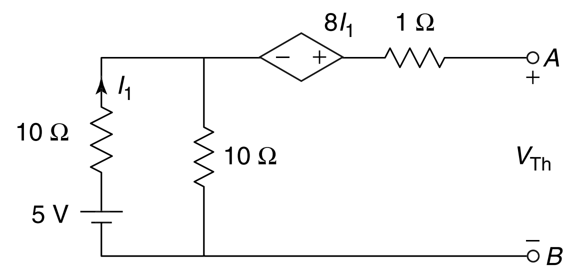


Fig. 3.212

Writing the V_{Th} equation,

$$5 - 10I_1 + 8I_1 - 0 - V_{Th} = 0$$

$$V_{Th} = 5 - 2I_1 = 5 - 2(0.25) = 4.5 \text{ V}$$

Step II Calculation of I_N (Fig. 3.213)
Applying KVL to Mesh 1,

$$5 - 10I_1 - 10(I_1 - I_2) = 0 \qquad \text{...(i)}$$

$$20I_1 - 10I_2 = 5$$

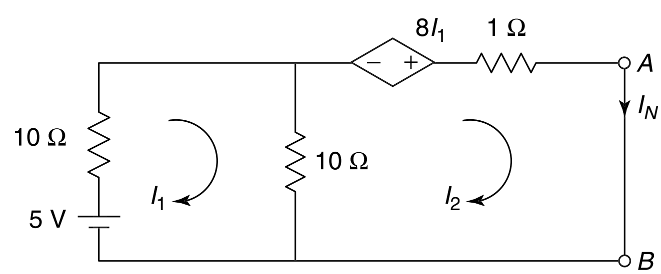


Fig. 3.213

Applying KVL to Mesh 2,

$$-10(I_2 - I_1) + 8I_1 - 1I_2 = 0 \qquad \text{...(ii)}$$

$$18I_1 - 11I_2 = 0$$

Solving Eqs (i) and (ii),

$$I_1 = 1.375 \text{ A}$$

$$I_2 = 2.25 \text{ A}$$

$$I_N = I_2 = 2.25 \text{ A}$$

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{4.5}{2.25} = 2\ \Omega$$

Step IV Thevenin's Equivalent Network (Fig. 3.214)

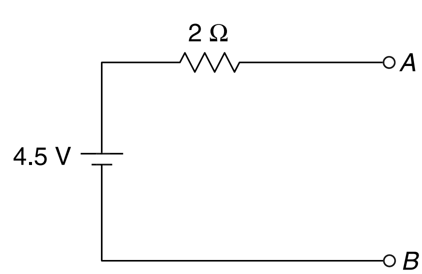


Fig. 3.214

Example 3.49 *Find V_{Th} and R_{Th} between terminals A and B of the network shown in Fig. 3.215 .*

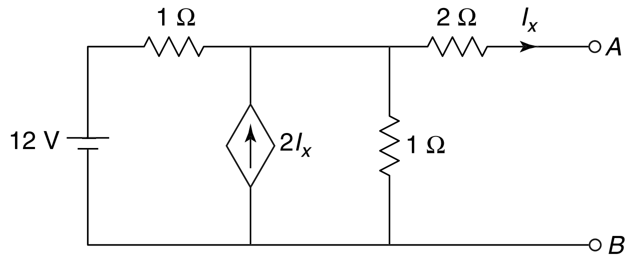


Fig. 3.215

Solution

Step I Calculation of V_{Th} (Fig. 3.216)

$$I_x = 0$$

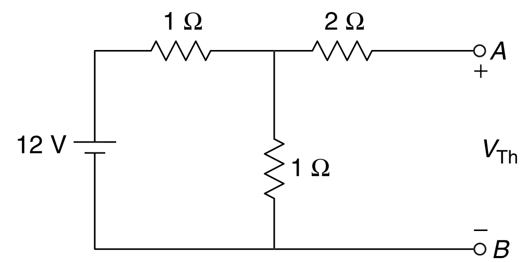


Fig. 3.216

The dependent source $2I_x$ depends on the controlling variable I_x. When $I_x = 0$, the dependent source vanishes, i.e., $2I_x = 0$ as shown in Fig. 3.216.

Writing the V_{Th} equation,

$$V_{\text{Th}} = 12 \times \frac{1}{1+1} = 6 \text{ V}$$

Step II Calculation of I_N (Fig. 3.217)
From Fig. 3.217,

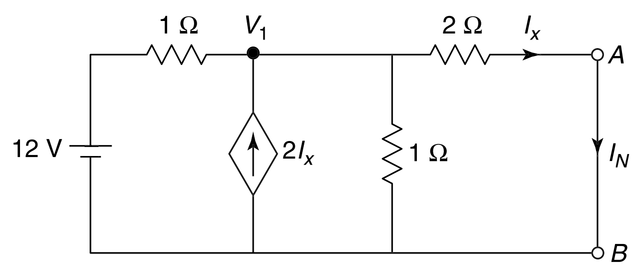


Fig. 3.217

$$I_x = \frac{V_1}{2}$$

Applying KCL at Node 1,

$$\frac{V_1 - 12}{1} + \frac{V_1}{1} + \frac{V_1}{2} = 2I_x$$

$$V_1 + V_1 + \frac{V_1}{2} - 12 = 2\left(\frac{V_1}{2}\right)$$

$$V_1 = 8 \text{ V}$$

$$I_N = \frac{V_1}{2} = \frac{8}{2} = 4 \text{ A}$$

Step III Calculation of R_{Th}

$$R_{\text{Th}} = \frac{V_{\text{Th}}}{I_N} = \frac{6}{4} = 1.5\ \Omega$$

Example 3.50 *Obtain the Thevenin equivalent network of Fig. 3.218 for the given network at terminals a and b .*

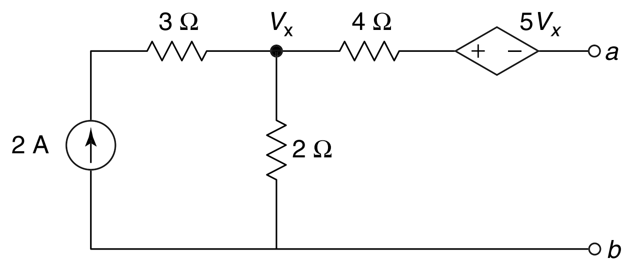


Fig. 3.218

Solution

Step I Calculation of V_{Th} (Fig. 3.219)
Applying KCL at Node x,

$$2 = \frac{V_x}{2}$$

$$V_x = 4 \text{ V}$$

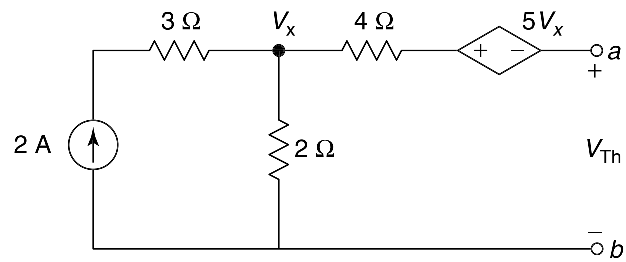


Fig. 3.219

Writing the V_{Th} equation,

$$V_{Th} = V_x - 5\,V_x = -4\,V_x$$

$$= -16 \text{ V} \quad (\text{the terminal } a \text{ is negative w.r.t. } b)$$

Step II Calculation of I_N (Fig. 3.220)
Applying KCL at Node x,

$$2 = \frac{V_x}{2} + \frac{V_x - 5\,V_x}{4}$$

$$2 = \frac{V_x}{2} - V_x = -\frac{V_x}{2}$$

$$V_x = -4 \text{ V}$$

$$I_N = \frac{V_x - 5\,V_x}{4} = -V_x = 4 \text{ A}$$

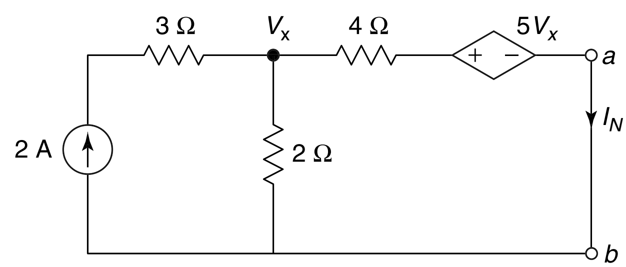


Fig. 3.220

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{-16}{4} = -4\ \Omega$$

Step IV Thevenin's Equivalent Network (Fig. 3.221)

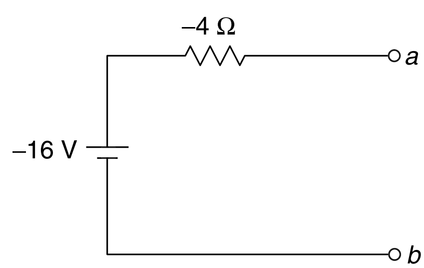


Fig. 3.221

Example 3.51 *Obtain the Thevenin equivalent network of Fig. 3.222 for the given network.*

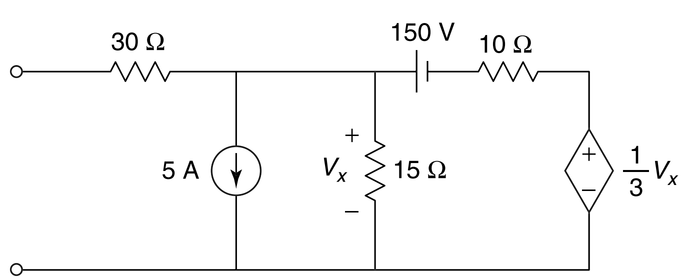


Fig. 3.222

Solution

Step I Calculation of V_{Th} (Fig. 3.223)

From Fig. 3.223

$$V_x = V_{Th}$$

Applying KCL at the node,

$$\frac{V_x - 150 - \frac{1}{3}V_x}{10} + \frac{V_x}{15} + 5 = 0$$

$$V_x = 75 \text{ V}$$

$$V_{Th} = 75 \text{ V}$$

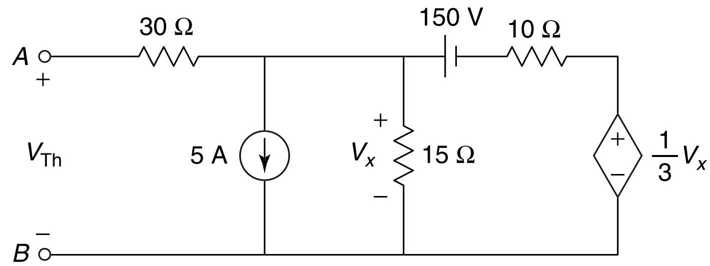


Fig. 3.223

Step II Calculation of I_N (Fig. 3.224)

Applying KCL at Node x,

$$\frac{V_x}{30} + 5 + \frac{V_x}{15} + \frac{V_x - 150 - \frac{1}{3}V_x}{10} = 0$$

$$\frac{V_x}{30} + \frac{V_x}{15} + \frac{V_x}{10} - \frac{V_x}{30} = 15 - 5$$

$$V_x = 60 \text{ V}$$

$$I_N = \frac{V_x}{30} = \frac{60}{30} = 2 \text{ A}$$

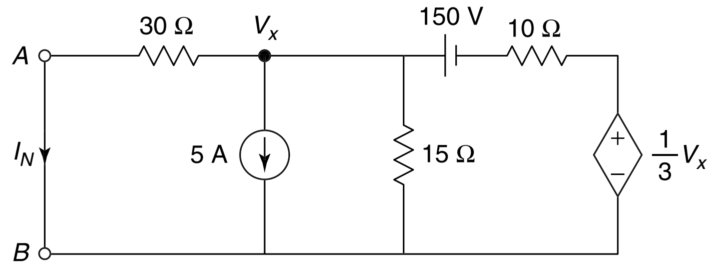


Fig. 3.224

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{75}{2} = 37.5\ \Omega$$

Step IV Thevenin's Equivalent Network (Fig. 3.225)

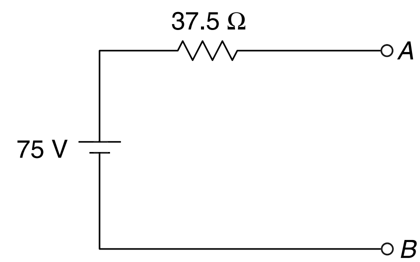


Fig. 3.225

Example 3.52 *Find the Thevenin's equivalent network of the network to the left of A-B in the Fig. 3.226.*

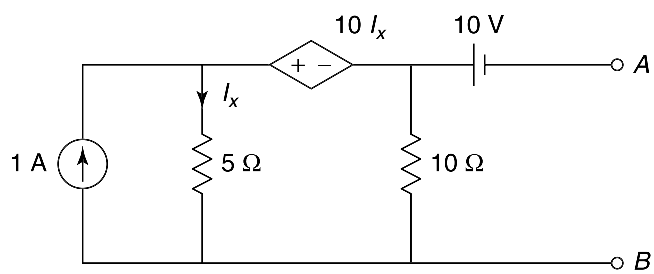


Fig. 3.226

Solution

Step I Calculation of V_{Th} (Fig. 3.227)

From Fig. 3.227,

$$I_x = I_1 - I_2 \qquad \ldots\text{(i)}$$

For Mesh 1,

$$I_1 = 1 \qquad \ldots\text{(ii)}$$

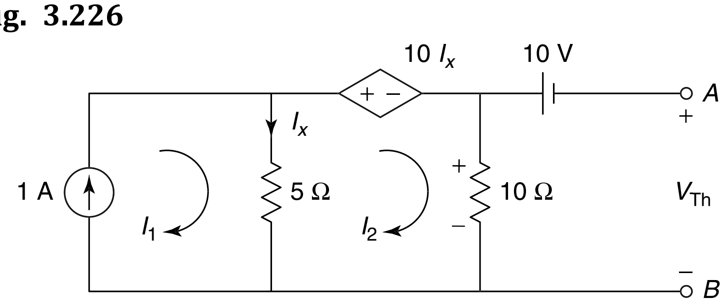


Fig. 3.227

Applying KVL to Mesh 2,

$$-5(I_2 - I_1) - 10I_x - 10I_2 = 0$$

$$-5(I_2 - I_1) - 10(I_1 - I_2) - 10I_2 = 0$$

$$5I_1 + 5I_2 = 0 \qquad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 1 \text{ A}$$

$$I_2 = -1 \text{ A}$$

$$I_x = I_1 - I_2 = 1 - (-1) = 2 \text{ A}$$

Writing the V_{Th} equation,

$$10I_2 - 10 - V_{\text{Th}} = 0$$

$$10(-1) - 10 - V_{\text{Th}} = 0$$

$$V_{\text{Th}} = -20 \text{ V}$$

Step II Calculation of I_N (Fig. 3.228)
From Fig. 3.228,

$$I_x = I_1 - I_2 \qquad \text{...(i)}$$

For Mesh 1,

$$I_1 = 1 \qquad \text{...(ii)}$$

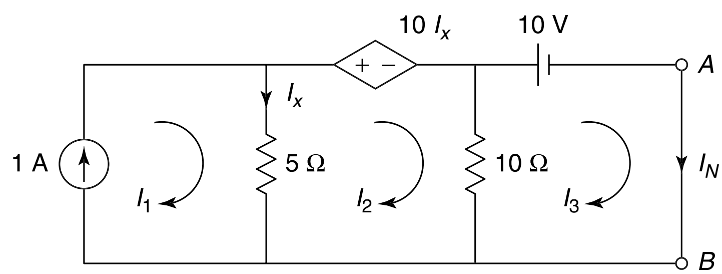


Fig. 3.228

Applying KVL to Mesh 2,

$$-5(I_2 - I_1) - 10I_x - 10(I_2 - I_3) = 0$$

$$-5(I_2 - I_1) - 10(I_1 - I_2) - 10(I_2 - I_3) = 0$$

$$-5I_1 - 5I_2 + 10I_3 = 0 \qquad \text{...(iii)}$$

Applying KVL to Mesh 3,

$$-10(I_3 - I_2) - 10 = 0$$

$$10I_2 - 10I_3 = 10 \qquad \text{...(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = 1 \text{ A}$$

$$I_2 = 3 \text{ A}$$

$$I_3 = 2 \text{ A}$$

$$I_N = I_3 = 2 \text{ A}$$

Step III Calculation of R_{Th}

$$R_{\text{Th}} = \frac{V_{\text{Th}}}{I_N} = \frac{-20}{2} = -10\ \Omega$$

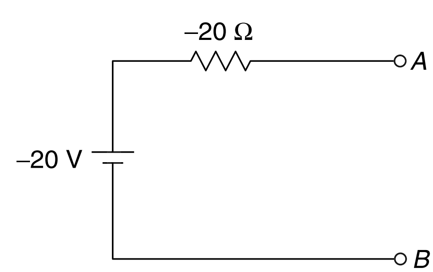


Fig. 3.229

Step IV Thevenin's Equivalent Network (Fig. 3.229)

Example 3.53 *Find Thevenin's equivalent network at terminals A and B in the network of Fig. 3.230.*

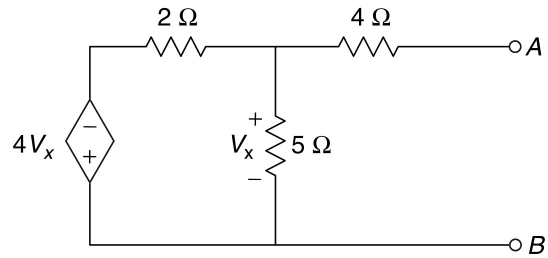


Fig. 3.230

Solution

Since the network does not contain any independent source,

$$V_{\text{Th}} = 0$$

$$I_N = 0$$

But the R_{Th} can be calculated by applying a known voltage source of 1 V at the terminals A and B as shown in Fig. 3.231.

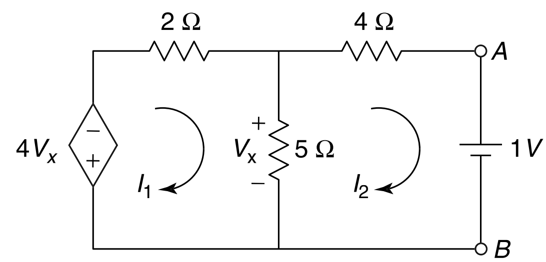


Fig. 3.231

$$R_{\text{Th}} = \frac{V}{I} = \frac{1}{I}$$

From Fig. 3.231,

$$V_x = 5(I_1 - I_2) \qquad ...(i)$$

Applying KVL to Mesh 1,

$$-4V_x - 2I_1 - 5(I_1 - I_2) = 0$$

$$-4[5(I_1 - I_2)] - 2I_1 - 5I_1 + 5I_2 = 0$$

$$-27I_1 + 25I_2 = 0 \qquad ...(ii)$$

Applying KVL to Mesh 2,

$$-5(I_2 - I_1) - 4I_2 - 1 = 0$$

$$5I_1 - 9I_2 = 1 \qquad ...(iii)$$

Solving Eqs (ii) and (iii),

$$I_1 = -0.21 \text{ A}$$

$$I_2 = -0.23 \text{ A}$$

Hence, current supplied by voltage source of 1 V is 0.23 A.

$$R_{\text{Th}} = \frac{1}{0.23} = 4.35\ \Omega$$

Hence, Thevenin's equivalent network is shown in Fig. 3.232.

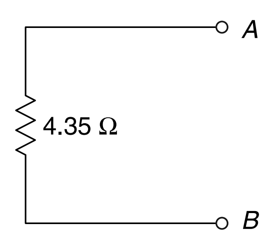


Fig. 3.232

Example 3.54 *Find the current in the 9 Ω resistor in Fig. 3.233.*

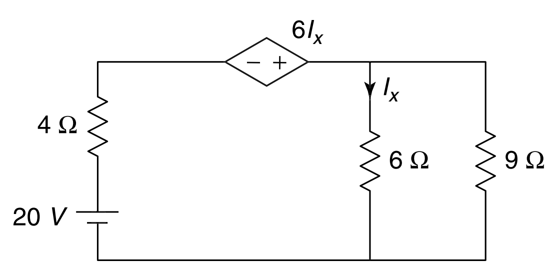


Fig. 3.233

Solution

Step I Calculation of V_{Th} (Fig. 3.234)
Applying KVL to the mesh,

$$20 - 4I_x + 6I_x - 6I_x = 0$$
$$I_x = 5 \text{ A}$$

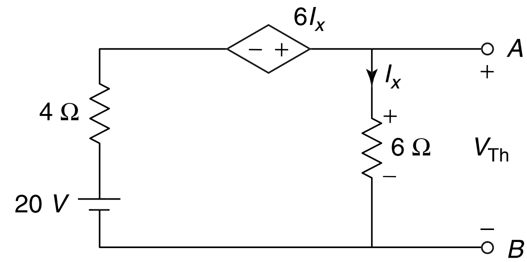


Fig. 3.234

Writing the V_{Th} equation,

$$6I_x - V_{\text{Th}} = 0$$
$$6(5) - V_{\text{Th}} = 0$$
$$V_{\text{Th}} = 30 \text{ V}$$

Step II Calculation of I_N (Fig. 3.235).
From Fig. 3.235,

$$I_x = 0$$

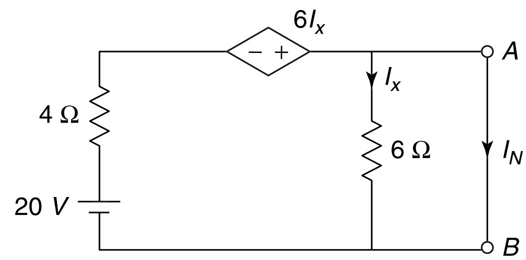


Fig. 3.235

The dependent source $6I_x$ depends on the controlling variable I_x. When $I_x = 0$, the dependent source vanishes, i.e., $6I_x = 0$ as shown in Fig. 3.236.

$$I_N = \frac{20}{4} = 5 \text{ A}$$

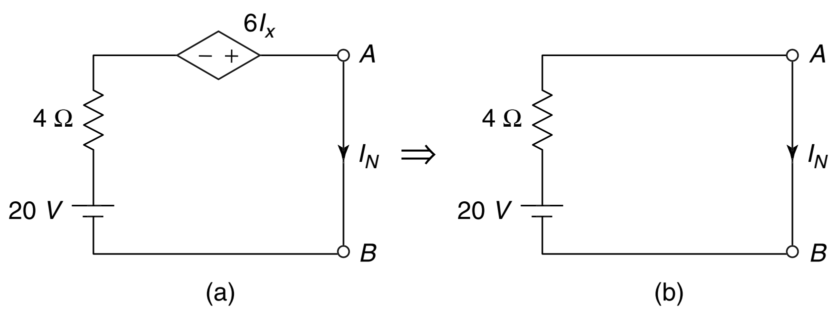


Fig. 3.236

Step III Calculation of R_{Th}

$$R_{\text{Th}} = \frac{V_{\text{Th}}}{I_N} = \frac{30}{5} = 6\ \Omega$$

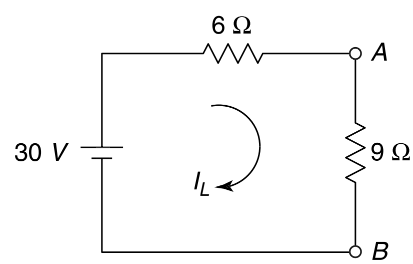


Fig. 3.237

Step IV Calculation of I_L (Fig. 3.237)

$$I_L = \frac{30}{6+9} = 2 \text{ A}$$

Example 3.55 *Determine the current in the 16 Ω resistor in Fig. 3.238.*

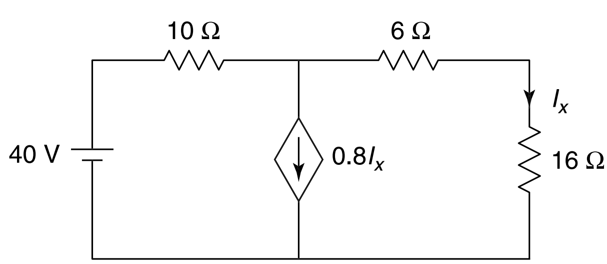


Fig 3.238

Solution

Step I Calculation of V_{Th} (Fig. 3.239)

From Fig. 3.239,

$$I_x = 0$$

The dependent source $0.8I_x$ depends on the controlling variable I_x. When $I_x = 0$, the dependent source vanishes, as shown in Fig. 3.240.

i.e.,
$$0.8I_x = 0$$
$$V_{Th} = 40 \text{ V}$$

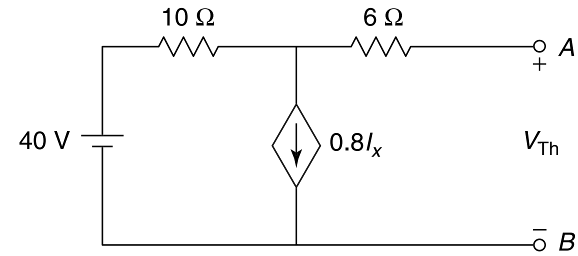


Fig. 3.239

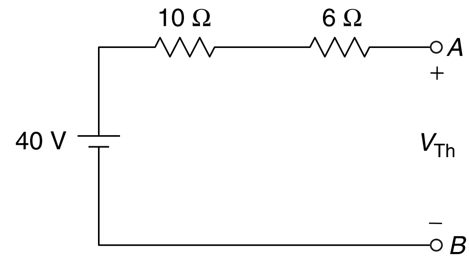


Fig. 3.240

Step II Calculation of I_N (Fig. 3.241)

From Fig. 3.241,

$$I_x = I_2 \quad \ldots\text{(i)}$$

Meshes 1 and 2 will form a supermesh,

Writing current equation for the supermesh,

$$I_1 - I_2 = 0.8\, I_x = 0.8\, I_2$$
$$I_1 - 1.8\, I_2 = 0 \quad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$40 - 10\, I_1 - 6\, I_2 = 0$$
$$10\, I_1 + 6\, I_2 = 40 \quad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 3 \text{ A}$$
$$I_2 = \frac{5}{3} \text{ A}$$
$$I_N = I_2 = \frac{5}{3} \text{ A}$$

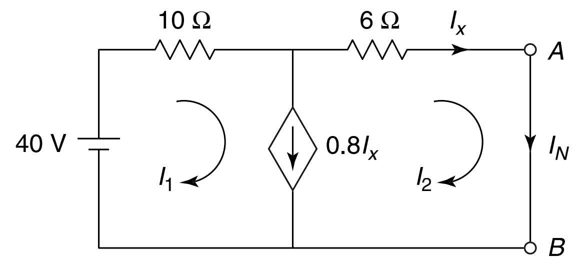


Fig. 3.241

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{40}{\frac{5}{3}} = 24\ \Omega$$

Step IV Calculation of I_L (Fig. 3.242)

$$I_L = \frac{40}{24 + 16} = 1 \text{ A}$$

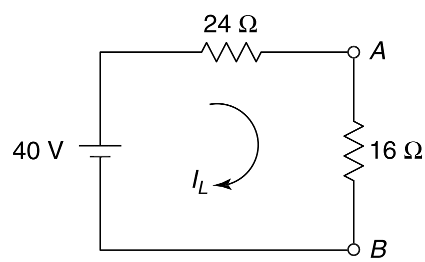


Fig. 3.242

Example 3.56 *Find the current in the 6 Ω resistor in Fig. 3.243.*

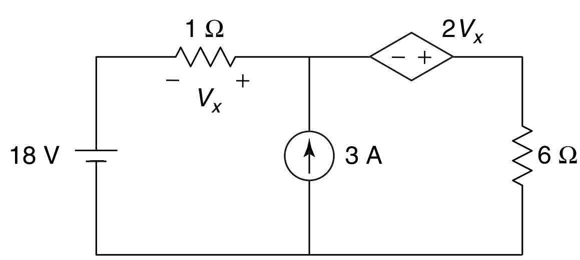


Fig. 3.243

Solution

Step I Calculation of V_{Th} (Fig. 3.244)

From Fig. 3.244,

$$V_x = -1I_1 = -I_1 \qquad \text{...(i)}$$

For Mesh 1,

$$I_1 = -3 \text{ A} \qquad \text{...(ii)}$$

$$V_x = 3 \text{ V}$$

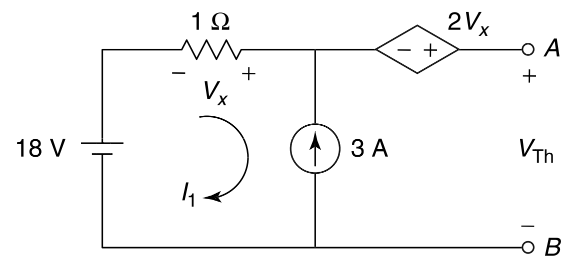


Fig. 3.244

Writing the V_{Th} equation,

$$18 - 1\,I_1 + 2\,V_x - V_{Th} = 0$$

$$18 + 3 + 2(3) - V_{Th} = 0$$

$$V_{Th} = 27 \text{ V}$$

Step II Calculation of I_N (Fig. 3.245)

From Fig. 3.245,

$$V_x = -I_1 \qquad \text{...(i)}$$

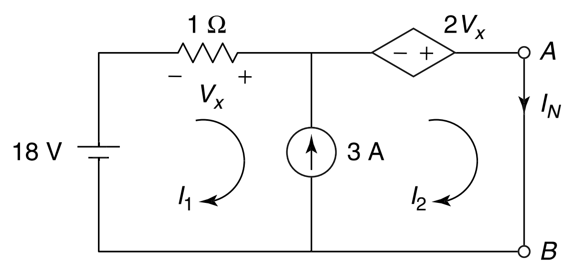


Fig. 3.245

Meshes 1 and 2 will form a supermesh,

Writing current equation for supermesh,

$$I_2 - I_1 = 3 \qquad \text{...(ii)}$$

Applying KVL to the outer path of the supermesh,

$$18 - 1I_1 + 2\,V_x = 0$$

$$18 - I_1 + 2(-I_1) = 0 \qquad \text{...(iii)}$$

$$I_1 = 6 \text{ A}$$

Solving Eqs (ii) and (iii),

$$I_2 = 9 \text{ A}$$

$$I_N = I_2 = 9 \text{ A}$$

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{27}{9} = 3\ \Omega$$

Step IV Calculation of I_L (Fig. 3.246)

$$I_L = \frac{27}{3+6} = 3 \text{ A}$$

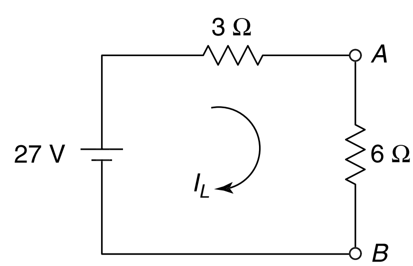


Fig. 3.246

Example 3.57 *Find the current in the 10 Ω resistor.*

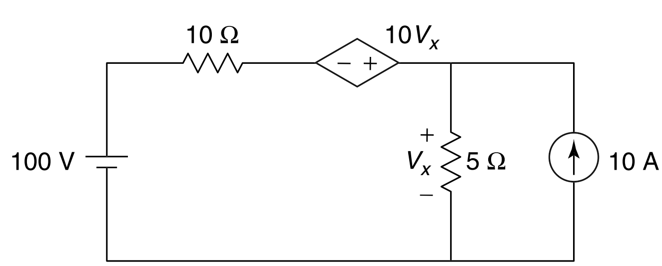


Fig. 3.247

Solution

Step I Calculation of V_{Th} (Fig. 3.248)

From the figure,

$$V_x = 10 \times 5 = 50\text{ V}$$

Writing the V_{Th} equation,

$$100 - V_{Th} + 10V_x - V_x = 0$$
$$100 - V_{Th} + 9V_x = 0$$
$$100 - V_{Th} + 9(50) = 0$$
$$V_{Th} = 550\text{ V}$$

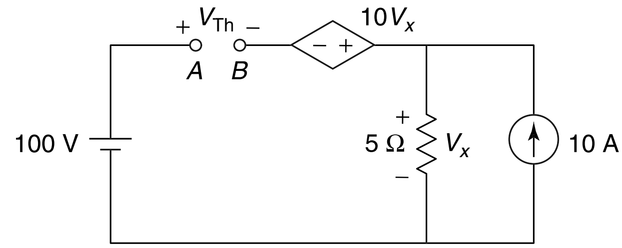


Fig. 3.248

Step II Calculation of I_N (Fig. 3.249)

From Fig. 3.249,

$$V_x = 5(I_N + 10)$$

Applying KVL to Mesh 1,

$$100 + 10V_x - V_x = 0$$
$$V_x = -\frac{100}{9}$$
$$-\frac{100}{9} = 5I_N + 50$$
$$I_N = -\frac{550}{45}\text{ A}$$

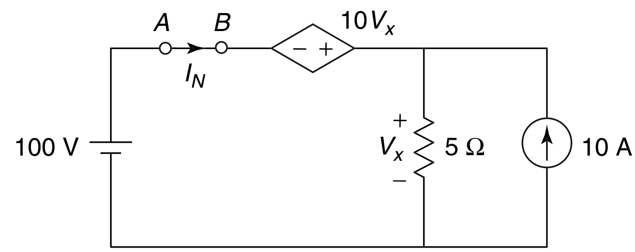


Fig. 3.249

Step III Calculation of R_{Th}

$$R_{Th} = \frac{550}{-\frac{550}{45}} = -45\ \Omega$$

Step IV Calculation of I_L (Fig. 3.250)

$$I_L = \frac{550}{-45+10} = -\frac{110}{7}\text{ A}$$

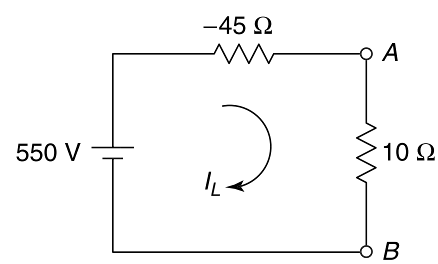


Fig. 3.250

3.4 NORTON'S THEOREM

It states that '*any two terminals of a network can be replaced by an equivalent current source and an equivalent parallel resistance.*' The constant current is equal to the current which would flow in a short circuit placed across the terminals. The parallel resistance is the resistance of the network when viewed from these open-circuited terminals after all voltage and current sources have been removed and replaced by internal resistances.

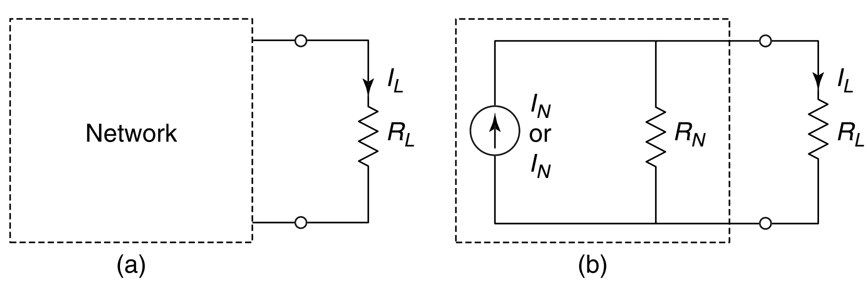


Fig. 3.251 *Network illustrating Norton's theorem*

Explanation Consider a simple network as shown in Fig.3.252

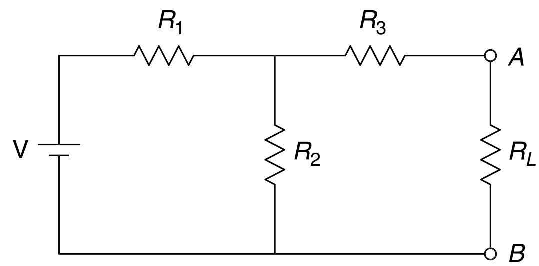


Fig. 3.252 *Network*

For finding load current through R_L, first remove the load resistor R_L from the network and calculate short circuit current I_{SC} or I_N which would flow in a short circuit placed across terminals A and B as shown in Fig. 3.253.

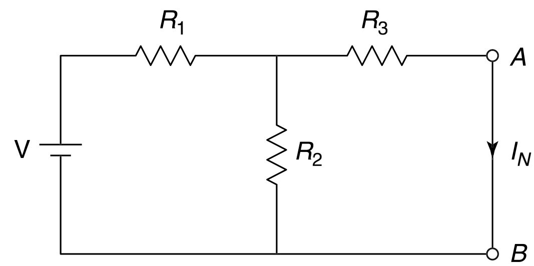


Fig. 3.253 *Calculation of I_N*

For finding parallel resistance R_N, replace the voltage source by a short circuit and calculate resistance between points *A* and *B* as shown in Fig. 3.254.

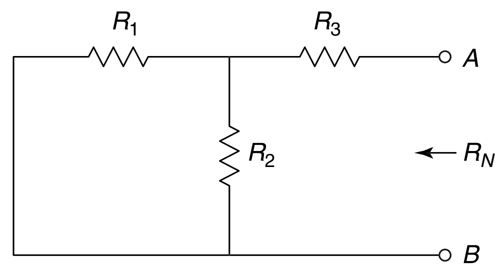


Fig. 3.254 *Calculation of R_N*

$$R_N = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

Norton's equivalent network is shown in Fig.3.255.

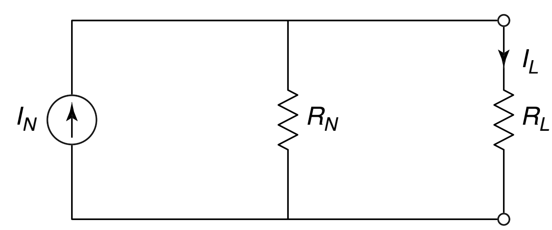


Fig. 3.255 *Norton's equivalent network*

$$I_L = I_N \frac{R_N}{R_N + R_L}$$

If the network contains both independent and dependent sources, Norton's resistances R_N is calculated as

$$R_N = \frac{V_{\text{Th}}}{I_N}$$

where V_{Th} is the open-circuit voltage across terminals *A* and *B*. If the network contains only dependent sources, then

$$V_{\text{Th}} = 0$$

$$I_N = 0$$

To find R_{Th} in such network, a known voltage *V* or current *I* is applied across the terminals *A* and *B*, and the current *I* or the voltage *V* is calculated respectively.

$$R_N = \frac{V}{I}$$

Norton's equivalent network for such a network is shown in Fig. 3.256.

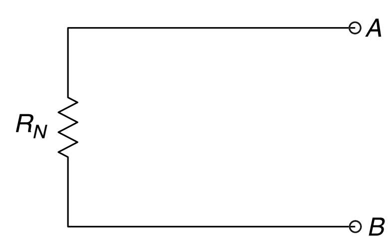


Fig. 3.256 *Norton's equivalent network*

Steps to be followed in Norton's Theorem

1. Remove the load resistance R_L and put a short circuit across the terminals.
2. Find the short-circuit current I_{SC} or I_N.
3. Find the resistance R_N as seen from points A and B.
4. Replace the network by a current source I_N in parallel with resistance R_N.
5. Find current through R_L by current–division rule.

$$I_L = \frac{I_N R_N}{R_N + R_L}$$

Example 3.58 *Find the current through the 10 Ω resistor in Fig. 3.257.*

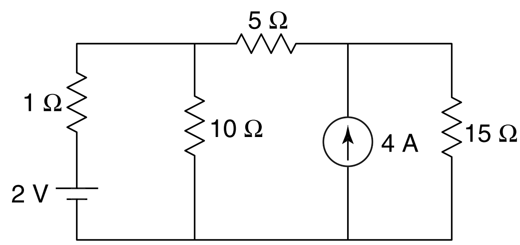


Fig. 3.257

Solution

Step I Calculation of I_N (Fig. 3.258)

Applying KVL to Mesh 1,

$$2 - 1I_1 = 0$$

$$I_1 = 2 \qquad \text{...(i)}$$

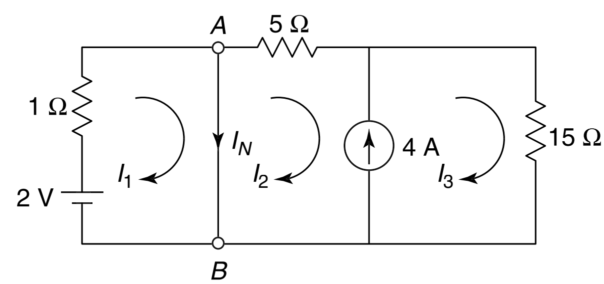


Fig. 3.258

Meshes 2 and 3 will form a supermesh.

Writing the current equation for the supermesh,

$$I_3 - I_2 = 4 \qquad \text{...(ii)}$$

Applying KVL to the supermesh,

$$-5I_2 - 15I_3 = 0 \qquad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1 = 2\text{ A}$$

$$I_2 = -3\text{ A}$$

$$I_3 = 1\text{ A}$$

$$I_N = I_1 - I_2 = 2 - (-3) = 5\text{ A}$$

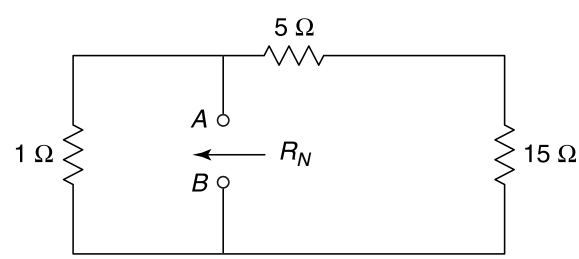


Fig. 3.259

Step II Calculation of R_N (Fig. 3.259)

$$R_N = 1 \,||\, (5+15) = 0.95\ \Omega$$

Step III Calculation of I_L (Fig.3.260)

$$I_L = 5 \times \frac{0.95}{0.95+10} = 0.43\text{ A}$$

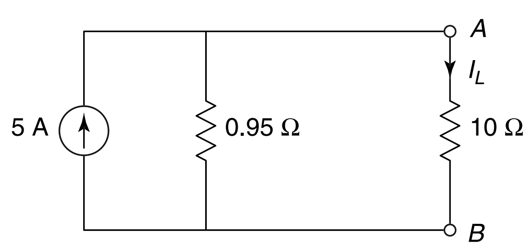


Fig. 3.260

Example 3.59

Find the current through the 10 Ω resistor in Fig. 3.258.

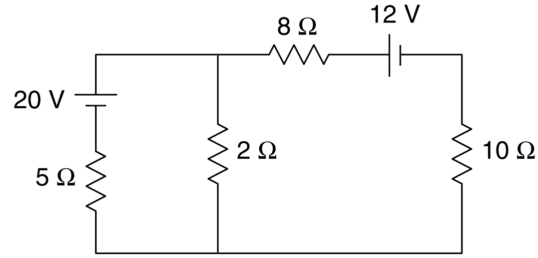


Fig. 3.261

Solution

Step I Calculation of I_N (Fig. 3.262)

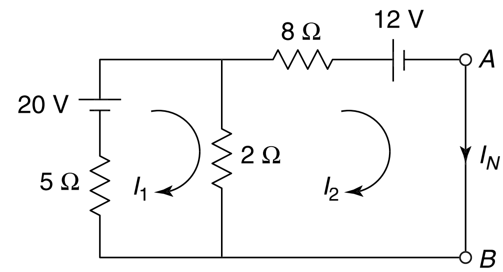


Fig. 3.262

Applying KVL to Mesh 1,

$$-5I_1 + 20 - 2(I_1 - I_2) = 0$$
$$7I_1 - 2I_2 = 20 \quad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-2(I_2 - I_1) - 8I_2 - 12 = 0$$
$$-2I_1 + 10I_2 = -12 \quad \text{...(ii)}$$

Solving Eqs (i) and (ii),

$$I_2 = -0.67\text{ A}$$
$$I_N = I_2 = -0.67\text{ A}$$

Step II Calculation of R_N (Fig. 3.263)

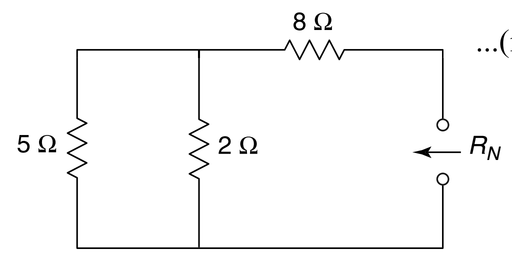


Fig. 3.263

$$R_N = (5 \,||\, 2) + 8 = 9.43\ \Omega$$

Step III Calculation of I_L (Fig. 3.264)

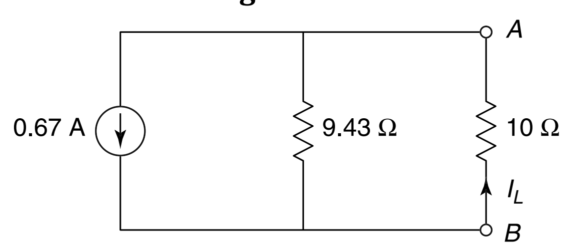


Fig. 3.264

$$I_L = 0.67 \times \frac{9.43}{9.43 + 10} = 0.33\text{ A}(\uparrow)$$

Example 3.60

Find the current in the 10 Ω resistor in Fig. 3.265.

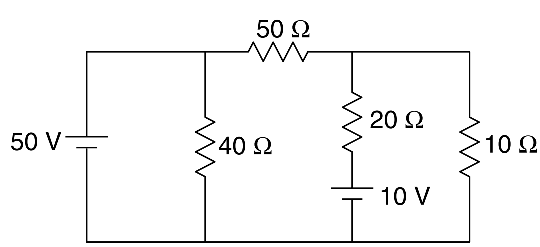


Fig. 3.265

Solution

Step I Calculation of I_N (Fig. 3.266)

The resistance of 40 Ω becomes redundant as it is connected across the 50 V source (Fig. 3.267).

Applying KVL to Mesh 1,

$$50 - 50I_1 - 20(I_1 - I_2) - 10 = 0$$
$$70I_1 - 20I_2 = 40 \quad \text{...(i)}$$

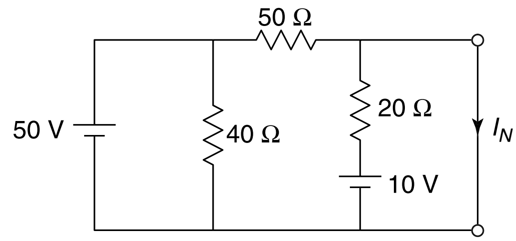


Fig. 3.266

Applying KVL to Mesh 2,

$$10 - 20(I_2 - I_1) = 0$$
$$-20I_1 + 20I_2 = 0 \quad \text{...(ii)}$$

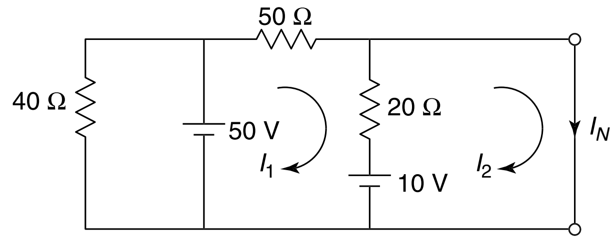


Fig. 3.267

Solving Eqs (i) and (ii),

$$I_1 = 1\,\text{A}$$
$$I_2 = 1.5\,\text{A}$$
$$I_N = I_2 = 1.5\,\text{A}$$

Step II Calculation of R_N (Fig. 3.268)

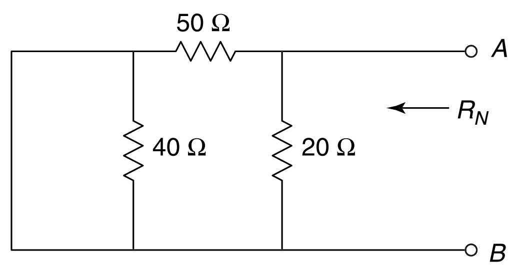


Fig. 3.268

$$R_N = 50 \,||\, 20 = 14.28\,\Omega$$

Step III Calculation of I_L (Fig. 3.269)

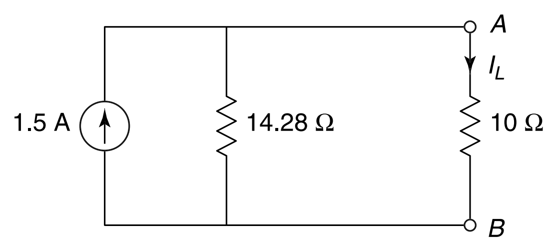


Fig. 3.269

$$I_L = 1.5 \times \frac{14.28}{14.28 + 10} = 0.88\,\text{A}$$

Example 3.61 *Find the current through the 10 Ω resistor in Fig. 3.270.*

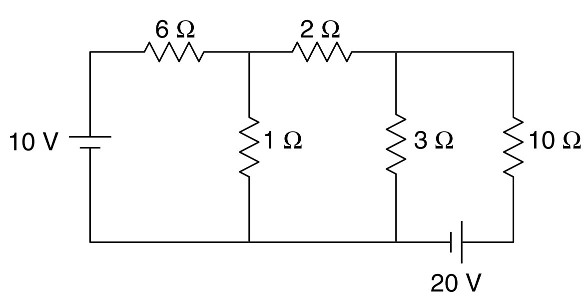


Fig. 3.270

Solution

Step I Calculation of I_N (Fig. 3.271)

Applying KVL to Mesh 1,

$$10 - 6I_1 - 1(I_1 - I_2) = 0$$
$$7I_1 - I_2 = 10 \quad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-1(I_2 - I_1) - 2I_2 - 3(I_2 - I_3) = 0$$
$$-I_1 + 6I_2 - 3I_3 = 0 \quad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-3(I_3 - I_2) - 20 = 0$$
$$3I_2 - 3I_3 = 20 \quad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_3 = -13.17 \text{ A}$$
$$I_N = I_3 = -13.17 \text{ A}$$

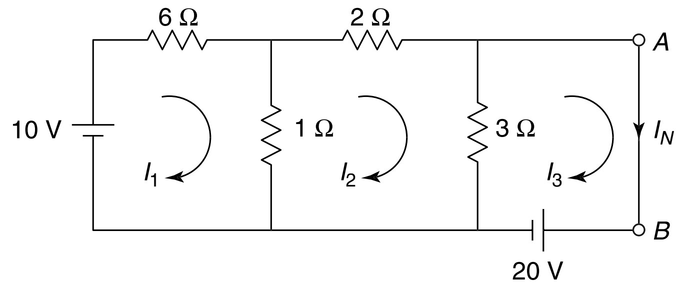


Fig. 3.271

Step II Calculation of R_N (Fig. 3.272)

$$R_N = [(6 \parallel 1) + 2] \parallel 3 = 1.46\ \Omega$$

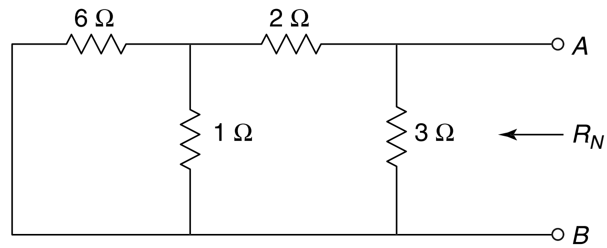


Fig. 3.272

Step III Calculation of I_L (Fig. 3.273)

$$I_L = 13.17 \times \frac{1.46}{1.46 + 10} = 1.68 \text{ A} (\uparrow)$$

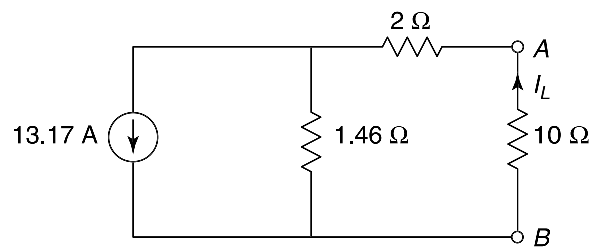


Fig. 3.273

Example 3.62 *Find the current through the 10 Ω resistor in Fig. 3.274.*

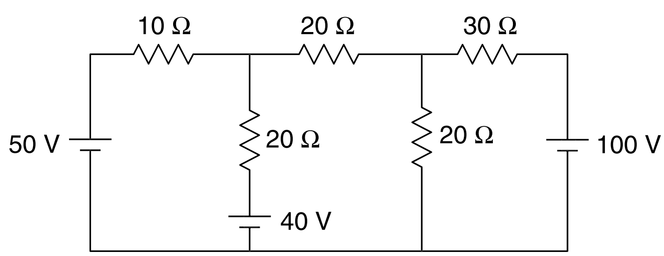


Fig. 3.274

Solution

Step I Calculation of I_N (Fig. 3.274)

Applying KVL to Mesh 1,

$$50 - 20(I_1 - I_2) - 40 = 0$$
$$20I_1 - 20I_2 = 10 \quad \text{...(i)}$$

Applying KVL to Mesh 2,

$$40 - 20(I_2 - I_1) - 20I_2 - 20(I_2 - I_3) = 0$$
$$-20I_1 + 60I_2 - 20I_3 = 40 \quad \text{...(ii)}$$

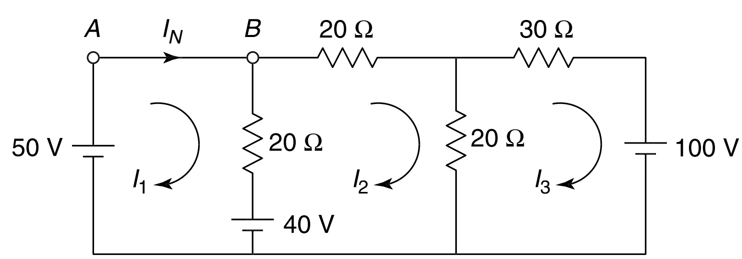


Fig. 3.275

Applying KVL to Mesh 3,

$$-20(I_3 - I_2) - 30I_3 - 100 = 0$$

$$-20I_2 + 50I_3 = -100 \qquad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1 = 0.81\,\text{A}$$

$$I_N = I_1 = 0.81\,\text{A}$$

Step II Calculation of R_N (Fig. 3.276)

$$R_N = [(20 \,||\, 30) + 20] \,||\, 20 = 12.3\,\Omega$$

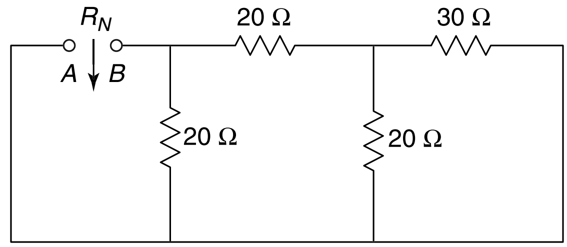


Fig. 3.276

Step III Calculation of I_L (Fig. 3.277)

$$I_L = 0.81 \times \frac{12.3}{12.3 + 10} = 0.45\,\text{A}$$

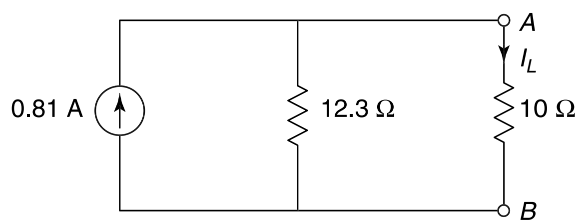


Fig. 3.277

Example 3.63 *Obtain Norton's equivalent network as seen by R_L in Fig. 3.278.*

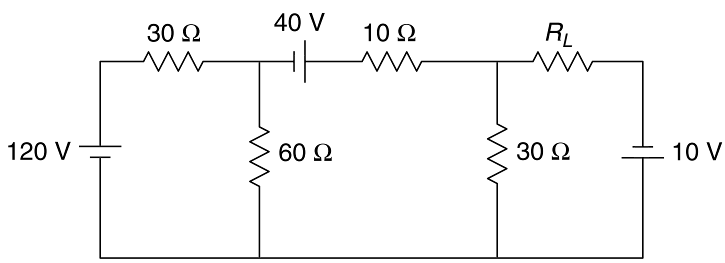


Fig. 3.278

Solution

Step I Calculation of I_N (Fig. 3.279)

Applying KVL to Mesh 1,

$$120 - 30I_1 - 60(I_1 - I_2) = 0$$

$$90I_1 - 60I_2 = 120 \qquad \text{...(i)}$$

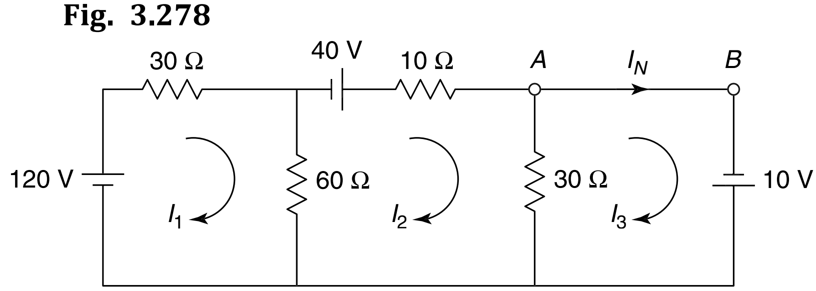


Fig. 3.279

Applying KVL to Mesh 2,

$$-60(I_2 - I_1) + 40 - 10I_2 - 30(I_2 - I_3) = 0$$

$$-60I_1 + 100I_2 - 30I_3 = 40 \qquad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-30(I_3 - I_2) + 10 = 0$$

$$30I_2 - 30I_3 = -10 \qquad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_3 = 4.67\text{ A}$$

$$I_N = I_3 = 4.67\text{ A}$$

Step II Calculation of R_N (Fig. 3.280)

$$R_N = [(30 \,||\, 60) + 10] \,||\, 30 = 15\,\Omega$$

Step III Norton's equivalent network (Fig. 3.281)

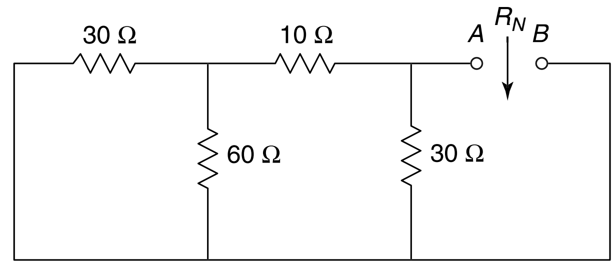


Fig. 3.280

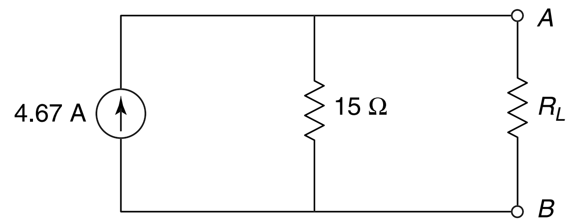


Fig. 3.281

Example 3.64 *Find the current through the 8 Ω resistor in Fig. 3.282.*

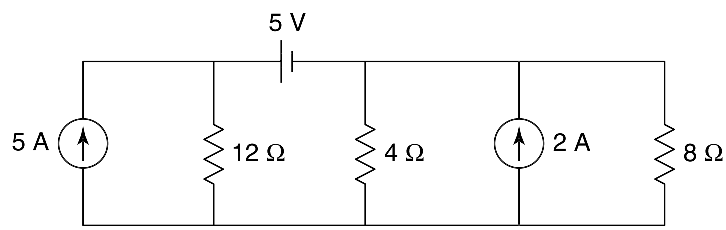


Fig. 3.282

Solution

Step I Calculation of I_N (Fig. 3.283)

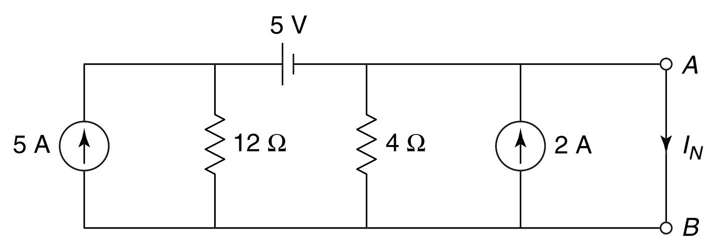


Fig. 3.283

The resistor of the 4 Ω gets shorted as it is in parallel with the short circuit. Simplifying the network by source transformation (Fig. 3.284),

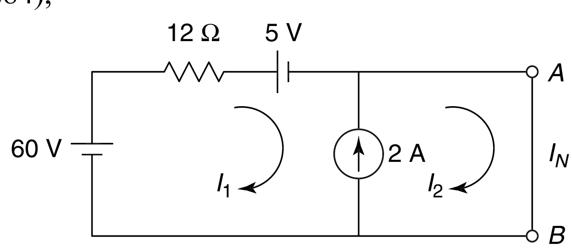


Fig. 3.284

Meshes 1 and 2 will form a supermesh.
Writing the current equation for the supermesh,

$$I_2 - I_1 = 2 \quad \text{...(i)}$$

Applying KVL to the supermesh,

$$60 - 12I_1 - 5 = 0$$
$$12I_1 = 55 \quad \text{...(ii)}$$

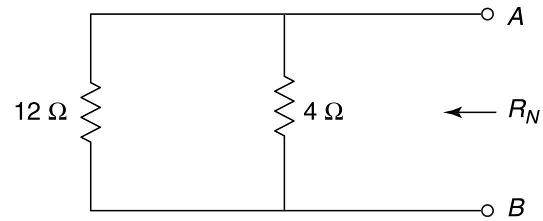


Fig. 3.285

Solving Eqs (i) and (ii),

$$I_1 = 4.58\,\text{A}$$
$$I_2 = 6.58\,\text{A}$$
$$I_N = I_2 = 6.58\,\text{A}$$

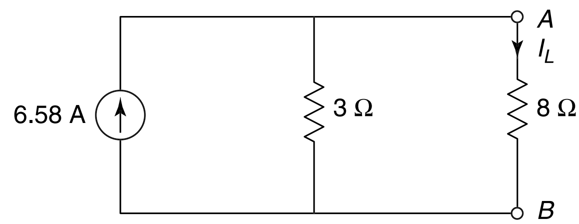


Fig. 3.286

Step II Calculation of R_N (Fig. 3.285)

$$R_N = 12 \,||\, 4 = 3\,\Omega$$

Step III Calculation of I_L (Fig. 3.286)

$$I_L = 6.58 \times \frac{3}{3+8} = 1.79\,\text{A}$$

Example 3.65 *Find the current through the 1 Ω resistor in Fig. 3.287.*

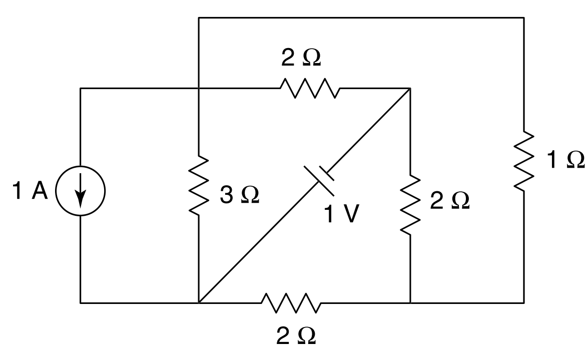


Fig. 3.287

Solution

Step I Calculation of I_N (Fig. 3.288)

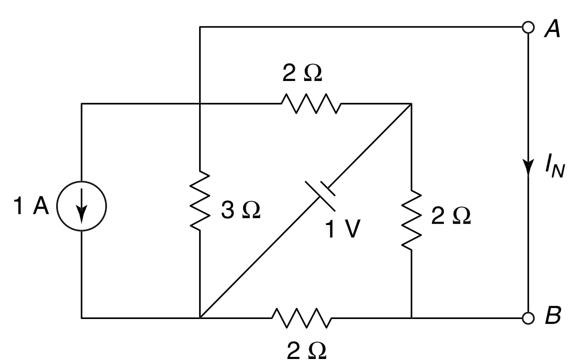


Fig. 3.288

By source transformation (Fig. 3.289),

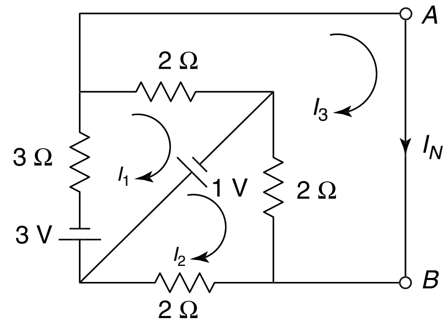


Fig. 3.289

Applying KVL to Mesh 1,

$$-3-3I_1-2(I_1-I_3)+1=0$$
$$5I_1-2I_3=-2 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-1-2(I_2-I_3)-2I_2=0$$
$$4I_2-2I_3=-1 \qquad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-2(I_3-I_1)-2(I_3-I_2)=0$$
$$-2I_1-2I_2+4I_3=0 \qquad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1=-0.64\text{ A}$$
$$I_2=-0.55\text{ A}$$
$$I_3=-0.59\text{ A}$$
$$I_N=I_3=-0.59\text{ A}$$

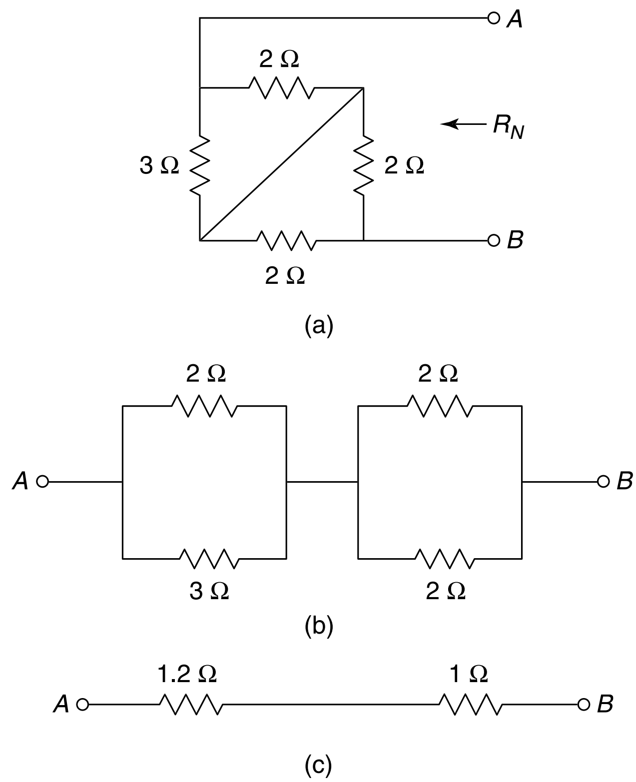


Fig. 3.290

Step II Calculation of R_N (Fig. 3.290)

$$R_N=2.2\ \Omega$$

Step III Calculation of I_L (Fig. 3.291)

$$I_L=0.59\times\frac{2.2}{2.2+1}=0.41\text{ A}$$

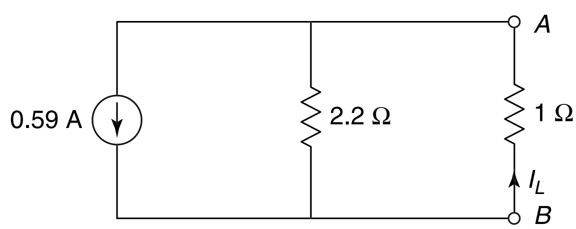


Fig. 3.291

EXAMPLES WITH DEPENDENT SOURCES

Example 3.66 *Find Norton's equivalent network across terminals A and B of Fig. 3.292.*

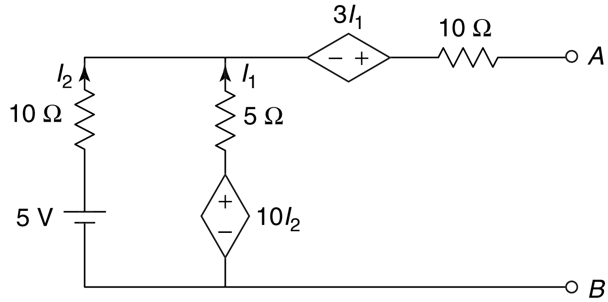


Fig. 3.292

Solution

Step I Calculation of V_{Th} (Fig. 3.293)

From Fig. 3.293,

$$I_2 = I_x$$
$$I_1 = -I_x$$

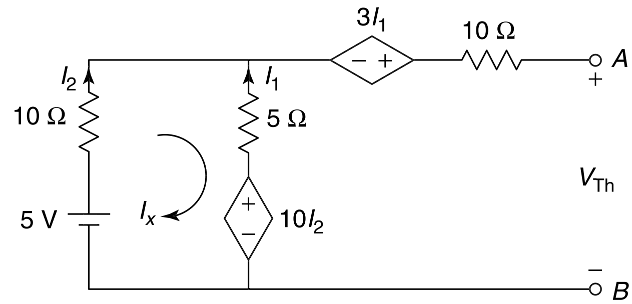


Fig. 3.293

Applying KVL to the mesh,

$$5 - 10I_x - 5I_x - 10I_2 = 0$$
$$5 - 10I_x - 5I_x - 10I_x = 0$$
$$I_x = 0.2 \text{ A}$$
$$I_1 = -0.2 \text{ A}$$

Writing the V_{Th} equation,

$$5 - 10I_x + 3I_1 - V_{Th} = 0$$
$$5 - 10(0.2) + 3(-0.2) - V_{Th} = 0$$
$$V_{Th} = 2.4 \text{ V}$$

Step II Calculation of I_N (Fig. 3.294)

From Fig. 3.294,

$$I_2 = I_x \quad \ldots\text{(i)}$$
$$I_1 = I_y - I_x \quad \ldots\text{(ii)}$$

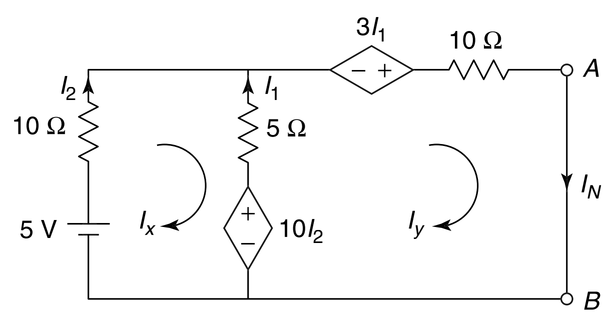


Fig. 3.294

Applying KVL to Mesh 1,

$$5 - 10I_x - 5(I_x - I_y) - 10I_2 = 0$$
$$5 - 10I_x - 5I_x + 5I_y - 10I_x = 0$$
$$25I_x - 5I_y = 5 \quad \ldots\text{(iii)}$$

Applying KVL to Mesh 2,

$$10I_2 - 5(I_y - I_x) + 3I_1 - 10I_y = 0$$
$$10I_x - 5I_y + 5I_x + 3(I_y - I_x) - 10I_y = 0$$
$$12I_x - 12I_y = 0 \quad \ldots\text{(iv)}$$

Solving Eqs (iii) and (iv),

$$I_x = 0.25 \text{ A}$$
$$I_y = 0.25 \text{ A}$$
$$I_N = I_y = 0.25 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{\text{Th}}}{I_N} = \frac{2.4}{0.25} = 9.6\ \Omega$$

Step IV Norton's Equivalent Network (Fig. 3.295)

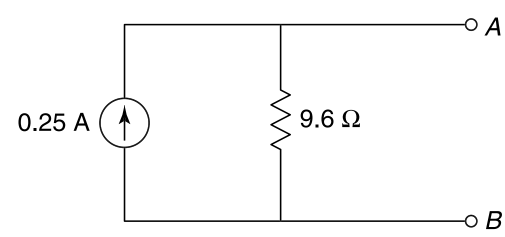


Fig. 3.295

Example 3.67 *For the network shown in Fig. 3.296, find Norton's equivalent network.*

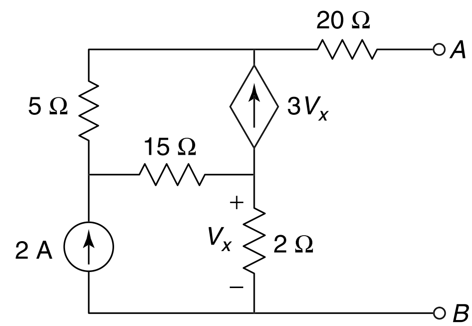


Fig. 3.296

Solution

Step I Calculation of V_{Th} (Fig. 3.297)
From Fig. 3.297,

$$V_x = 2I_2 \qquad \text{...(i)}$$

For Mesh 1,

$$I_1 = -3V_x = -3(2I_2) = -6I_2 \qquad \text{...(ii)}$$

For Mesh 2,

$$I_2 = 2 \qquad \text{...(iii)}$$

$$I_1 = -6I_2 = -6(2) = -12 \text{ A}$$

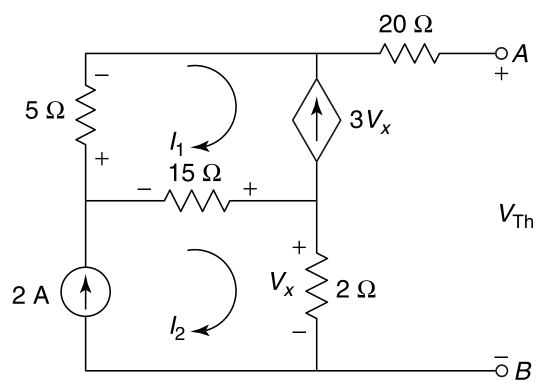


Fig. 3.297

Writing the V_{Th} equation,

$$V_{\text{Th}} - 0 + 5I_1 + 15(I_1 - I_2) - 2I_2 = 0$$
$$V_{\text{Th}} + 5(-12) + 15(-12-2) - 2(2) = 0$$
$$V_{\text{Th}} = 274 \text{ V}$$

Step II Calculation of I_N (Fig. 3.298)
From Fig. 3.298,

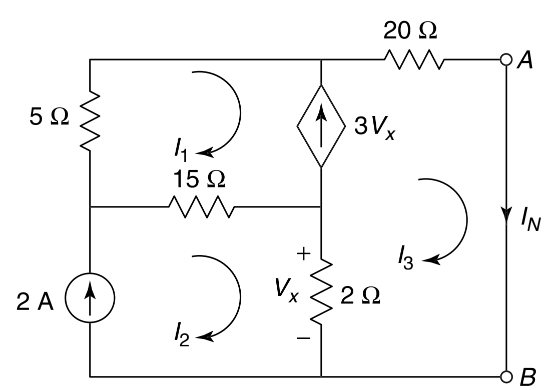


Fig. 3.298

$$V_x = 2(I_2 - I_3) \qquad \text{...(i)}$$

For Mesh 2,

$$I_2 = 2 \qquad \text{...(ii)}$$

Meshes 1 and 3 will form a supermesh.
Writing the current equation for the supermesh,

$$I_3 - I_1 = 3V_x = 3[2(I_2 - I_3)] = 6I_2 - 6I_3$$

$$I_1 + 6I_2 - 7I_3 = 0 \qquad \ldots\text{(iii)}$$

Applying KVL to the outer path of the supermesh,

$$-5I_1 - 20I_3 - 2(I_3 - I_2) - 15(I_1 - I_2) = 0$$

$$-20I_1 + 17I_2 - 22I_3 = 0 \qquad \ldots\text{(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = -0.16 \text{ A}$$
$$I_2 = 2 \text{ A}$$
$$I_3 = 1.69 \text{ A}$$
$$I_N = I_3 = 1.69 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{Th}}{I_N} = \frac{274}{1.69} = 162.13\ \Omega$$

Step IV Norton's Equivalent Network (Fig. 3.299)

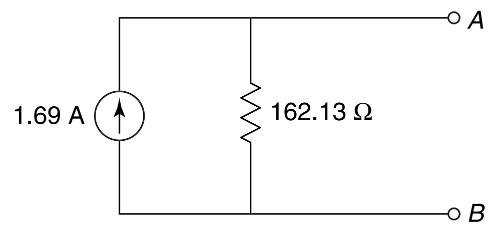


Fig. 3.299

Example 3.68 *Obtain Norton's equivalent network across A-B in the network of Fig. 3.300.*

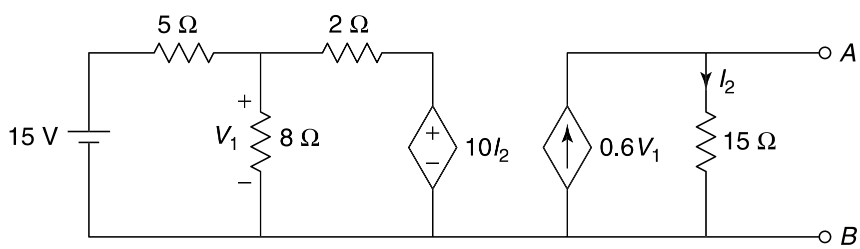


Fig. 3.300

Solution

Step I Calculation of V_{Th} (Fig. 3.301)
From Fig. 3.301,

$$V_1 = 8(I_x - I_y) \qquad \ldots\text{(i)}$$

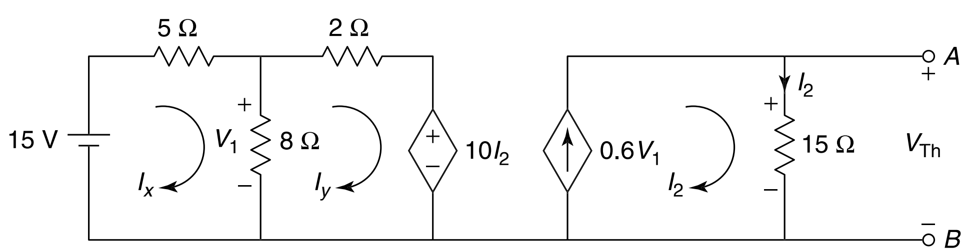


Fig. 3.301

Applying KVL to Mesh 1,

$$15 - 5I_x - 8(I_x - I_y) = 0$$

$$13I_x - 8I_y = 15 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 2,

$$-8(I_y - I_x) - 2I_y - 10I_2 = 0$$

$$8I_x - 10I_y - 10I_2 = 0 \qquad \ldots\text{(iii)}$$

For Mesh 3,

$$I_2 = 0.6V_1 = 0.6\left[8(I_x - I_y)\right]$$

$$4.8I_x - 4.8I_y - I_2 = 0 \qquad \text{...(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_x = 3.28 \text{ A}$$
$$I_y = 3.45 \text{ A}$$
$$I_2 = -0.83 \text{ A}$$

Writing the V_{Th} equation,

$$15I_2 - V_{Th} = 0$$
$$15(-0.83) - V_{Th} = 0$$
$$V_{Th} = -12.45 \text{ V}$$

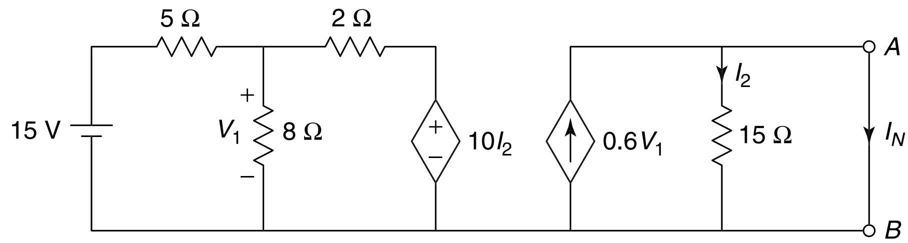


Fig. 3.302

Step II Calculation of I_N (Fig. 3.302)
From Fig. 3.302,

$$I_2 = 0$$

The dependent source of 10 I_2 depends on the controlling variable I_2. When $I_2 = 0$, the dependent source vanishes, i.e. $10I_2 = 0$ as shown in Fig. 3.303.

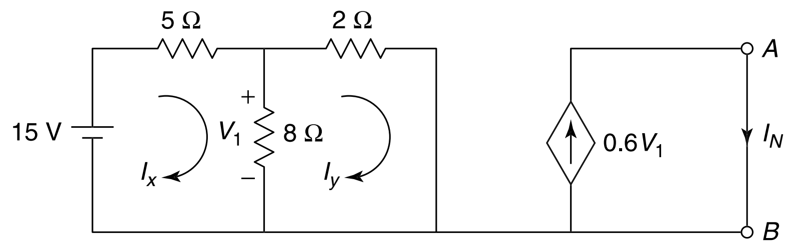


Fig. 3.303

From Fig. 3.303,

$$V_1 = 8(I_x - I_y) \qquad \text{...(i)}$$

Applying KVL to Mesh 1,

$$15 - 5I_x - 8(I_x - I_y) = 0$$
$$13I_x - 8I_y = 15 \qquad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$-8(I_y - I_x) - 2I_y = 0$$
$$-8I_x + 10I_y = 0 \qquad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_x = 2.27 \text{ A}$$
$$I_y = 1.82 \text{ A}$$
$$V_1 = 8(I_x - I_y) = 8(2.27 - 1.82) = 3.6 \text{ V}$$

For Mesh 3,

$$I_N = 0.6V_1 = 0.6(3.6) = 2.16 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{Th}}{I_N} = \frac{-12.45}{2.16} = -5.76\ \Omega$$

Step IV Norton's Equivalent Network (Fig. 3.304)

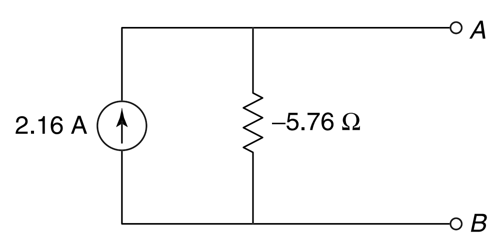


Fig. 3.304

Example 3.69 *Find Norton's equivalent network of Fig. 3.305.*

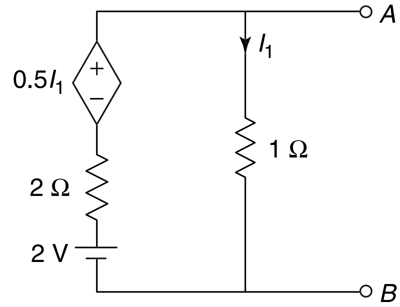


Fig. 3.305

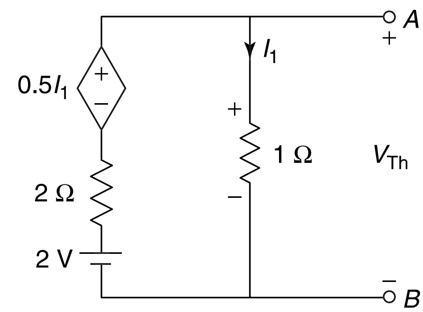


Fig. 3.306

Solution

Step I Calculation of V_{Th} (Fig. 3.306)

Applying KVL to the mesh,

$$2 - 2I_1 + 0.5I_1 - 1I_1 = 0$$

$$2 - 2.5I_1 = 0$$

$$I_1 = 0.8\text{ A}$$

Writing the V_{Th} equation,

$$1I_1 - V_{Th} = 0$$

$$1(0.8) - V_{Th} = 0$$

$$V_{Th} = 0.8\text{ V}$$

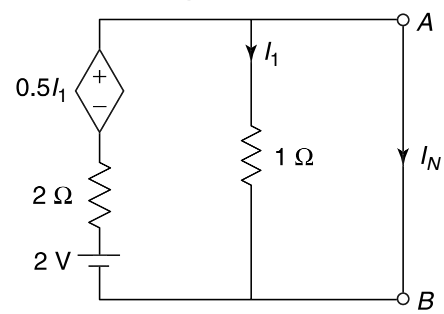


Fig. 3.307

Step II Calculation of I_N (Fig. 3.307)

When a short circuit is placed across the 1 Ω resistor, it gets shorted.

$$I_1 = 0$$

The dependent source of $0.5I_1$ depends on the controlling variable I_1. When $I_1 = 0$, the dependent source vanishes, i.e. $0.5\,I_1 = 0$ as shown in Fig. 3.308.

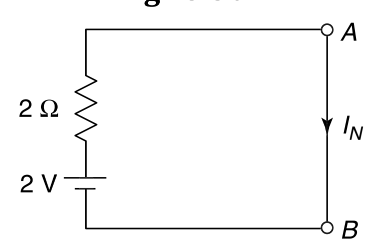


Fig. 3.308

$$I_N = \frac{2}{2} = 1\text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{Th}}{I_N} = \frac{0.8}{1} = 0.8\ \Omega$$

Step IV Norton's Equivalent Network (Fig. 3.309)

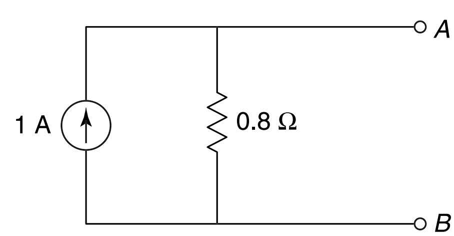


Fig. 3.309

Example 3.70 *Find Norton's equivalent network at the terminals A and B of Fig. 3.310.*

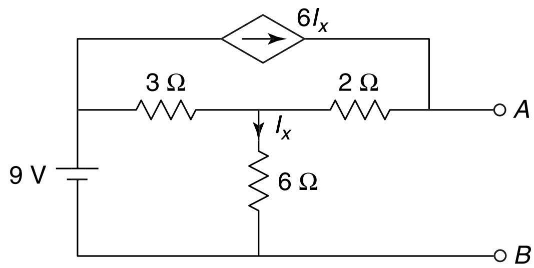


Fig. 3.310

Solution

Step I Calculation of V_{Th} (Fig. 3.311)

From Fig. 3.311,

$$I_x = I_1 \qquad \ldots\text{(i)}$$

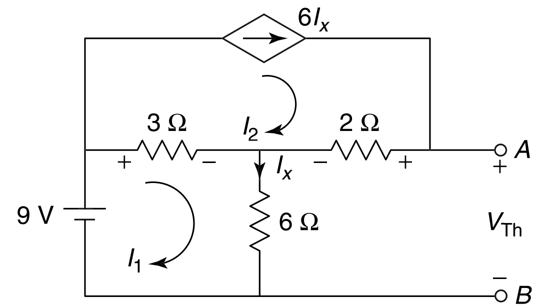


Fig. 3.311

Applying KVL to Mesh 1,

$$9 - 3(I_1 - I_2) - 6I_1 = 0$$
$$9I_1 - 3I_2 = 9 \qquad \ldots\text{(ii)}$$

For Mesh 2,

$$I_2 = 6I_x = 6I_1$$
$$6I_1 - I_2 = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = -1 \text{ A}$$
$$I_2 = -6 \text{ A}$$

Writing the V_{Th} equation,

$$9 - 3(I_1 - I_2) + 2I_2 - V_{\text{Th}} = 0$$
$$9 - 3(-1 + 6) + 2(-6) - V_{\text{Th}} = 0$$
$$V_{\text{Th}} = -18 \text{ V}$$

Step II Calculation of I_N (Fig. 3.312)

From Fig. 3.312,

$$I_x = I_1 - I_3 \qquad \ldots\text{(i)}$$

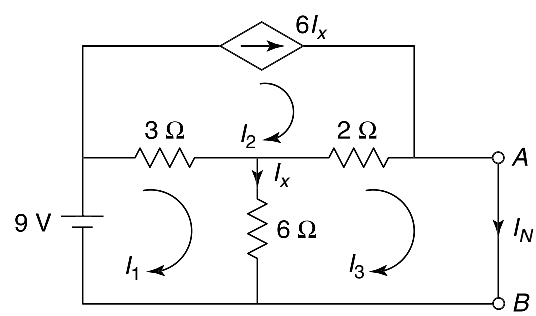


Fig. 3.312

Applying KVL to Mesh 1,

$$9 - 3(I_1 - I_2) - 6(I_1 - I_3) = 0$$
$$9I_1 - 3I_2 - 6I_3 = 9 \qquad \ldots\text{(ii)}$$

For Mesh 2,

$$I_2 = 6I_x = 6(I_1 - I_3)$$
$$6I_1 - I_2 - 6I_3 = 0 \qquad \ldots\text{(iii)}$$

Applying KVL to Mesh 3,

$$-6(I_3 - I_1) - 2(I_3 - I_2) = 0$$
$$-6I_1 - 2I_2 + 8I_3 = 0 \qquad \ldots\text{(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = 5 \text{ A}$$
$$I_2 = 3 \text{ A}$$
$$I_3 = 4.5 \text{ A}$$
$$I_N = I_3 = 4.5 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{\text{Th}}}{I_N} = \frac{-18}{4.5} = -4\ \Omega$$

Step IV Norton's Equivalent Network (Fig. 3.313)

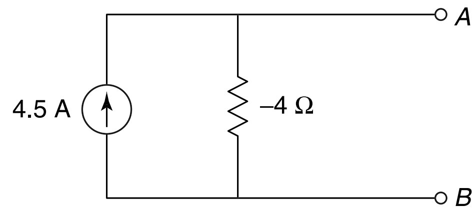


Fig. 3.313

Example 3.71 *Find Norton's equivalent network to the left of terminal A-B in Fig. 3.314.*

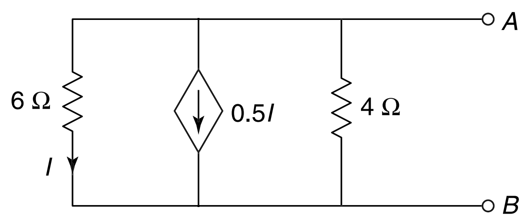


Fig. 3.314

Solution Since the network does not contain any independent source,

$$V_{\text{Th}} = 0$$
$$I_N = 0$$

But R_N can be calculated by applying a known current source of 1 A at the terminals A and B as shown in Fig. 3.315.

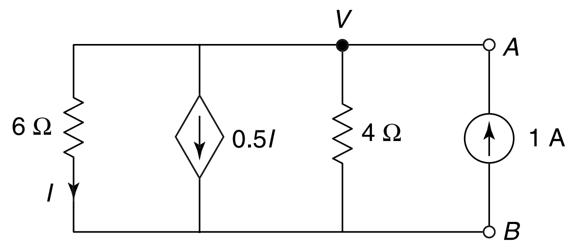


Fig. 3.315

From Fig. 3.315,

$$I = \frac{V}{6}$$

Applying KCL at the node,

$$\frac{V}{6} + 0.5I + \frac{V}{4} = 1$$

$$\frac{V}{6} + 0.5\left(\frac{V}{6}\right) + \frac{V}{4} = 1$$

$$\left(\frac{1}{6} + \frac{0.5}{6} + \frac{1}{4}\right)V = 1$$

$$V = 2$$

$$R_N = \frac{V}{1} = \frac{2}{1} = 2\ \Omega$$

Hence, Norton's equivalent network is shown in Fig. 3.316.

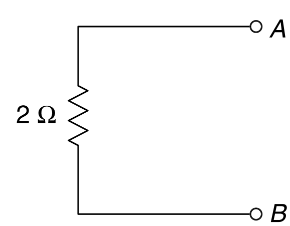


Fig. 3.316

Example 3.72 *Find the current through the 2 Ω resistor in the network shown in Fig. 3.317.*

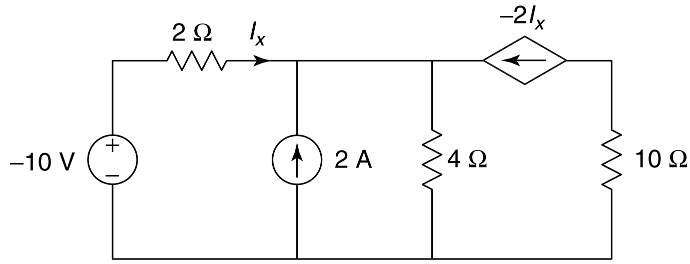


Fig. 3.317

Solution

Step I Calculation of V_{Th} (Fig. 3.318)

From Fig. 3.318,

$$I_x = 0$$

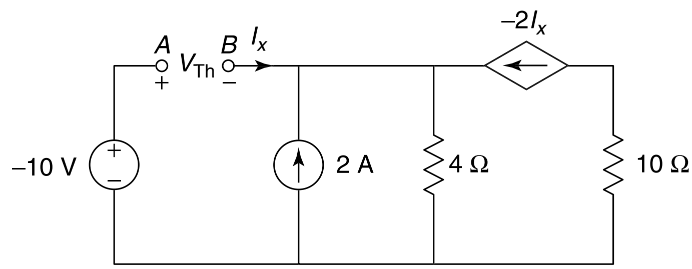


Fig. 3.318

The dependent source of $-2\,I_x$ depends on the controlling variable I_x. When $I_x = 0$, the dependent source vanishes, i.e. $-2I_x = 0$ as shown in Fig. 3.319.

$$I_1 = 2$$

Writing the V_{Th} equation,

$$-10 - V_{\text{Th}} - 4I_1 = 0$$

$$-10 - V_{\text{Th}} - 4(2) = 0$$

$$V_{\text{Th}} = -18 \text{ V}$$

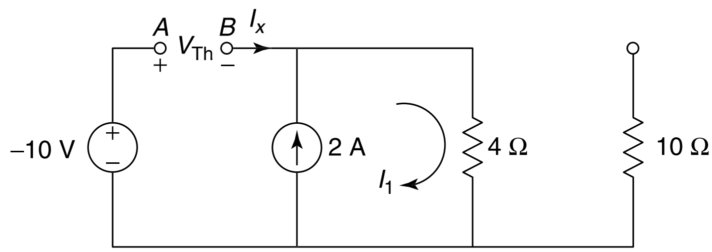


Fig. 3.319

Step II Calculation of I_N (Fig. 3.320)

From Fig. 3.320,

$$I_x = I_1 \qquad \ldots\text{(i)}$$

Mesh 1 and 2 will form a supermesh.

Writing the current equation for the supermesh,

$$I_2 - I_1 = 2 \qquad \ldots\text{(ii)}$$

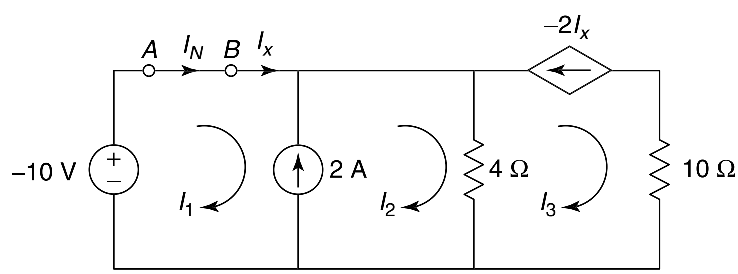


Fig. 3.320

Applying KVL to the outer path of the supermesh,

$$-10 - 4(I_2 - I_3) = 0$$

$$-4I_2 + 4I_3 = 10 \qquad \ldots\text{(iii)}$$

For Mesh 3,

$$I_3 = -(-2I_x) = 2I_x = 2I_1$$

$$2I_1 - I_3 = 0 \qquad \ldots\text{(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = 4.5 \text{ A}$$
$$I_2 = 6.5 \text{ A}$$
$$I_3 = 9 \text{ A}$$
$$I_N = I_1 = 4.5 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{\text{Th}}}{I_N} = \frac{-18}{4.5} = -4\ \Omega$$

Step IV Calculation of I_L (Fig. 3.321)

$$I_L = 4.5 \times \frac{-4}{-4+2} = 9 \text{ A}$$

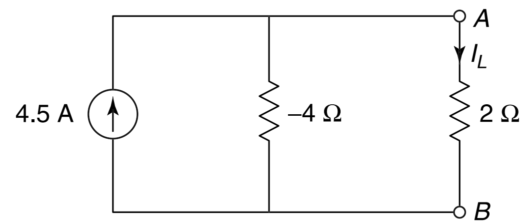


Fig. 3.321

Example 3.73 *Find the current through the 2Ω resistor in the network of Fig. 3.322.*

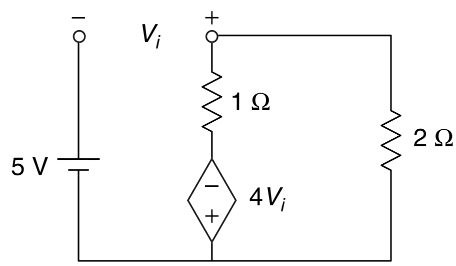


Fig. 3.322

Solution

Step I Calculation of V_{Th} (Fig. 3.322)

From Fig. 3.323,

$$5 + V_i + 4V_i = 0$$
$$V_i = -1 \text{ V}$$

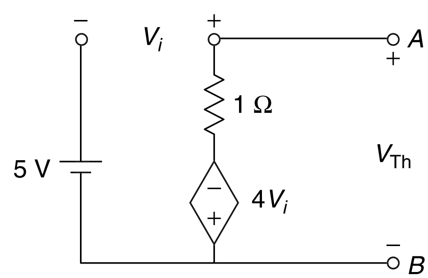


Fig. 3.323

Writing the V_{Th} equation,

$$-4V_i - V_{\text{Th}} = 0$$
$$V_{\text{Th}} = -4V_i = -4(-1) = 4 \text{ V}$$

Step II Calculation of I_N (Fig. 3.324)

From Fig. 3.324,

$$5 + V_i = 0$$
$$V_i = -5 \text{ V}$$

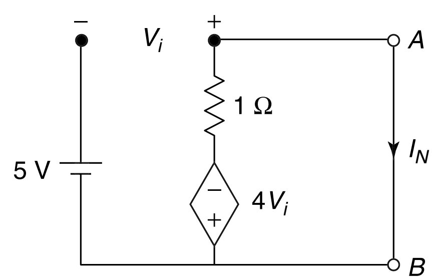


Fig. 3.324

Applying KVL to the mesh,

$$-4V_i - 1I_N = 0$$
$$I_N = -4V_i = -4(-5) = 20 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{\text{Th}}}{I_N} = \frac{4}{20} = 0.2\ \Omega$$

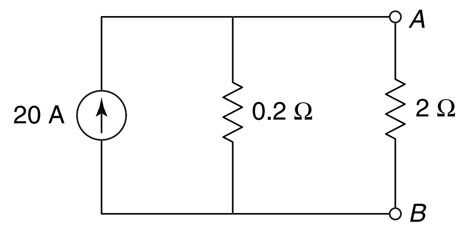


Fig. 3.325

Step IV Calculation of I_L (Fig. 3.325)

$$I_L = 20 \times \frac{0.2}{0.2+2} = 1.82\ \text{A}$$

Example 3.74 *Find the current in the 2 Ω resistor in the network of Fig. 3.326.*

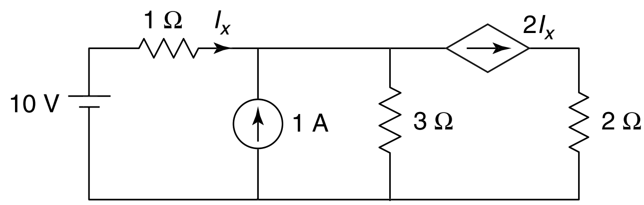


Fig. 3.326

Solution

Step I Calculation of V_{Th} (Fig. 3.327)

Meshes 1 and 2 will form a supermesh.

Writing current equation for the supermesh,

$$I_2 - I_1 = 1 \qquad \text{...(i)}$$

Applying KVL to the outer path of the supermesh,

$$10 - 1I_1 - 3I_2 = 0$$
$$I_1 + 3I_2 = 10 \qquad \text{...(ii)}$$

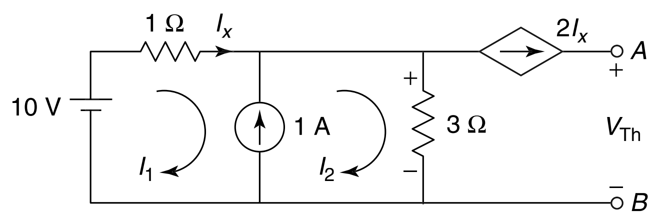


Fig. 3.327

Solving Eqs (i) and (ii),

$$I_1 = 1.75\ \text{A}$$
$$I_2 = 2.75\ \text{A}$$

Writing the V_{Th} equation,

$$3I_2 - V_{\text{Th}} = 0$$
$$3(2.75) - V_{\text{Th}} = 0$$
$$V_{\text{Th}} = 8.25\ \text{V}$$

Step II Calculation of I_N (Fig. 3.328)

From Fig. 3.328,

$$I_x = I_1 \qquad \text{...(i)}$$

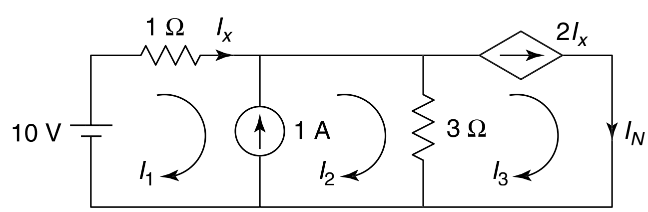


Fig. 3.328

Meshes 1 and 2 will form a supermesh.

Writing the current equation for the supermesh,

$$I_2 - I_1 = 1 \qquad \text{...(ii)}$$

Applying KVL to the outer path of the supermesh,

$$10 - 1I_1 - 3(I_2 - I_3) = 0$$

$$I_1 + 3I_2 - 3I_3 = 10 \qquad \ldots\text{(iii)}$$

For Mesh 3,

$$I_3 = 2I_x = 2I_1$$

$$2I_1 - I_3 = 0 \qquad \ldots\text{(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = -3.5 \text{ A}$$

$$I_2 = -2.5 \text{ A}$$

$$I_3 = -7 \text{ A}$$

$$I_N = I_3 = -7 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{\text{Th}}}{I_N} = \frac{8.25}{-7} = -1.18\ \Omega$$

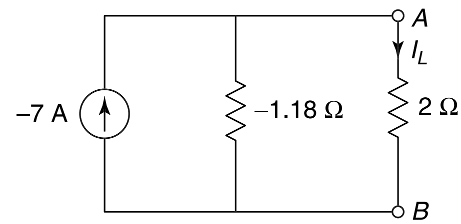


Fig. 3.329

Step IV Calculation of I_L (Fig. 3.329)

$$I_L = -7 \times \frac{-1.18}{-1.18 + 2} = 10.07 \text{ A}$$

Example 3.75 *Find the current through the 10 Ω resistor for the network of Fig. 3.330.*

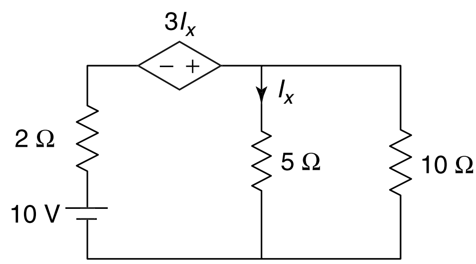


Fig. 3.330

Solution

Step I Calculation of V_{Th} (Fig. 3.331)
Applying KVL to the mesh,

$$10 - 2I_x + 3I_x - 5I_x = 0$$

$$I_x = 2.5 \text{ A}$$

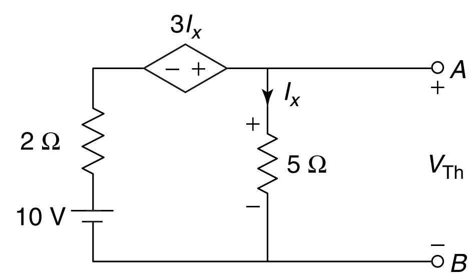


Fig. 3.331

Writing the V_{Th} equation,

$$5I_x - V_{\text{Th}} = 0$$

$$5(2.5) - V_{\text{Th}} = 0$$

$$V_{\text{Th}} = 12.5 \text{ V}$$

Step II Calculation of I_N (Fig. 3.332)
From Fig. 3.332,

$$I_x = 0$$

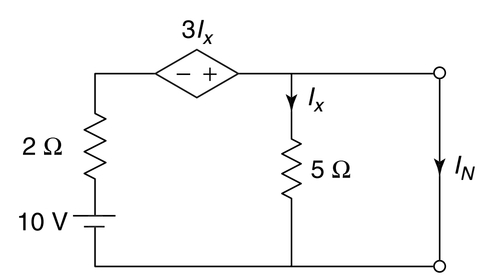


Fig. 3.332

The dependent source of 3 I_x depends on the controlling variable I_x. When $I_x = 0$, the dependent source 3 I_x vanishes, i.e. 3 $I_x = 0$ as shown in Fig. 3.333.

$$I_N = \frac{10}{2} = 5 \text{ A}$$

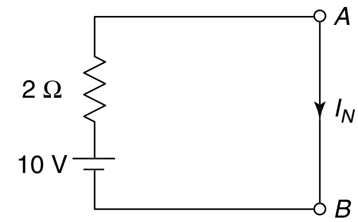


Fig. 3.333

Step III Calculation of R_N

$$R_N = \frac{V_{\text{Th}}}{I_N} = \frac{12.5}{5} = 2.5\ \Omega$$

Step IV Calculation of I_L (Fig. 3.334)

$$I_L = 5 \times \frac{2.5}{2.5 + 10} = 1 \text{ A}$$

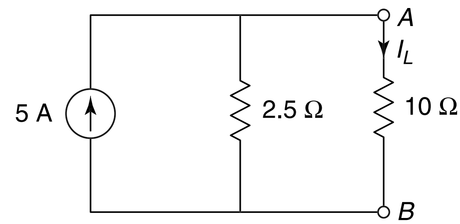


Fig. 3.334

Example 3.76 *Find the current through the 5 Ω resistor in the network of Fig. 3.335.*

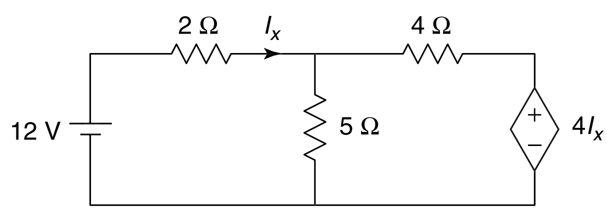


Fig. 3.335

Solution

Step I Calculation of V_{Th} (Fig. 3.336)

Applying KVL to the mesh,

$$12 - 2I_x - 4I_x - 4I_x = 0$$
$$12 - 10I_x = 0$$
$$I_x = 1.2 \text{ A}$$

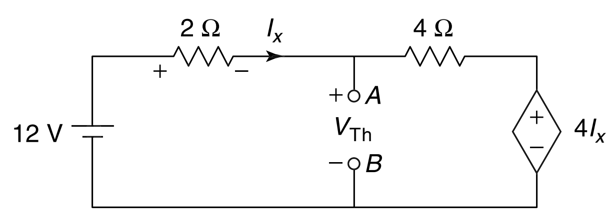


Fig. 3.336

Writing the V_{Th} equation,

$$12 - 2I_x - V_{\text{Th}} = 0$$
$$12 - 2(1.2) - V_{\text{Th}} = 0$$
$$V_{\text{Th}} = 9.6 \text{ V}$$

Step II Calculation of I_N (Fig. 3.337)

From Fig. 3.337,

$$I_x = I_1 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 1,

$$12 - 2I_1 = 0$$
$$I_1 = 6 \text{ A} \qquad \ldots\text{(ii)}$$

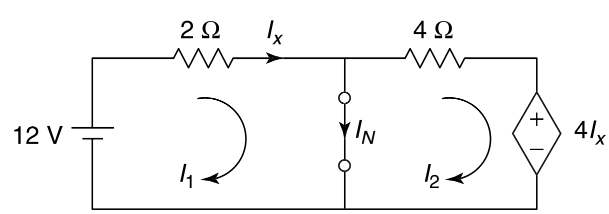


Fig. 3.337

Applying KVL to Mesh 2,

$$-4I_2 - 4I_x = 0$$
$$-4I_2 - 4I_1 = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_2 = -6 \text{ A}$$
$$I_N = I_1 - I_2 = 6 - (-6) = 12 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{\text{Th}}}{I_N} = \frac{9.6}{12} = 0.8\ \Omega$$

Step IV Calculation of I_L (Fig. 3.338)

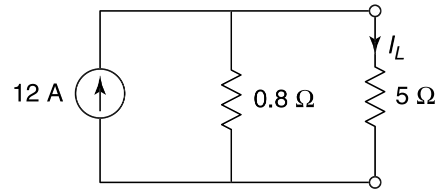


Fig. 3.338

$$I_L = 12 \times \frac{0.8}{0.8+5} = 1.66\ \text{A}$$

Example 3.77 *Find the current through the 10 Ω resistor for the network of Fig. 3.339.*

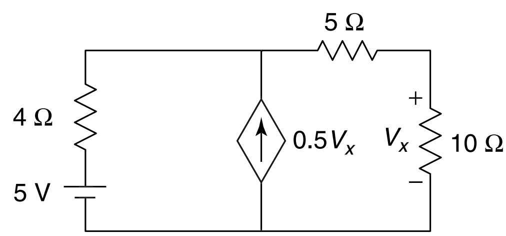


Fig. 3.339

Solution

Step I Calculation of V_{Th} (Fig. 3.340)

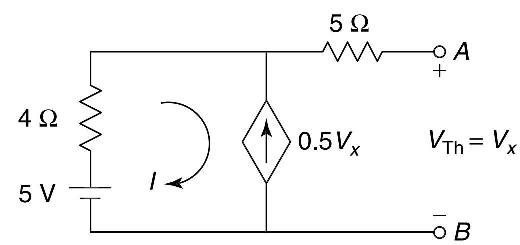


Fig. 3.340

For the mesh,

$$I = -0.5V_x = -0.5V_{\text{Th}}$$

Writing the V_{Th} equation,

$$5 - 4I - 0 - V_{\text{Th}} = 0$$

$$5 - 4(-0.5V_{\text{Th}}) - V_{\text{Th}} = 0$$

$$V_{\text{Th}} = -5\ \text{V}$$

Step II Calculation of I_N (Fig. 3.341)

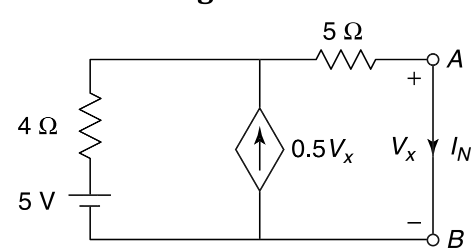


Fig. 3.341

From Fig. 3.341,

$$V_x = 0$$

The dependent source of $0.5V_x$ depends on the controlling variable V_x. When $V_x = 0$, the dependent source vanishes, i.e. $0.5\ V_x = 0$ as shown in Fig. 3.342.

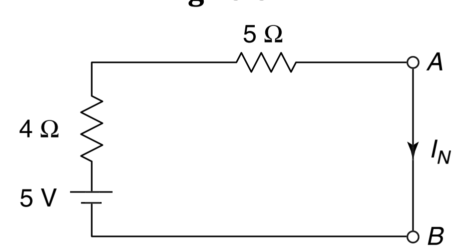


Fig. 3.342

$$I_N = \frac{5}{4+5} = \frac{5}{9}\ \text{A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{\text{Th}}}{I_N} = \frac{-5}{\frac{5}{9}} = -9\ \Omega$$

Step IV Calculation of I_L (Fig. 3.343)

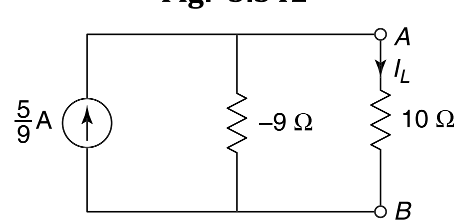


Fig. 3.343

$$I_L = \frac{5}{9} \times \frac{-9}{-9+10} = -5\ \text{A}$$

Example 3.78 *Find the current through the 10 Ω resistor in the network shown in Fig. 3.344.*

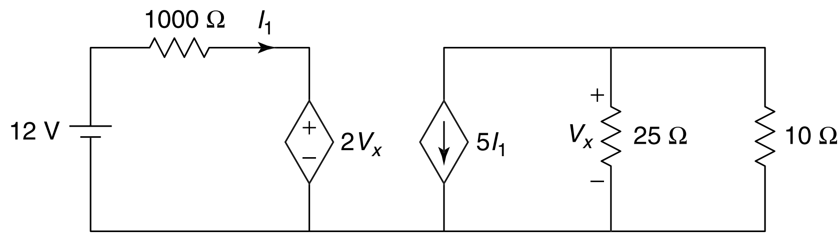


Fig. 3.344

Solution

Step I Calculation of V_{Th} (Fig. 3.345)

From Fig. 3.345,

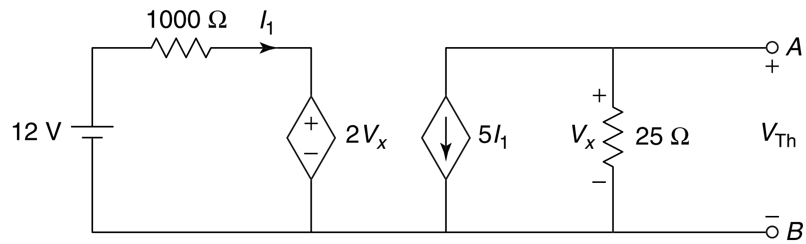


Fig. 3.345

$$V_x = -25(5I_1) = -125\,I_1 \quad \ldots\text{(i)}$$

Applying KVL to Mesh 1,

$$12 - 1000I_1 - 2V_x = 0$$

$$12 - 1000I_1 - 2(-125I_1) = 0 \qquad \ldots\text{(ii)}$$

$$I_1 = 0.016 \text{ A}$$

$$V_x = -125I_1 = -125(0.016) = -2 \text{ V}$$

Writing the V_{Th} equation,

$$V_{Th} = V_x = -2 \text{ V}$$

Step II Calculation of I_N (Fig. 3.346)

From Fig. 3.346,

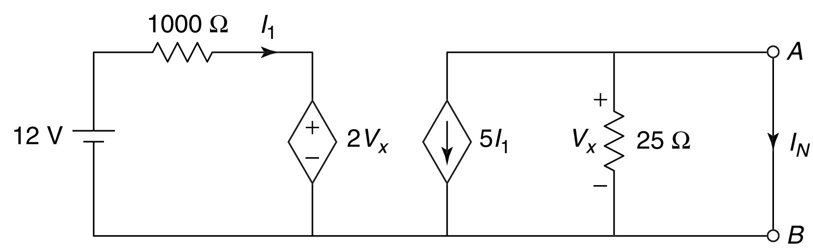


Fig. 3.346

$$V_x = 0$$

The dependent source of $2V_x$ depends on the controlling variable V_x. When $V_x = 0$, the dependent source vanishes, i.e. $2\,V_x = 0$ as shown in Fig. 3.347.

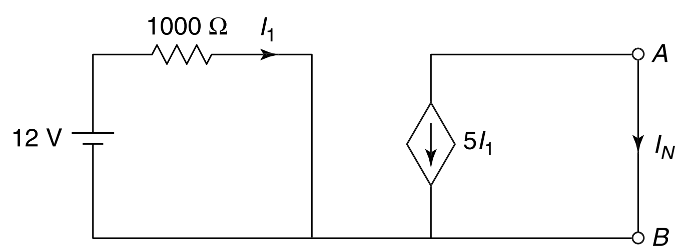


Fig. 3.347

$$I_1 = \frac{12}{1000} = 0.012 \text{ A}$$

$$I_N = -5I_1 = -5(0.012) = -0.06 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{Th}}{I_N} = \frac{-2}{-0.06} = 33.33\ \Omega$$

Step IV Calculation of I_L (Fig. 3.348)

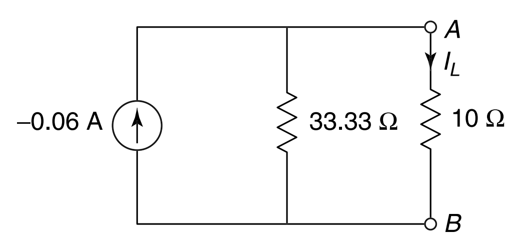


Fig. 3.348

$$I_L = -0.06 \times \frac{33.33}{33.33 + 10} = -0.046 \text{ A}$$

Example 3.79 *Find the current through the 5 Ω resistor for the network of Fig. 3.349.*

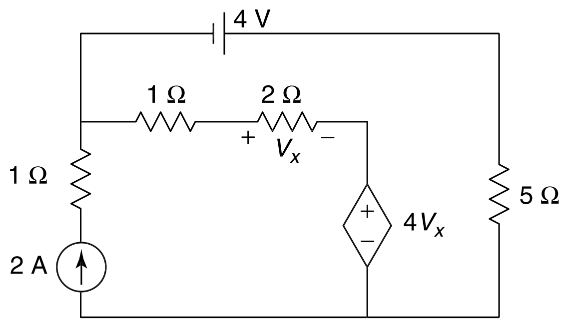


Fig. 3.349

Solution

Step I Calculation of V_{Th} (Fig. 3.350)

From Fig. 3.350,

$$V_x = 2I \quad \ldots\text{(i)}$$

For the mesh,

$$I = 2 \quad \ldots\text{(ii)}$$

$$V_x = 2(2) = 4 \text{ V}$$

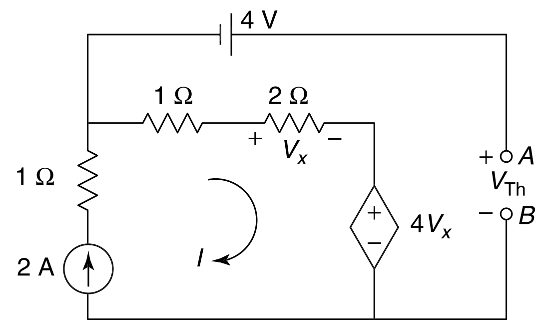


Fig. 3.350

Writing the V_{Th} equation,

$$4V_x + 2I + 1I + 4 - V_{Th} = 0$$

$$4(4) + 2(2) + 2 + 4 - V_{Th} = 0$$

$$V_{Th} = 26 \text{ V}$$

Step II Calculation of I_N (Fig. 3.351)

From Fig. 3.351,

$$V_x = 2(I_1 - I_2) \quad \ldots\text{(i)}$$

For Mesh 1,

$$I_1 = 2 \quad \ldots\text{(ii)}$$

Applying KVL to Mesh 2,

$$4V_x - 2(I_2 - I_1) - 1(I_2 - I_1) + 4 = 0$$

$$4[2(I_1 - I_2)] - 2I_2 + 2I_1 - I_2 + I_1 + 4 = 0$$

$$11I_1 - 11I_2 = -4 \quad \ldots\text{(iii)}$$

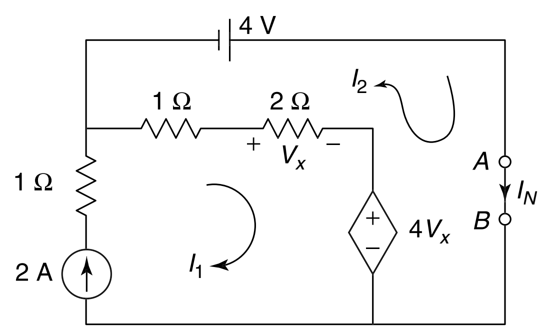


Fig. 3.351

Solving Eqs (ii) and (iii),

$$I_1 = 2 \text{ A}$$

$$I_2 = 2.36 \text{ A}$$

$$I_N = I_2 = 2.36 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{Th}}{I_N} = \frac{26}{2.36} = 11.02\ \Omega$$

Step IV Calculation of I_L (Fig. 3.352)

$$I_L = 2.36 \times \frac{11.02}{11.02 + 5} = 1.62 \text{ A}$$

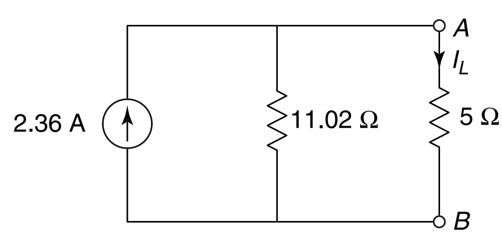


Fig. 3.352

Example 3.80 *Find the current through the 1 Ω resistor in the network of Fig. 3.353.*

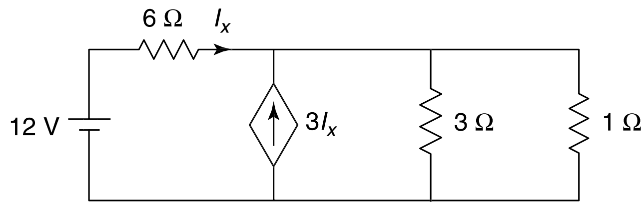


Fig. 3.353

Solution

Step I Calculation of V_{Th} (Fig. 3.354)

From Fig. 3.354,

$$I_x = I_1 \quad \ldots\text{(i)}$$

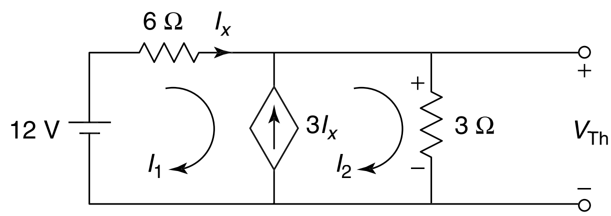


Fig. 3.354

Meshes 1 and 2 will form a supermesh.

Writing the current equation for the supermesh,

$$I_2 - I_1 = 3I_x = 3I_1$$

$$4I_1 - I_2 = 0 \quad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$12 - 6I_1 - 3I_2 = 0$$

$$6I_1 + 3I_2 = 12 \quad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 0.67 \text{ A}$$

$$I_2 = 2.67 \text{ A}$$

Writing the V_{Th} equation,

$$3I_2 - V_{\text{Th}} = 0$$

$$3(2.67) - V_{\text{Th}} = 0$$

$$V_{\text{Th}} = 8 \text{ V}$$

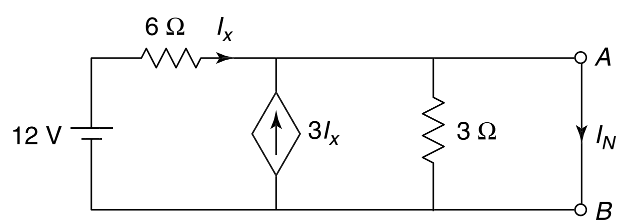


Fig. 3.355

Step II Calculation of I_N (Fig. 3.355)

When a short circuit is placed across a 3 Ω resistor, it gets shorted as shown in Fig. 3.356.

From Fig. 3.356,

$$I_x = I_1 \quad \ldots\text{(i)}$$

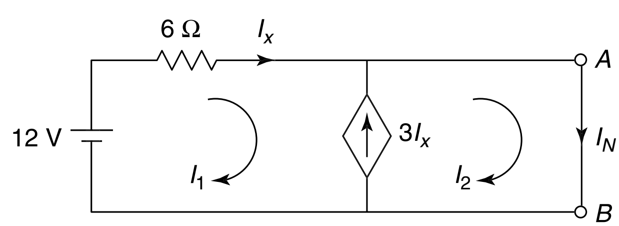


Fig. 3.356

Meshes 1 and 2 will form a supermesh.

Writing the current equation for the supermesh,

$$I_2 - I_1 = 3I_x = 3I_1$$

$$4I_1 - I_2 = 0 \quad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$12 - 6I_1 = 0$$

$$I_1 = 2 \quad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 2 \text{ A}$$
$$I_2 = 8 \text{ A}$$
$$I_N = I_2 = 8 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{Th}}{I_N} = \frac{8}{8} = 1\ \Omega$$

Step IV Calculation of I_L (Fig. 3.357)

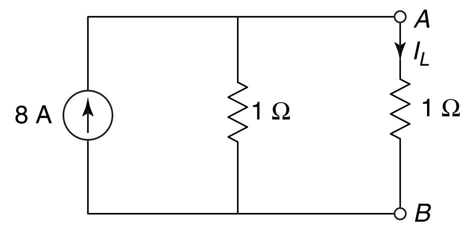


Fig. 3.357

$$I_L = 8 \times \frac{1}{1+1} = 4 \text{ A}$$

Example 3.81 *Find the current through the 1.6 Ω resistor in the network of Fig. 3.358.*

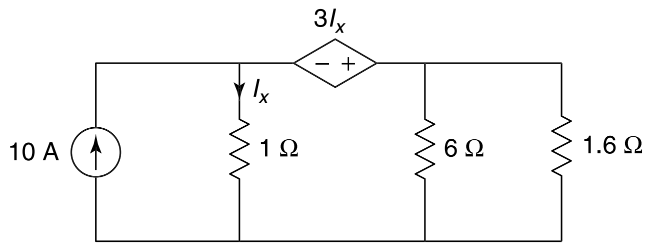


Fig. 3.358

Solution

Step I Calculation of V_{Th} (Fig. 3.359)

From Fig. 3.359,

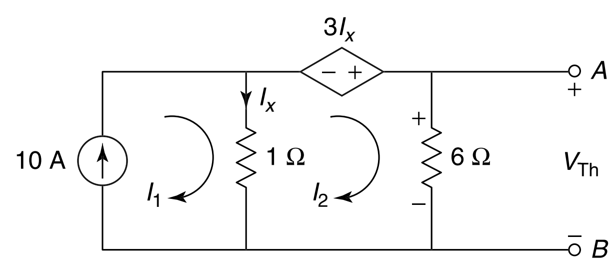


Fig. 3.359

$$I_x = I_1 - I_2 \quad \ldots\text{(i)}$$

For Mesh 1,

$$I_1 = 10 \quad \ldots\text{(ii)}$$

Applying KVL to Mesh 2,

$$-1(I_2 - I_1) + 3I_x - 6I_2 = 0$$
$$-I_2 + I_1 + 3(I_1 - I_2) - 6I_2 = 0$$
$$4I_1 - 10I_2 = 0 \quad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 10 \text{ A}$$
$$I_2 = 4 \text{ A}$$

Writing the V_{Th} equation,

$$6I_2 - V_{Th} = 0$$
$$6(4) - V_{Th} = 0$$
$$V_{Th} = 24 \text{ V}$$

Step II Calculation of I_N (Fig. 3.360)

When a short circuit is placed across the 3 Ω resistor, it gets shorted as shown in Fig. 3.361.

From Fig. 3.361,

$$I_x = I_1 - I_2 \qquad \ldots\text{(i)}$$

For Mesh 1,

$$I_1 = 10 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 2,

$$-1(I_2 - I_1) + 3I_x = 0$$
$$-I_2 + I_1 + 3(I_1 - I_2) = 0$$
$$4I_1 - 4I_2 = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 10 \text{ A}$$
$$I_2 = 10 \text{ A}$$
$$I_N = I_2 = 10 \text{ A}$$

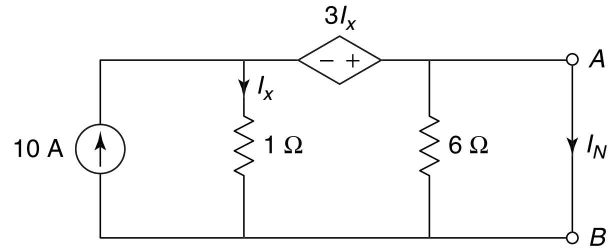


Fig. 3.360

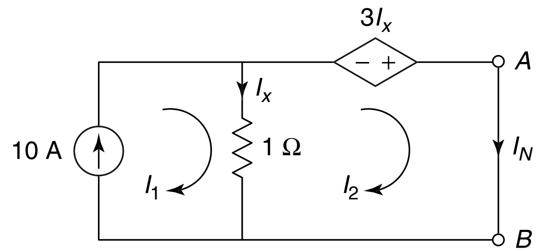


Fig. 3.361

Step III Calculation of R_N

$$R_N = \frac{V_{\text{Th}}}{I_N} = \frac{24}{10} = 2.4\ \Omega$$

Step IV Calculation of I_L (Fig. 3.362)

$$I_L = 10 \times \frac{2.4}{2.4 + 1.6} = 6 \text{ A}$$

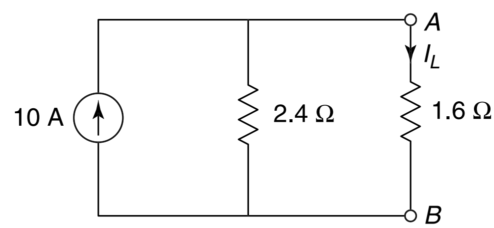


Fig. 3.362

3.5 MAXIMUM POWER TRANSFER THEOREM

It states that '*the maximum power is delivered from a source to a load when the load resistance is equal to the source resistance.*'

Proof From Fig. 3.363,

$$I = \frac{V}{R_s + R_L}$$

Power delivered to the load $R_L = P = I^2\ R_L = \dfrac{V^2 R_L}{(Rs + R_L)^2}$

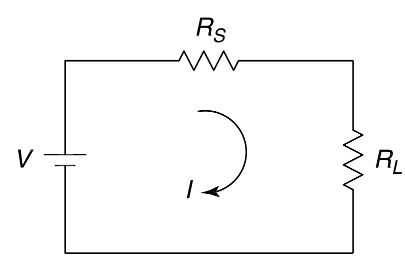


Fig. 3.363 *Network illustrating maximum power transfer theorem*

To determine the value of R_L for maximum power to be transferred to the load,

$$\frac{dP}{dR_L} = 0$$

$$\frac{dP}{dR_L} = \frac{d}{dR_L}\frac{V^2}{(R_s + R_L)^2}\ R_L$$

$$= \frac{V^2[(R_s + R_L)^2 - (2R_L)(R_s + R_L)]}{(R_s + R_L)^4}$$

$$(R_s + R_L)^2 - 2\,R_L(R_s + R_L) = 0$$

$$R_s^{\,2} + R_L^{\,2} + 2R_sR_L - 2R_LR_s - 2R_L^2 = 0$$

$$R_s = R_L$$

Hence, the maximum power will be transferred to the load when load resistance is equal to the source resistance.

Steps to be followed in Maximum Power Transfer Theorem

1. Remove the variable load resistor R_L.
2. Find the open circuit voltage V_{Th} across points A and B.
3. Find the resistance R_{Th} as seen from points A and B.
4. Find the resistance R_L for maximum power transfer.

$$R_L = R_{Th}$$

5. Find the maximum power (Fig. 3.364).

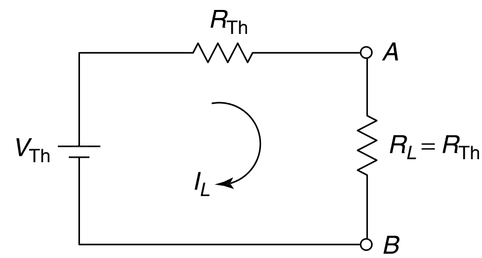


Fig. 3.364 *Thevenin's equivalent network*

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{V_{Th}}{2R_{Th}}$$

$$P_{max} = I_L^2\,R_L = \frac{V_{Th}^2}{4R_{Th}^2} \times R_{Th} = \frac{V_{Th}^2}{4R_{Th}}$$

Example 3.82 *Find the value of resistance R_L in Fig. 3.365 for maximum power transfer and calculate maximum power.*

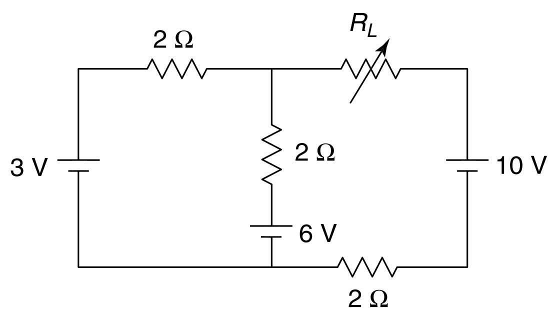


Fig. 3.365

Solution

Step I Calculation of V_{Th} (Fig. 3.366)

Applying KVL to the mesh,

$$3 - 2I - 2I - 6 = 0$$

$$I = -0.75\text{ A}$$

Writing the V_{Th} equation,

$$6 + 2I - V_{Th} - 10 = 0$$

$$V_{Th} = 6 + 2I - 10 = 6 + 2(-0.75) - 10 = -5.5\text{ V}$$

$$= 5.5\text{ V(terminal } B \text{ is positive w.r.t } A)$$

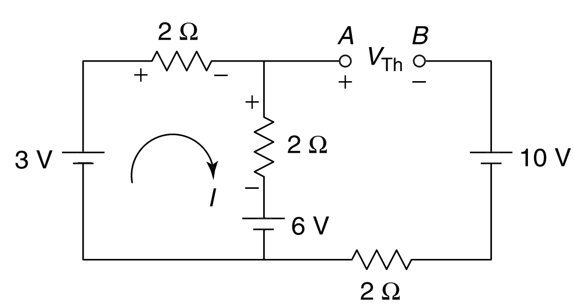


Fig. 3.366

Step II Calculation of R_{Th} (Fig. 3.367)

$$R_{Th} = (2 \parallel 2) + 2 = 3\,\Omega$$

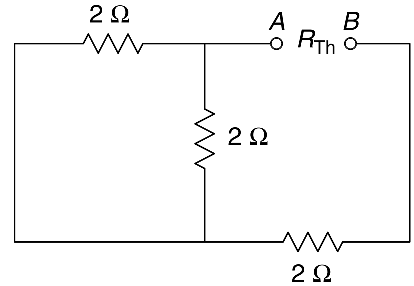


Fig. 3.367

Step III Calculation of R_L

For maximum power transfer,

$$R_L = R_{Th} = 3\,\Omega$$

Step IV Calculation of P_{max} (Fig. 3.368)

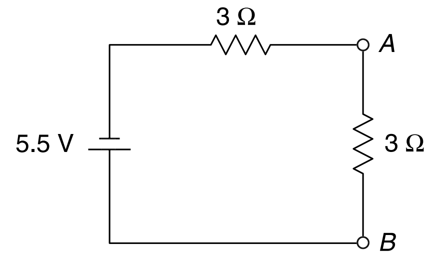


Fig. 3.368

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(5.5)^2}{4 \times 3} = 2.52\,\text{W}$$

Example 3.83 *Find the value of resistance R_L in Fig. 3.369 for maximum power transfer and calculate maximum power.*

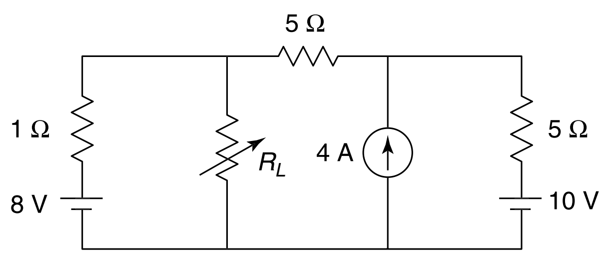


Fig. 3.369

Solution

Step I Calculation of V_{Th} (Fig. 3.370)

$$I_2 - I_1 = 4 \quad \text{...(i)}$$

Applying KVL to the outer path,

$$8 - 1I_1 - 5I_1 - 5I_2 - 10 = 0$$

$$-6I_1 - 5I_2 = 2 \quad \text{...(ii)}$$

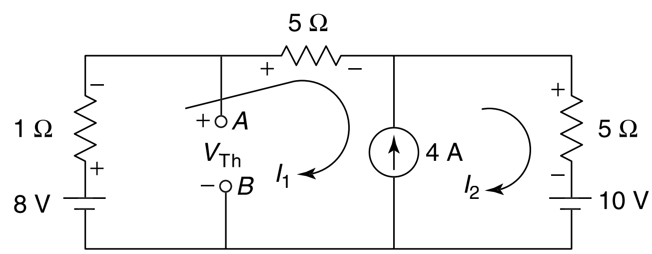


Fig. 3.370

Solving Eqs (i) and (ii),

$$I_1 = -2\,\text{A}$$

$$I_2 = 2\,\text{A} \quad \text{...(ii)}$$

Writing the V_{Th} equation,

$$8 - 1I_1 - V_{Th} = 0$$

$$V_{Th} = 8 - I_1 = 8 - (-2) = 10\,\text{V}$$

Step II Calculation of R_{Th} (Fig. 3.371)

$$R_{Th} = 10 \parallel 1 = 0.91\,\Omega$$

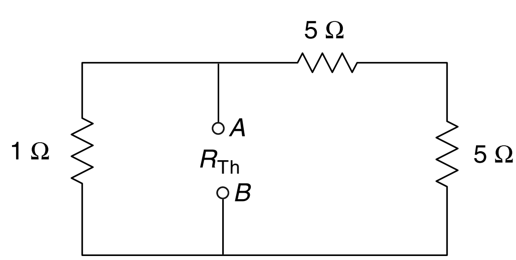


Fig. 3.371

Step III Calculation of R_L

For maximum power transfer,

$$R_L = R_{\text{Th}} = 0.91\ \Omega$$

Step IV Calculation of P_{max}

$$P_{\text{max}} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(10)^2}{4 \times 0.91} = 27.47\ \text{W}$$

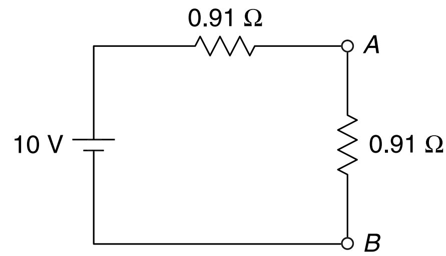


Fig. 3.372

Example 3.84 *Find the value of the resistance R_L in Fig. 3.373 for maximum power transfer and calculate the maximum power.*

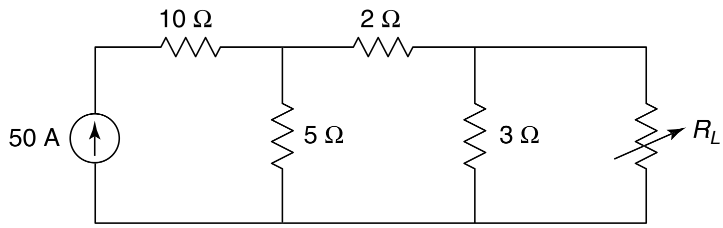


Fig. 3.373

Solution

Step I Calculation of V_{Th} (Fig. 3.374)

For Mesh 1,

$$I_1 = 50$$

Applying KVL to Mesh 2,

$$-5(I_2 - I_1) - 2I_2 - 3I_2 = 0$$
$$5I_1 - 10I_2 = 0$$
$$I_1 = 2I_2$$
$$I_2 = 25\ \text{A}$$
$$V_{\text{Th}} = 3I_2 = 3(25) = 75\ \text{V}$$

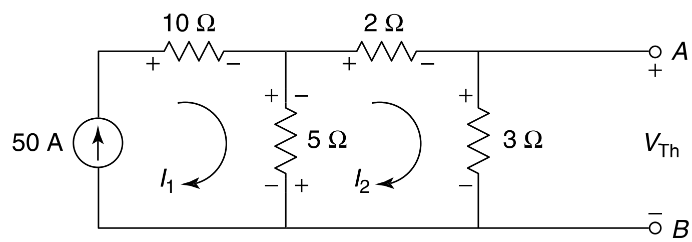


Fig. 3.374

Step II Calculation of R_{Th} (Fig. 3.375)

$$R_{\text{Th}} = (5 + 2) \,||\, 3 = 2.1\ \Omega$$

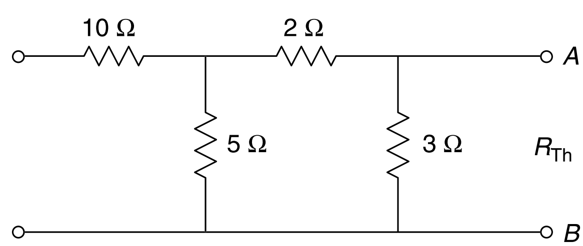


Fig. 3.375

Step III Calculation of R_L

For maximum power transfer,

$$R_L = R_{\text{Th}} = 2.1\ \Omega$$

Step IV Calculation of P_{max} (Fig. 3.376)

$$P_{\text{max}} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(75)^2}{4 \times 2.1} = 669.64\ \text{W}$$

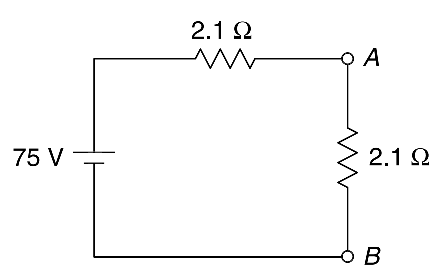


Fig. 3.376

Example 3.85 *Find the value of resistance R_L in Fig. 3.377 for maximum power transfer and calculate maximum power.*

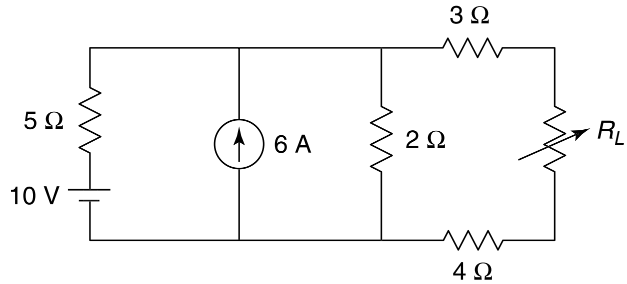


Fig. 3.377

Solution

Step I Calculation of V_{Th} (Fig. 3.378)
Writing the current equation for the supermesh,

$$I_2 - I_1 = 6 \quad \text{...(i)}$$

Applying KVL to the supermesh,

$$10 - 5I_1 - 2I_2 = 0$$
$$5I_1 + 2I_2 = 10 \quad \text{...(ii)}$$

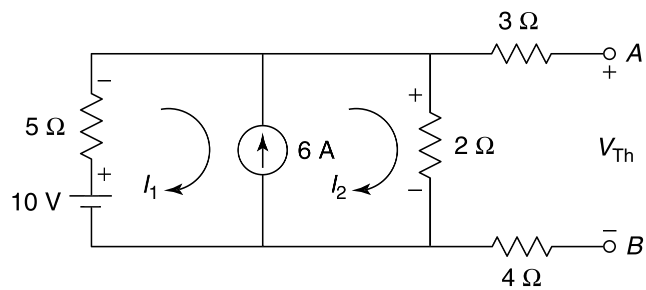


Fig. 3.378

Solving Eqs (i) and (ii),

$$I_1 = -0.29\text{ A}$$
$$I_2 = 5.71\text{ A}$$

Writing the V_{Th} equation,

$$V_{Th} = 2I_2 = 11.42\text{ V}$$

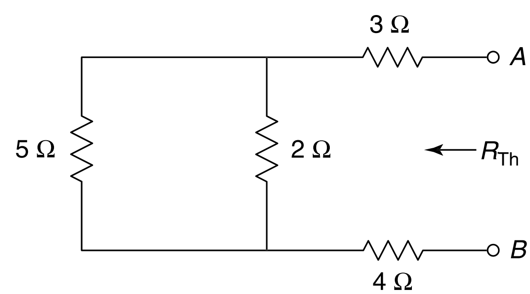


Fig. 3.379

Step II Calculation of R_{Th} (Fig. 3.379)

$$R_{Th} = (5 \,||\, 2) + 3 + 4 = 8.43\ \Omega$$

Step III Calculation of R_L
For maximum power transfer,

$$R_L = R_{Th} = 8.43\ \Omega$$

Step IV Calculation of P_{max} (Fig. 3.380)

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(11.42)^2}{4 \times 8.43} = 3.87\text{ W}$$

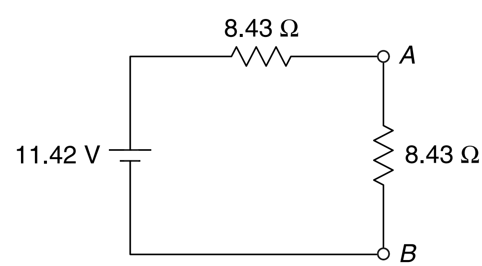


Fig. 3.380

Example 3.86 *Find the value of resistance R_L in Fig. 3.381 for maximum power transfer and calculate the maximum power.*

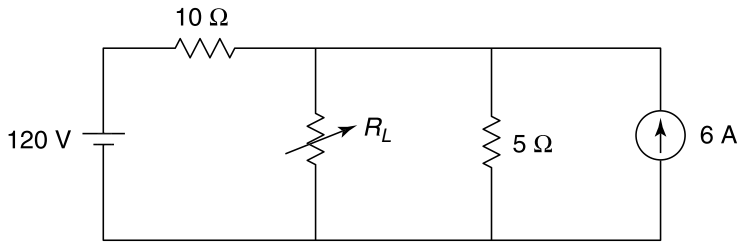


Fig. 3.381

Solution

Step I Calculation of V_{Th} (Fig. 3.382)
Applying KVL to Mesh 1,

$$120 - 10I_1 - 5(I_1 - I_2) = 0$$

$$15I_1 - 5I_2 = 120 \qquad \text{...(i)}$$

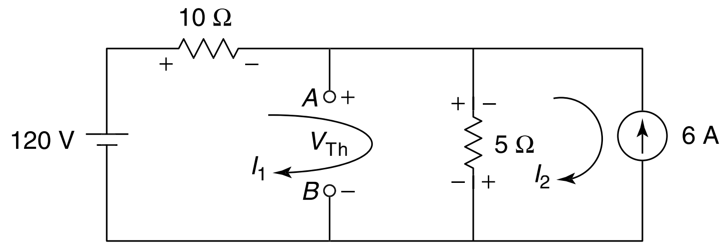


Fig. 3.382

Writing current equation for Mesh 2,

$$I_2 = -6 \qquad \text{...(ii)}$$

Solving Eqs (i) and (ii),

$$I_1 = 6\text{ A}$$

Writing the V_{Th} equation,

$$120 - 10I_1 - V_{Th} = 0$$

$$V_{Th} = 120 - 10(6) = 60\text{ V}$$

Step II Calculation of R_{Th} (Fig. 3.383)

$$R_{Th} = 10 \parallel 5 = 3.33\ \Omega$$

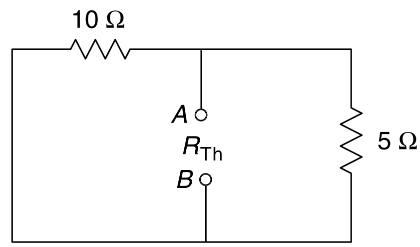


Fig. 3.383

Step III Calculation of R_L
For maximum power transfer,

$$R_L = R_{Th} = 3.33\ \Omega$$

Step IV Calculation of P_{max} (Fig. 3.384)

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(60)^2}{4 \times 3.33} = 270.27\text{ W}$$

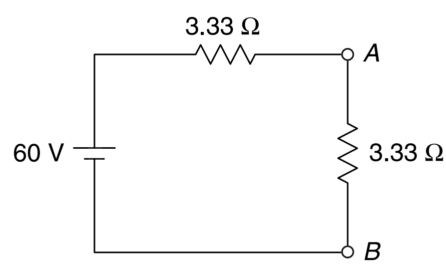


Fig. 3.384

Example 3.87 *Find the value of resistance R_L in Fig. 3.385 for maximum power transfer and calculate the maximum power.*

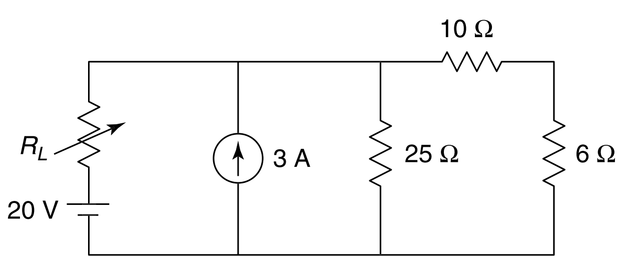


Fig. 3.385

Solution

Step I Calculation of V_{Th} (Fig. 3.386)

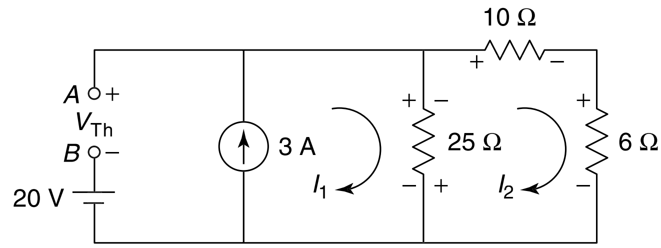


Fig. 3.386

$$I_1 = 3 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-25(I_2 - I_1) - 10I_2 - 6I_2 = 0$$

$$-25I_1 + 41I_2 = 0 \qquad \ldots\text{(ii)}$$

Solving Eqs (i) and (ii),

$$I_2 = 1.83 \text{ A}$$

Writing the V_{Th} equation,

$$20 + V_{Th} - 10I_2 - 6I_2 = 0$$

$$V_{Th} = -20 + 10\,(1.83) + 6\,(1.83) = 9.28 \text{ V}$$

Step II Calculation of R_{Th} (Fig. 3.387)

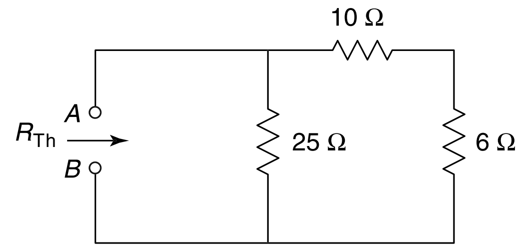


Fig. 3.387

$$R_{Th} = 25 \,||\, (10 + 6) = 9.76\ \Omega$$

Step III Calculation of R_L

For maximum power transfer,

$$R_L = R_{Th} = 9.76\ \Omega$$

Step IV Calculation of P_{max} (Fig. 3.388)

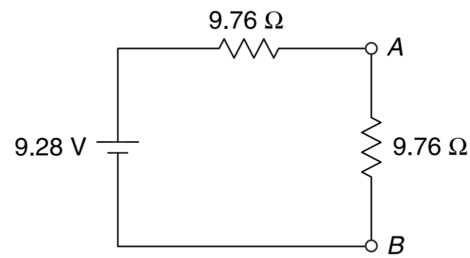


Fig. 3.388

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(9.28)^2}{4 \times 9.76} = 2.21 \text{ W}$$

Example 3.88 *Find the value of resistance R_L in Fig. 3.389 for maximum power transfer and calculate maximum power.*

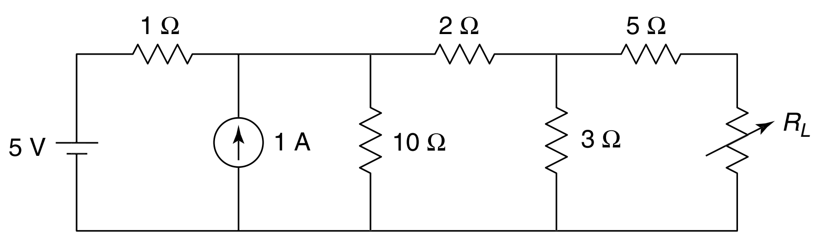


Fig. 3.389

Solution

Step I Calculation of V_{Th} (Fig. 3.390)

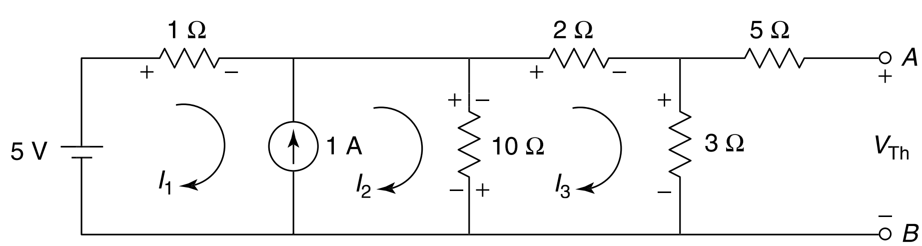


Fig. 3.390

Meshes 1 and 2 will form a supermesh.
Writing the current equation for the supermesh,

$$I_2 - I_1 = 1 \qquad \text{...(i)}$$

Writing the voltage equation for the supermesh,

$$5 - 1I_1 - 10(I_2 - I_3) = 0$$
$$I_1 + 10I_2 - 10I_3 = 5 \qquad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-10(I_3 - I_2) - 2I_3 - 3I_3 = 0$$
$$-10I_2 + 15I_3 = 0 \qquad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1 = 0.38 \text{ A}$$
$$I_2 = 1.38 \text{ A}$$
$$I_3 = 0.92 \text{ A}$$

Writing the V_{Th} equation,

$$V_{Th} = 3I_3 = 2.76 \text{ V}$$

Step II Calculation of R_{Th} (Fig. 3.391)

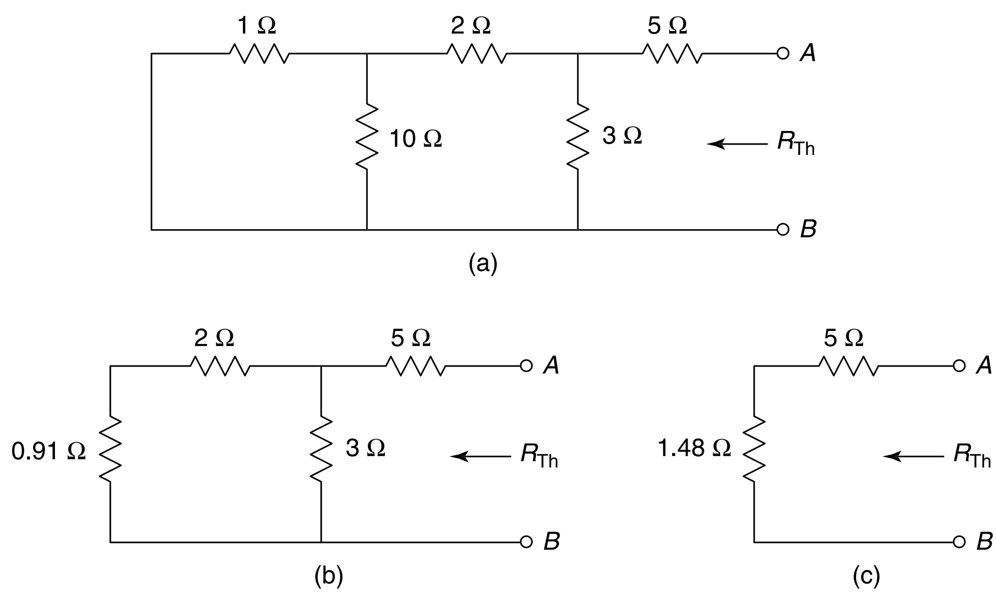


Fig. 3.391

$$R_{Th} = 6.48\ \Omega$$

Step III Calculation of R_L
For maximum power transfer,

$$R_L = R_{Th} = 6.48\ \Omega$$

Step IV Calculation of P_{max} (Fig. 3.392)

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(2.76)^2}{4 \times 6.48} = 0.29 \text{ W}$$

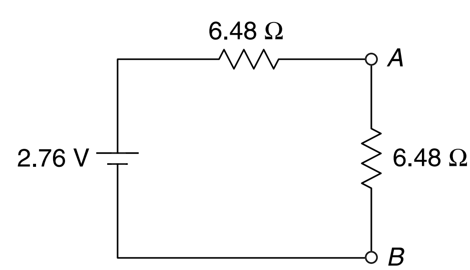


Fig. 3.392

Example 3.89 *For the network shown in Fig. 3.393, find the value of the resistance R_L for maximum power transfer and calculate the maximum power.*

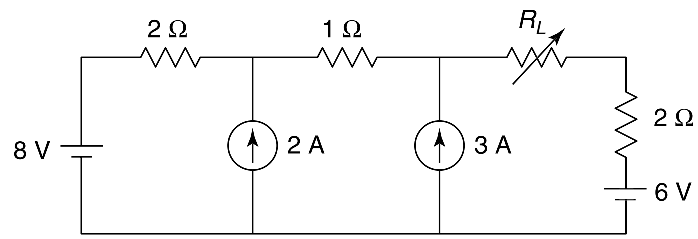


Fig. 3.393

Solution

Step I Calculation of V_{Th} (Fig. 3.394)

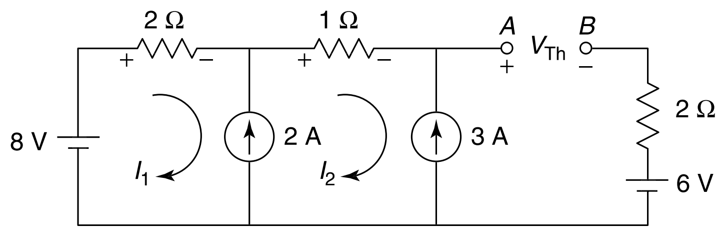


Fig. 3.394

$$I_2 - I_1 = 2 \quad \text{...(i)}$$
$$I_2 = -3 \text{ A} \quad \text{...(ii)}$$

Solving Eqs (i), and (ii),

$$I_1 = -5 \text{ A}$$

Writing the V_{Th} equation,

$$8 - 2I_1 - 1I_2 - V_{Th} - 6 = 0$$
$$V_{Th} = 8 - 2(-5) - (-3) - 6 = 15 \text{ V}$$

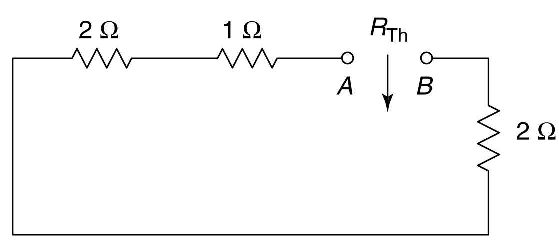


Fig. 3.395

Step II Calculation of R_{Th} (Fig. 3.395)

$$R_{Th} = 5\ \Omega$$

Step III Calculation of R_L

For maximum power transfer,

$$R_L = R_{Th} = 5\ \Omega$$

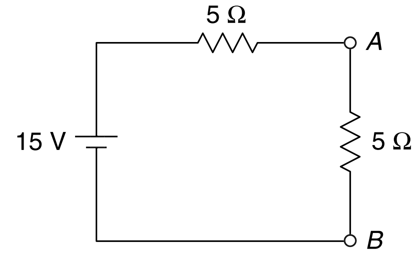


Fig. 3.396

Step IV Calculation of P_{max} (Fig. 3.396)

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(15)^2}{4 \times 5} = 11.25 \text{ W}$$

Example 3.90 *For the value of resistance R_L in Fig. 3.397 for maximum power transfer and calculate the maximum power.*

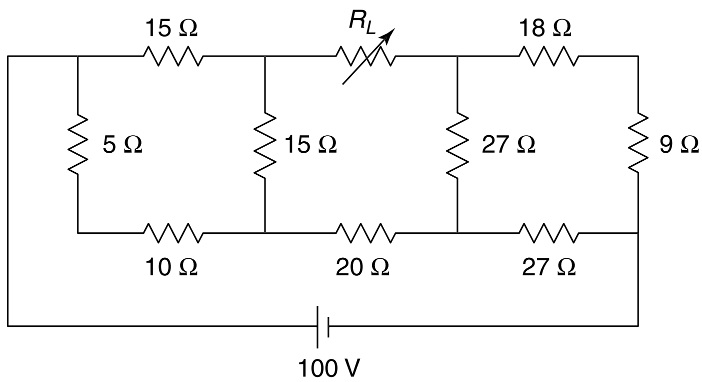


Fig. 3.397

Solution

Step I Calculation of V_{Th} (Fig. 3.398)

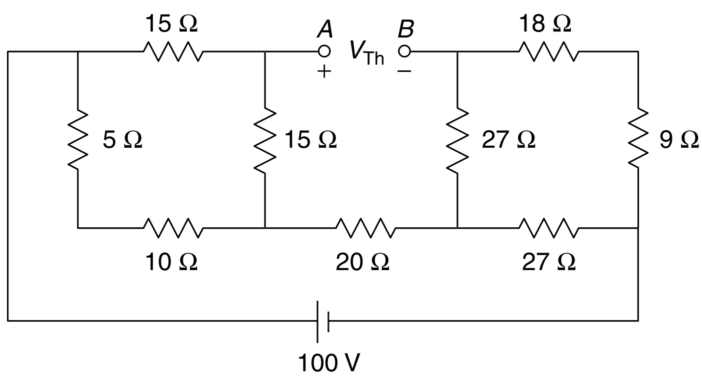


Fig. 3.398

By star-delta transformation (Fig. 3.399),

$$I = \frac{100}{5+5+20+9+9} = 2.08 \text{ A}$$

Writing the V_{Th} equation,

$$100 - 5I - V_{Th} - 9I = 0$$

$$V_{Th} = 100 - 14I$$

$$= 100 - 14(2.08)$$

$$= 70.88 \text{ V}$$

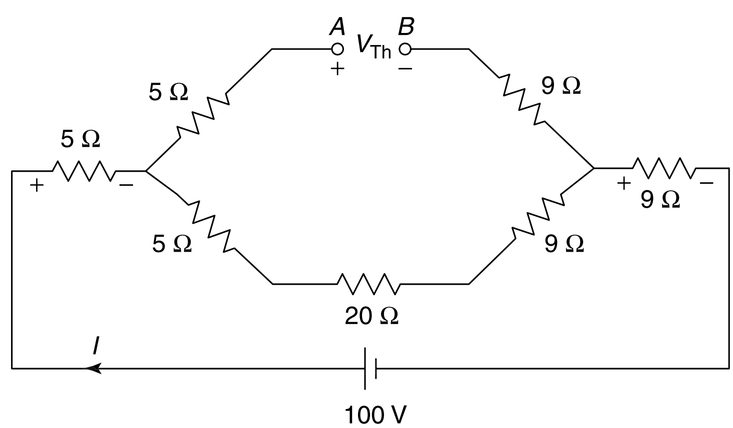


Fig. 3.399

Step II Calculation of R_{Th} (Fig. 3.400)

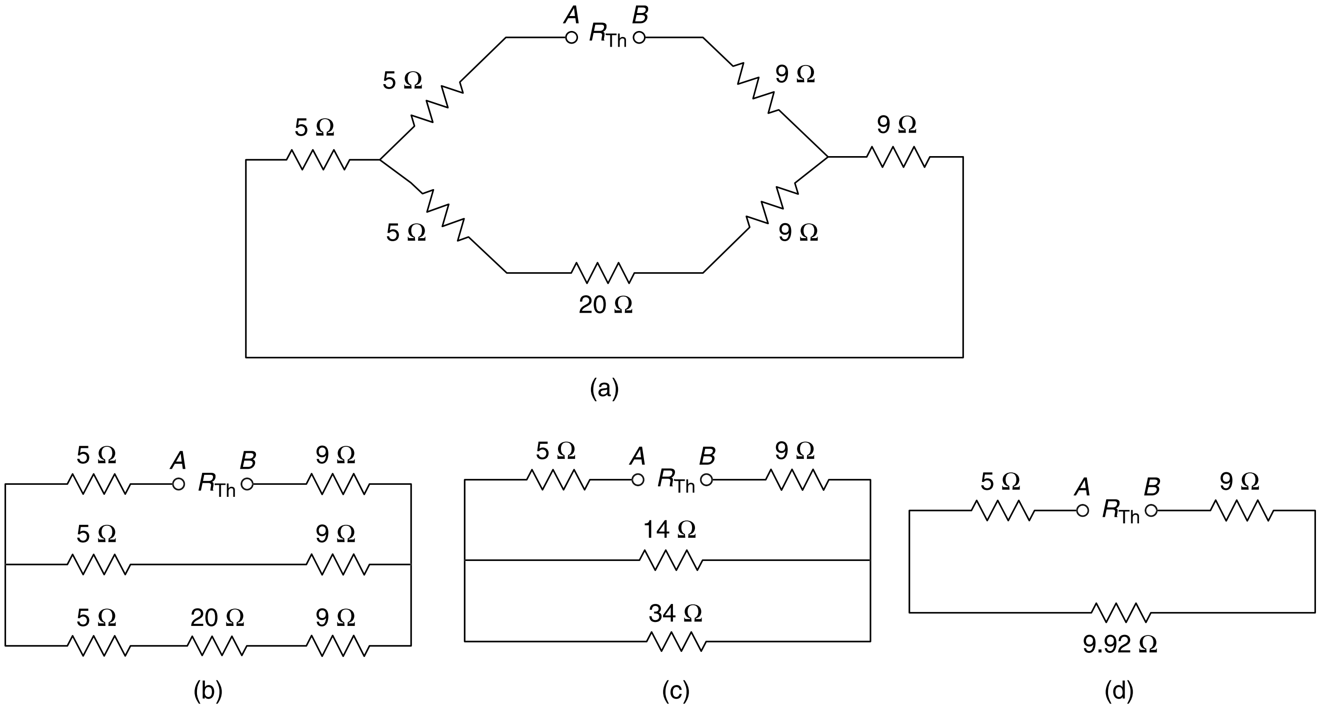


Fig. 3.400

$$R_{Th} = 23.92\ \Omega$$

Step III Calculation of R_L

For maximum power transfer,

$$R_L = R_{Th} = 23.92\ \Omega$$

Step IV Calculation of P_{max} (Fig. 3.401)

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(70.88)^2}{4 \times 23.92} = 52.51\ \text{W}$$

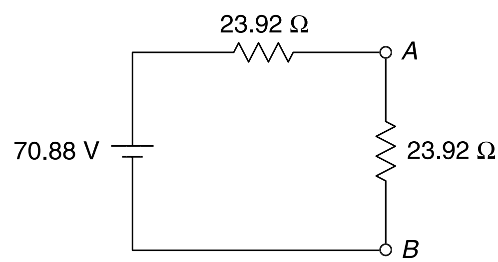


Fig. 3.401

Example 3.91 *For the value of resistance R_L in Fig. 3.402 for maximum power transfer and calculate the maximum power.*

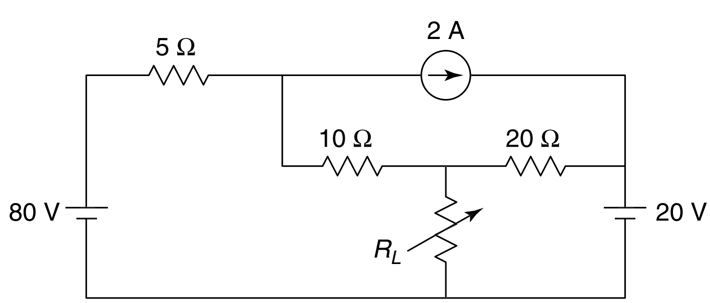


Fig. 3.402

Solution

Step I Calculation of V_{Th} (Fig. 3.403)

Applying KVL to Mesh 1,

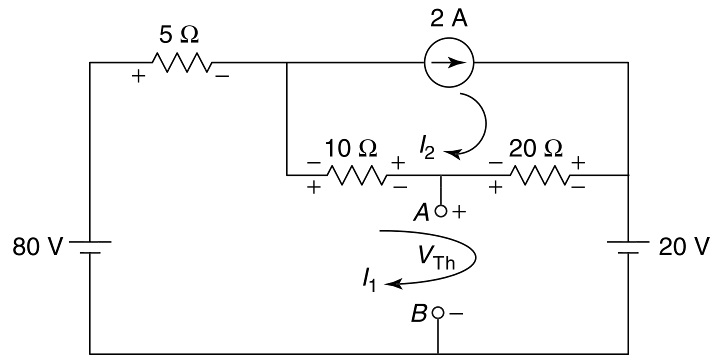


Fig. 3.403

$$80-5I_1-10(I_1-I_2)-20(I_1-I_2)-20=0$$

$$35I_1-30I_2=60 \quad \ldots\text{(i)}$$

Writing the current equation for Mesh 2,

$$I_2 = 2 \quad \ldots\text{(ii)}$$

Solving Eqs (i) and (ii),

$$I_1 = 3.43 \text{ A}$$

Writing the V_{Th} equation,

$$V_{Th} - 20\,(I_1 - I_2) - 20 = 0$$

$$V_{Th} = 20(3.43-2)+20 = 48.6 \text{ V}$$

Step II Calculation of R_{Th} (Fig. 3.404)

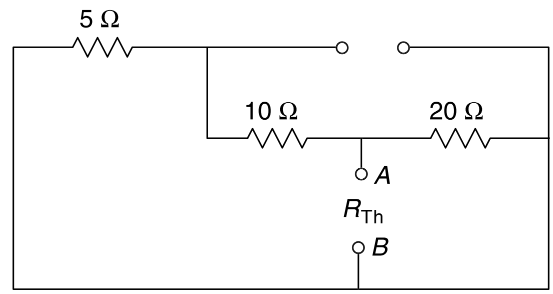


Fig. 3.404

$$R_{Th} = 15 \,||\, 20 = 8.57\ \Omega$$

Step III Calculation of R_L

For maximum power transfer,

$$R_L = R_{Th} = 8.57\ \Omega$$

Step IV Calculation of P_{max} (Fig. 3.405)

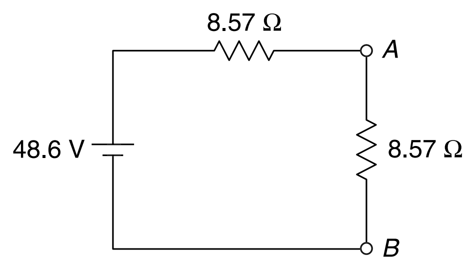


Fig. 3.405

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(48.6)^2}{4\times 8.57} = 68.9 \text{ W}$$

Example 3.92 *For the value of resistance R_L in Fig. 3.406 for maximum power transfer and calculate the maximum power.*

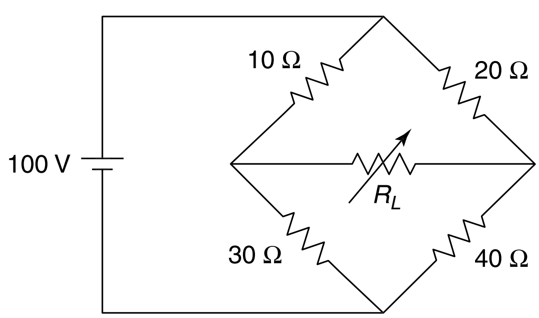


Fig. 3.406

Solution

Step I Calculation of V_{Th} (Fig. 3.407)

$$I_1 = \frac{100}{10+30} = 2.5 \text{ A}$$

$$I_2 = \frac{100}{20+40} = 1.66 \text{ A}$$

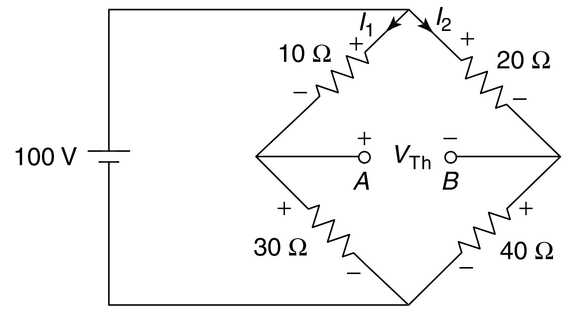


Fig. 3.407

Writing the V_{Th} equation,

$$V_{Th} + 10\, I_1 - 20 I_2 = 0$$

$$V_{Th} = 20 I_2 - 10 I_1 = 20(1.66) - 10(2.5) = 8.2 \text{ V}$$

Step II Calculation of R_{Th} (Fig. 3.408)

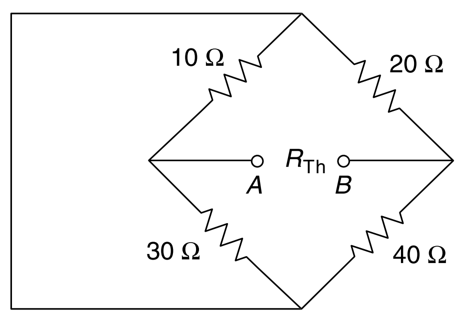


Fig. 3.408

Redrawing the network (Fig. 3.409),

$$R_{Th} = (10 \,||\, 30) + (20 \,||\, 40) = 20.83\ \Omega$$

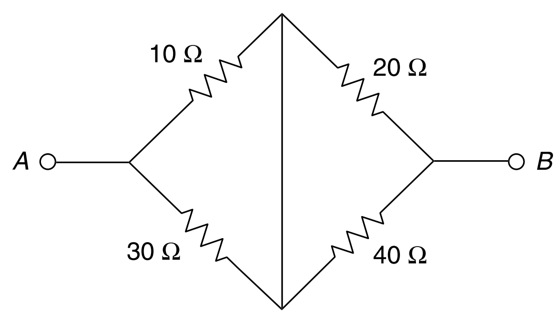


Fig. 3.409

Step III Value of R_L

For maximum power transfer,

$$R_L = R_{Th} = 20.83\ \Omega$$

Step IV Calculation of P_{max} (Fig. 3.410)

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(8.2)^2}{4 \times 20.83} = 0.81 \text{ W}$$

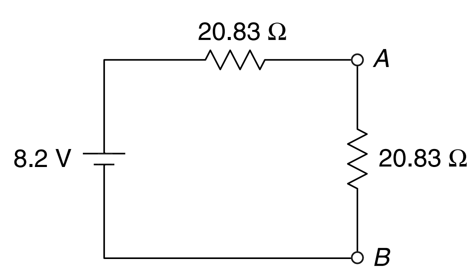


Fig. 3.410

Example 3.93 *For the value of resistance R_L in Fig. 3.411 for maximum power transfer and calculate the maximum power.*

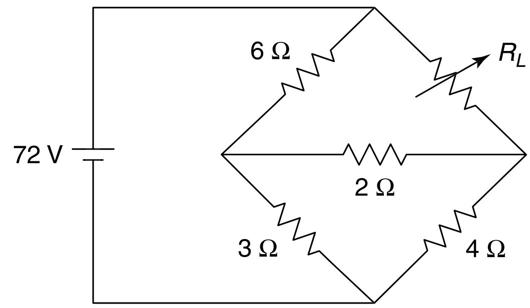


Fig. 3.411

Solution

Step I Calculation of V_{Th} (Fig. 3.412)

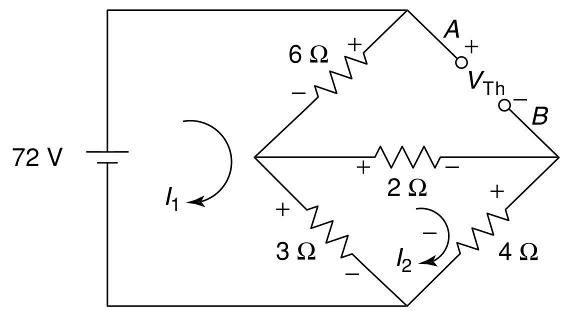


Fig. 3.412

Applying KVL to Mesh 1,

$$72 - 6I_1 - 3(I_1 - I_2) = 0$$

$$9I_1 - 3I_2 = 72 \quad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-3(I_2 - I_1) - 2I_2 - 4I_2 = 0$$

$$-3I_1 + 9I_2 = 0 \quad \ldots\text{(ii)}$$

Solving Eqs (i) and (ii),

$$I_1 = 9 \text{ A}$$

$$I_2 = 3 \text{ A}$$

Writing the V_{Th} equation,

$$V_{\text{Th}} - 6I_1 - 2I_2 = 0$$

$$V_{\text{Th}} = 6I_1 + 2I_2 = 6(9) + 2(3) = 60 \text{ V}$$

Step II Calculation of R_{Th} (Fig. 3.413)

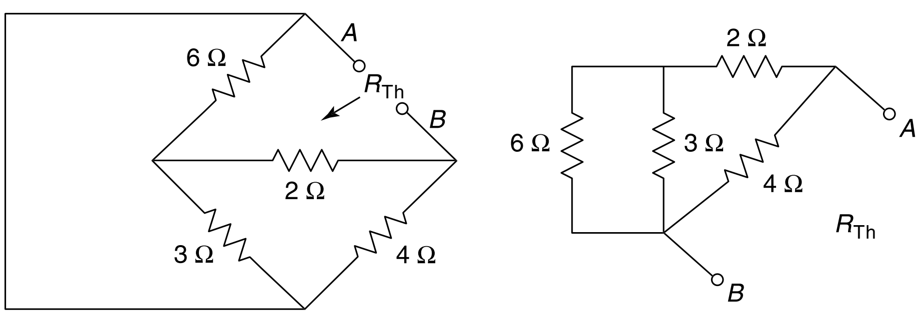


Fig. 3.413

$$R_{\text{Th}} = [(6 \,||\, 3) + 2] \,||\, 4 = 2\ \Omega$$

Step III Calculation of R_L

For maximum power transfer,

$$R_L = R_{Th} = 2\ \Omega$$

Step IV Calculation of P_{max} (Fig. 3.414)

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(60)^2}{4\times 2} = 450\ \text{W}$$

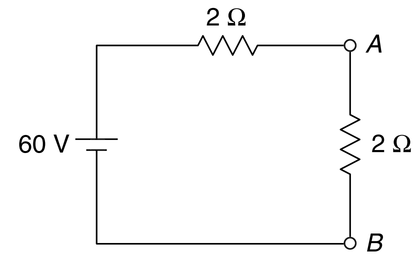


Fig. 3.414

Example 3.94 *For the network shown in Fig. 3.415 find the value of the resistance R_L for maximum power transfer and calculate maximum power.*

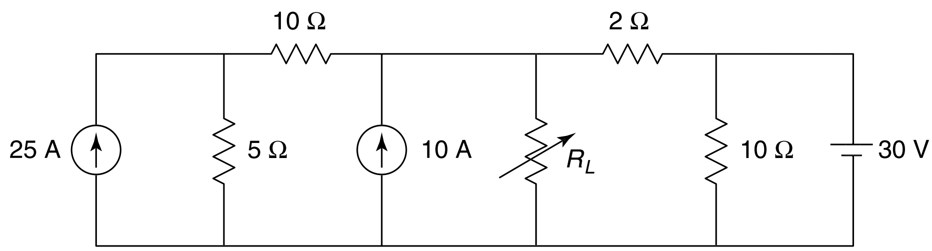


Fig. 3.415

Solution

Step I Calculation of V_{Th} (Fig. 3.416)

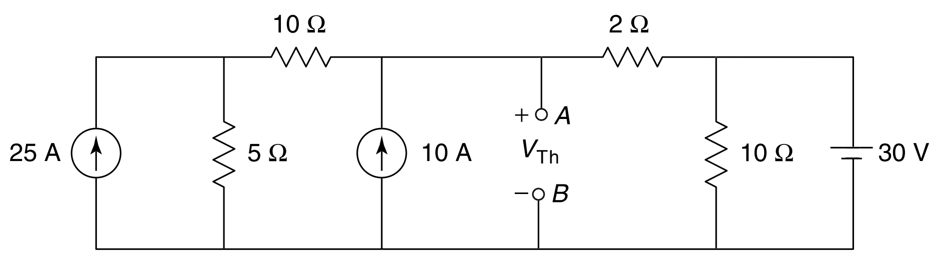


Fig. 3.416

By source transformation, the current source of 25 A and the 5 Ω resistor is converted into an equivalent voltage source of 125 V and a series resistor of 5 Ω. Also the voltage source of 30 V is connected across the 10 Ω resistor. Hence, the 10 Ω resistor becomes redundant (Fig. 3.417).

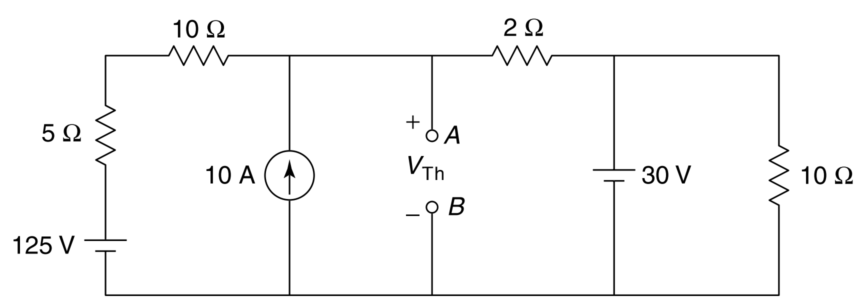


Fig. 3.417

Applying KCL at the node,

$$\frac{V_{Th}-125}{15}-10+\frac{V_{Th}-30}{2}=0$$

$$V_{Th}=58.81\text{ V}$$

Step II Calculation of R_{Th} (Fig. 3.418)

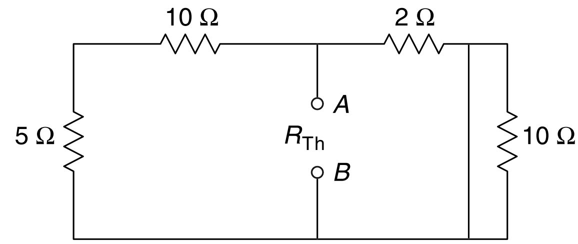


Fig. 3.418

$$R_{Th}=15\,||\,2=1.76\ \Omega$$

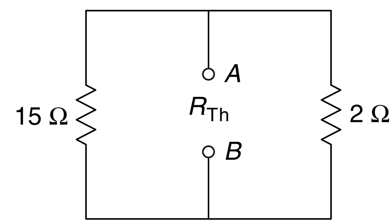


Fig. 3.419

Step III Value of R_L

For maximum power transfer

$$R_L=R_{Th}=1.76\ \Omega$$

Step IV Calculation of P_{max} (Fig. 3.420)

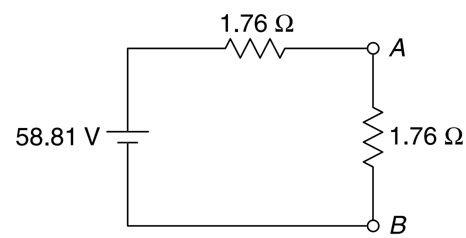


Fig. 3.420

$$P_{max}=\frac{V_{Th}^2}{4R_{Th}}=\frac{(58.81)^2}{4\times1.76}=491.28\text{ W}$$

Example 3.95 *For the network shown in Fig. 3.421, find the value of the resistance R_L for maximum power transfer and calculate maximum power.*

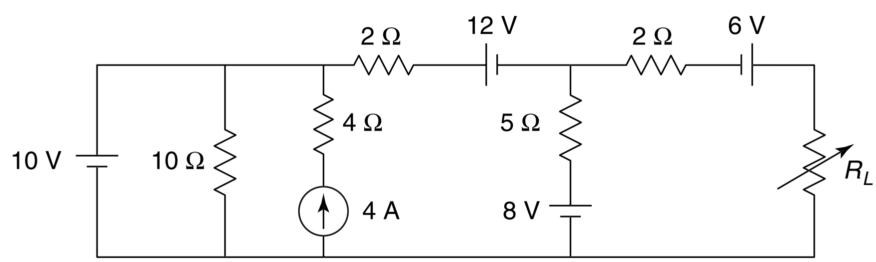


Fig. 3.421

Solution

Step I Calculation of V_{Th} (Fig. 3.422)

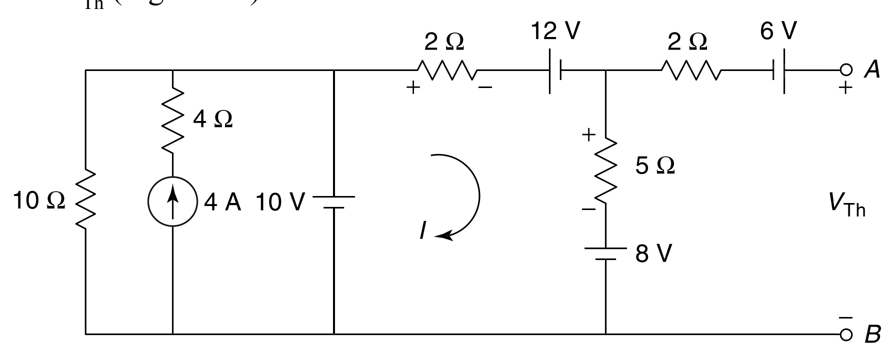


Fig. 3.422

Applying KVL to the outer path,

$$10 - 2I - 12 - 5I - 8 = 0$$

$$I = -\frac{10}{7} = -1.43 \text{ A}$$

Writing the V_{Th} equation,

$$8 + 5I + 6 - V_{Th} = 0$$

$$V_{Th} = 8 + 6 + 5I = 8 + 6 + 5(-1.43) = 6.85 \text{ V}$$

Step II Calculation of R_{Th} (Fig. 3.423)

$$R_{Th} = (2 \| 5) + 2$$
$$= 3.43\ \Omega$$

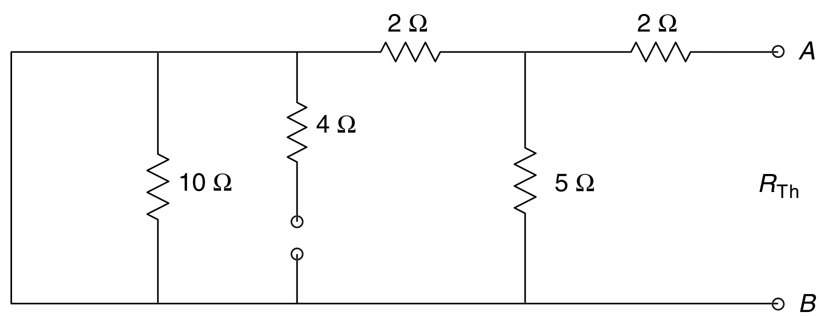


Fig. 3.423

Step III Value of R_L

For maximum power transfer,

$$R_L = R_{Th} = 3.43\ \Omega$$

Step IV Calculation of P_{max} (Fig. 3.424)

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(6.85)^2}{4 \times 3.43} = 3.42 \text{ W}$$

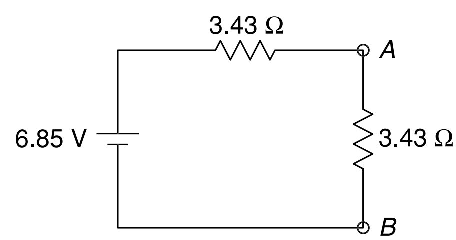


Fig. 3.424

EXAMPLES WITH DEPENDENT SOURCES

Example 3.96 *For the network shown in Fig. 3.425, find the value of R_L for maximum power transfer. Also, calculate maximum power.*

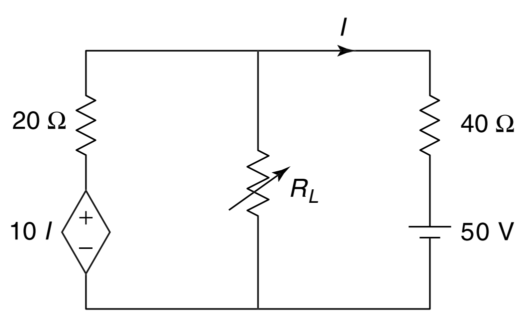


Fig. 3.425

Solution

Step I Calculation of V_{Th} (Fig. 3.426)

Applying KVL to the mesh,

$$10I - 20I - 40I - 50 = 0$$
$$I = -1 \text{ A}$$

Writing the V_{Th} equation,

$$V_{\text{Th}} - 40I - 50 = 0$$
$$V_{\text{Th}} - 40(-1) - 50 = 0$$
$$V_{\text{Th}} = 10 \text{ V}$$

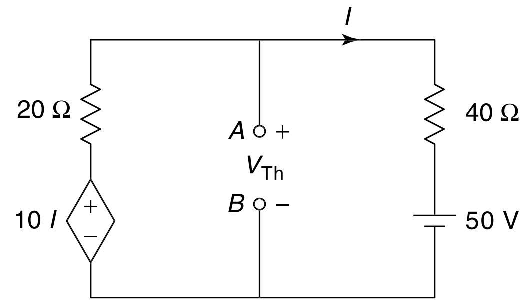


Fig. 3.426

Step II Calculation of I_N (Fig. 3.427)

From Fig. 3.427,

$$I = I_2 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 1,

$$10I - 20I_1 = 0$$
$$10I_2 - 20I_1 = 0 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 2,

$$-40I_2 - 50 = 0$$
$$I_2 = -1.25 \text{ A} \qquad \ldots\text{(iii)}$$

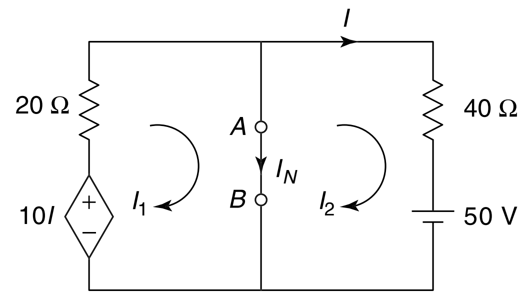


Fig. 3.427

Solving Eqs (i), (ii) and (iii),

$$I_1 = -0.625 \text{ A}$$
$$I_N = I_1 - I_2 = -0.625 + 1.25 = 0.625 \text{ A}$$

Step III Calculation of R_N

$$R_{\text{Th}} = \frac{V_{\text{Th}}}{I_N} = \frac{10}{0.625} = 16\ \Omega$$

Step IV Calculation of R_L

For maximum power transfer,

$$R_L = R_{\text{Th}} = 16\ \Omega$$

Step V Calculation of P_{max} (Fig. 3.428)

$$P_{\text{max}} = \frac{V_{\text{Th}}}{4\,R_{\text{Th}}} = \frac{(10)^2}{4 \times 16} = 1.56 \text{ W}$$

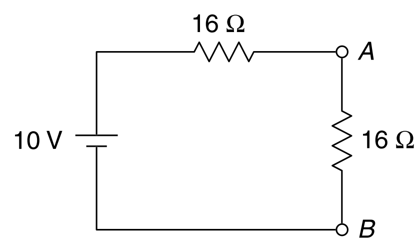


Fig. 3.428

Example 3.97 *For the network shown in Fig. 3.429, calculate the maximum power that may be dissipated in the load resistor R_L.*

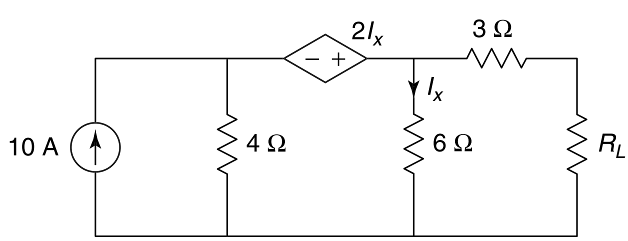


Fig. 3.429

Solution

Step I Calculation of V_{Th} (Fig. 3.430)

From Fig. 3.430,

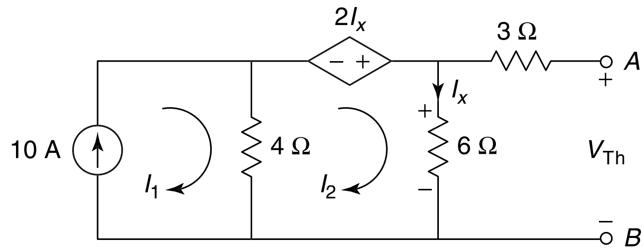


Fig. 3.430

$$I_x = I_2 \quad \text{...(i)}$$

For Mesh 1,

$$I_1 = 10 \quad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$-4(I_2 - I_1) + 2I_x - 6I_2 = 0$$
$$-4I_2 + 4I_1 + 2I_2 - 6I_2 = 0$$
$$4I_1 - 8I_2 = 0 \quad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 10 \text{ A}$$
$$I_2 = 5 \text{ A}$$

Writing the V_{Th} equation,

$$6I_2 - 0 - V_{Th} = 0$$
$$V_{Th} = 6I_2 = 6(5) = 30 \text{ V}$$

Step II Calculation of I_N (Fig. 3.431)

From Fig. 3.431,

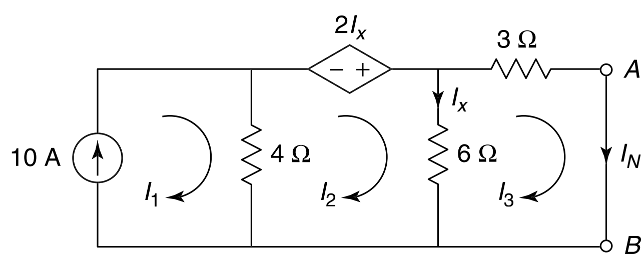


Fig. 3.431

$$I_x = I_2 - I_3 \quad \text{...(i)}$$

For Mesh 1,

$$I_1 = 10 \quad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$-4(I_2 - I_1) + 2I_x - 6(I_2 - I_3) = 0$$
$$-4I_2 + 4I_1 + 2(I_2 - I_3) - 6I_2 + 6I_3 = 0$$
$$4I_1 - 8I_2 + 4I_3 = 0 \quad \text{...(iii)}$$

Applying KVL to Mesh 3,

$$-6(I_3 - I_2) - 3I_3 = 0$$
$$6I_2 - 9I_3 = 0 \quad \text{...(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = 10 \text{ A}$$
$$I_2 = 7.5 \text{ A}$$
$$I_3 = 5 \text{ A}$$
$$I_N = I_3 = 5 \text{ A}$$

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{30}{5} = 6\ \Omega$$

Step IV Calculation of R_L
For maximum power transfer,

$$R_L = R_{Th} = 6\ \Omega$$

Step V Calculation of P_{max}

$$P_{max} = \frac{V_{Th}^2}{4\,R_{Th}} = \frac{(30)^2}{4\times 16} = 37.5\ \text{W}$$

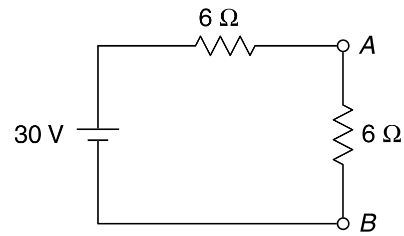


Fig. 3.432

Example 3.98 *For the network shown in Fig. 3.433, find the value of R_L for maximum power transfer. Also, find maximum power.*

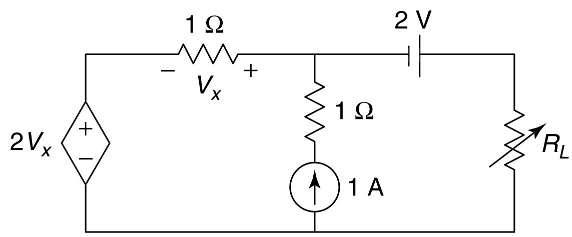


Fig. 3.433

Solution

Step I Calculation of V_{Th} (Fig. 3.434)
From Fig. 3.434,

$$V_x = -1I = -I \qquad \text{...(i)}$$

For Mesh 1,

$$I = -1 \qquad \text{...(ii)}$$
$$V_x = 1\ \text{V}$$

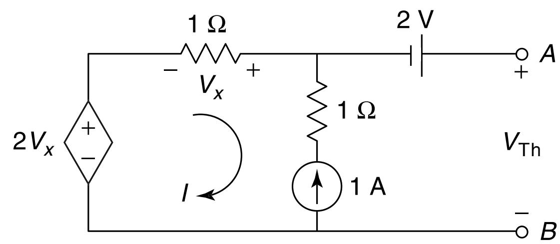


Fig. 3.434

Writing the V_{Th} equation,

$$2V_x - 1I + 2 - V_{Th} = 0$$
$$2(1) - (-1) + 2 - V_{Th} = 0$$
$$V_{Th} = 5\ \text{V}$$

Step II Calculation of I_N (Fig. 3.435)
From Fig. 3.435,

$$V_x = -1I_1 = -I_1 \qquad \text{...(i)}$$

Meshes 1 and 2 will form a supermesh.
Writing the current equation for the supermesh,

$$I_2 - I_1 = 1 \qquad \text{...(ii)}$$

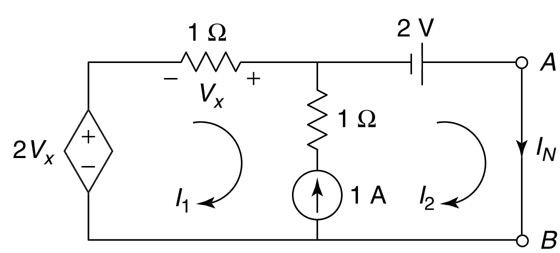


Fig. 3.435

Applying KVL to the outer path of the supermesh,

$$2V_x - 1I_1 + 2 = 0$$
$$2(-I_1) - I_1 + 2 = 0$$
$$3I_1 = 0 \qquad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 0.67 \text{ A}$$
$$I_2 = 1.67 \text{ A}$$
$$I_N = I_2 = 1.67 \text{ A}$$

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{5}{1.67} = 3\ \Omega$$

Step IV Calculation of R_L

For maximum power transfer,

$$R_L = R_{Th} = 3\ \Omega$$

Step V Calculation of P_{max} (Fig. 3.436)

$$P_{max} = \frac{V_{Th}^2}{4\,R_{Th}} = \frac{(5)^2}{4 \times 3} = 2.08 \text{ W}$$

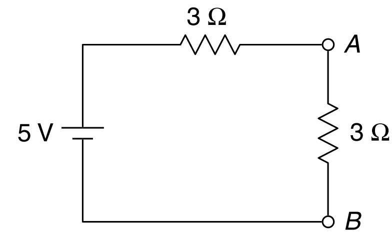


Fig. 3.436

Example 3.99 *What will be the value of R_L in Fig. 3.437 to get maximum power delivered to it? What is the value of this power?*

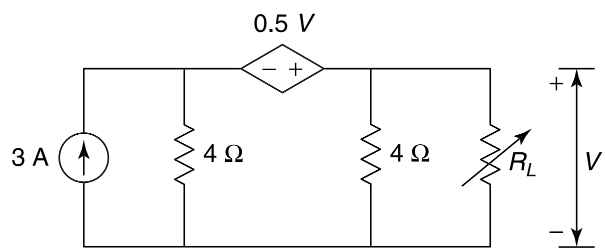


Fig. 3.437

Solution

Step I Calculation of V_{Th} (Fig. 3.438)

By source transformation,

From Fig. 3.438,

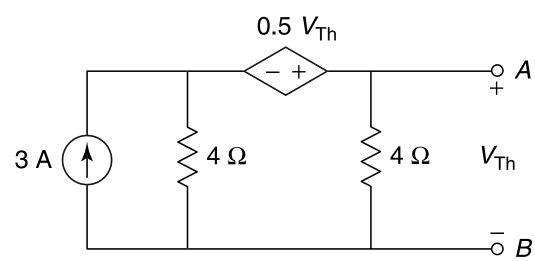


Fig. 3.438

$$V_{Th} = 4I$$

Applying KVL to the mesh,

$$12 - 4I + 0.5\,V_{Th} - 4I = 0$$
$$12 - V_{Th} + 0.5\,V_{Th} - V_{Th} = 0$$
$$V_{Th} = 8 \text{ V}$$

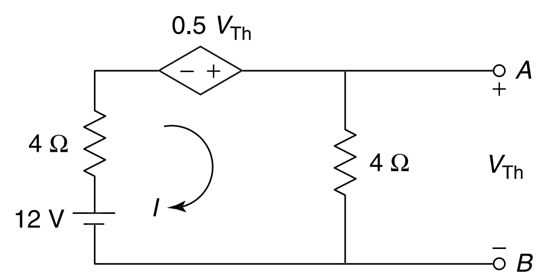


Fig. 3.439

Step II Calculation of I_N (Fig. 3.439)

If two terminals A and B are shorted, the 4 Ω resistor gets shorted.

$$V = 0$$

Dependent source 0.5 V depends on the controlling variable V. When $V = 0$, the dependent source vanishes, ie. 0.5 $V = 0$ as shown in Fig. 3.441.

$$I_N = \frac{12}{4} = 3 \text{ A}$$

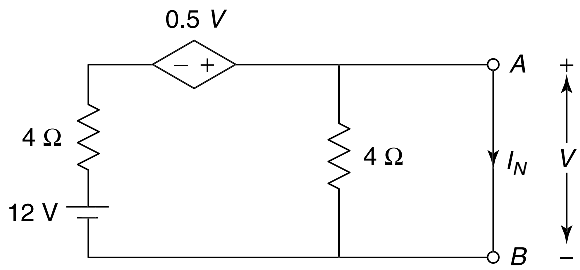


Fig. 3.440

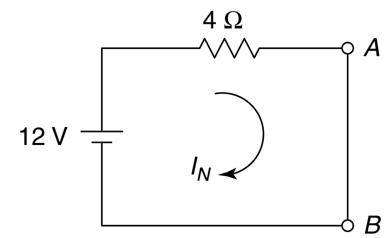


Fig. 3.441

Step III Calculation of R_{Th}

$$R_{\text{Th}} = \frac{V_{\text{Th}}}{I_N} = \frac{8}{3} = 2.67\ \Omega$$

Step IV *Calculation of R_L*
For maximum power transfer,

$$R_L = R_{\text{Th}} = 2.67\ \Omega$$

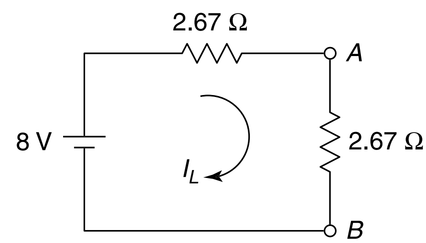


Fig. 3.442

Step V Calculation of P_{max} (Fig. 3.442)

$$P_{\text{max}} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(8)^2}{4 \times 2.67} = 6 \text{ W}$$

3.6 RECIPROCITY THEOREM

It states that '*in a linear, bilateral, active, single source network, the ratio of excitation to response remains same when the positions of excitation and response are interchanged.*'

In other words, it may be stated as 'if a single voltage source V_a in the branch 'a' produces a current I_b in the branch 'b' then if the voltage source V_a is removed and inserted in the branch 'b', it will produce a current I_b in branch 'a''.

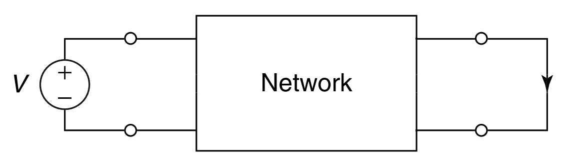


Fig. 3.443 *Network*

Explanation Consider a network shown in Fig. 3.443.

When the voltage source V is applied at the port 1, it produces a current I at the port 2. If the positions of the excitation (source) and response are interchanged, i.e., if the voltage source is applied at the port 2 then it produces a current I at the port 1.

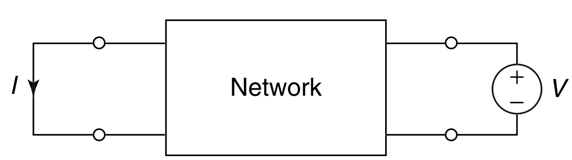


Fig. 3.444 *Network when excitation and response are interchanged*

The limitation of this theorem is that it is applicable only to a single-source network. This theorem is not applicable in the network which has a dependent source. This is applicable only in linear and bilateral networks. In the reciprocity theorem, position of any passive element (R, L, C) do not change. Only the excitation and response are interchanged.

Steps to be followed in Reciprocity Theorem

1. Identify the branches between which reciprocity is to be established.
2. Find the current in the branch when excitation and response are not interchanged.
3. Find the current in the branch when excitation and response are interchanged.

Example 3.100 *Calculate current I and verify the reciprocity theorem for the network shown in Fig. 3.445.*

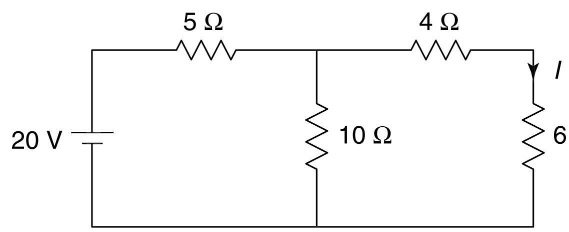


Fig. 3.445

Solution

Case I Calculation of current I when excitation and response are not interchanged (Fig. 3.446)

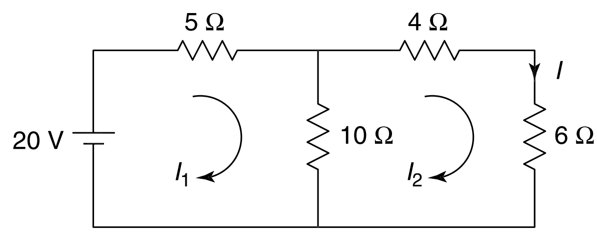


Fig. 3.446

Applying KVL to Mesh 1,

$$20-5I_1-10(I_1-I_2)=0$$

$$15I_1-10I_2=20 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-10(I_2-I_1)-4I_2-6I_2=0$$

$$-10I_1+20I_2=0 \qquad \ldots\text{(ii)}$$

Solving. Eqs (i) and (ii),

$$I_1=2\text{ A}$$

$$I_2=1\text{ A}$$

$$I=I_2=1\text{ A}$$

Case II Calculation of current I when excitation and response are interchanged (Fig. 3.447).

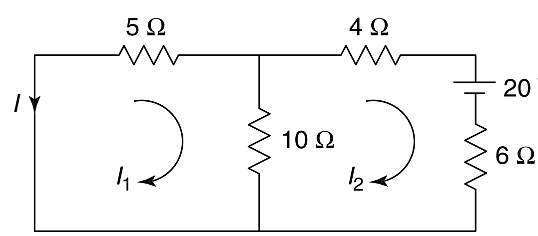


Fig. 3.447

Applying KVL to Mesh 1,

$$-5I_1-10(I_1-I_2)=0$$

$$15I_1-10I_2=0 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-10(I_2-I_1)-4I_2-20-6I_2=0$$

$$-10I_1+20I_2=-20 \qquad \ldots\text{(ii)}$$

Solving Eqs (i) and (ii),

$$I_1=-1\text{ A}$$

$$I_2=-1.5\text{ A}$$

$$I=-I_1=1\text{ A}$$

Since the current I remains the same in both the cases, reciprocity theorem is verified.

Example 3.101 *Find the current I and verify reciprocity theorem for the network shown in Fig. 3.446.*

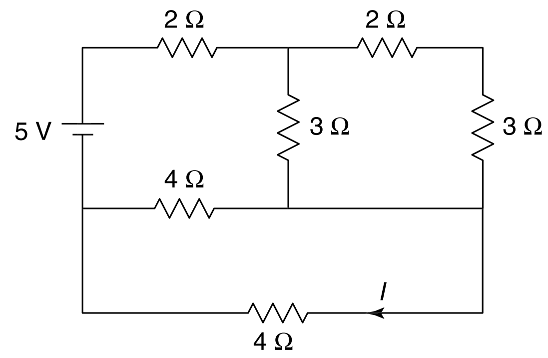


Fig. 3.448

Solution

Case I Calculation of the current I when excitation and response are not interchanged (Fig. 3.449)

Applying KVL to Mesh 1,

$$5 - 2I_1 - 3(I_1 - I_2) - 4(I_1 - I_3) = 0$$

$$9I_1 - 3I_2 - 4I_3 = 5 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-3(I_2 - I_1) - 2I_2 - 3I_2 = 0$$

$$-3I_1 + 8I_2 = 0 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-4(I_3 - I_1) - 4I_3 = 0$$

$$-4I_1 + 8I_3 = 0 \qquad \ldots\text{(iii)}$$

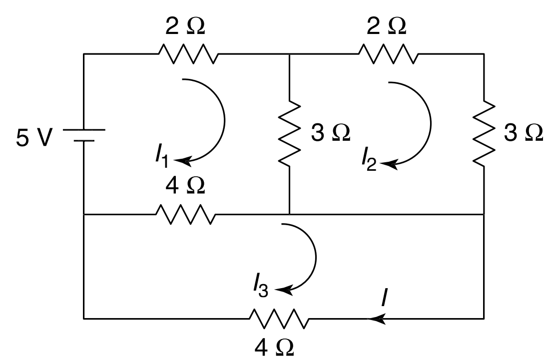


Fig. 3.449

Solving Eqs (i), (ii) and (iii),

$$I_1 = 0.85 \text{ A}$$

$$I_2 = 0.32 \text{ A}$$

$$I_3 = 0.43 \text{ A}$$

$$I = I_3 = 0.43 \text{ A}$$

Case II Calculation of current I when excitation and response are interchanged (Fig. 3.450).

Applying KVL to Mesh 1,

$$-2I_1 - 3(I_1 - I_2) - 4(I_1 - I_3) = 0$$

$$9I_1 - 3I_2 - 4I_3 = 0 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-3(I_2 - I_1) - 2I_2 - 3I_2 = 0$$

$$-3I_1 + 8I_2 = 0 \qquad \ldots\text{(ii)}$$

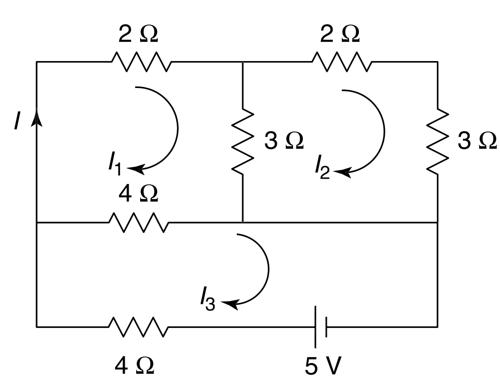


Fig. 3.450

Applying KVL to Mesh 3,

$$-4(I_3 - I_1) + 5 - 4I_3 = 0$$

$$-4I_1 + 8I_3 = 5 \quad \ldots\text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1 = 0.43 \text{ A}$$

$$I_2 = 0.16 \text{ A}$$

$$I_3 = 0.84 \text{ A}$$

$$I = I_1 = 0.43 \text{ A}$$

Since the current I remains the same in both the cases, reciprocity theorem is verified.

Example 3.102 *Find the voltage V and verify reciprocity theorem for the network shown in Fig. 3.451.*

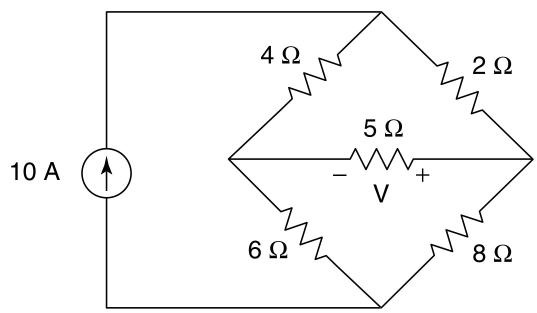


Fig. 3.451

Solution

Case I Calculation of the voltage V when excitation and response are not interchanged (Fig. 3.451)

For Mesh 1,

$$I_1 = 10 \quad \ldots\text{(i)}$$

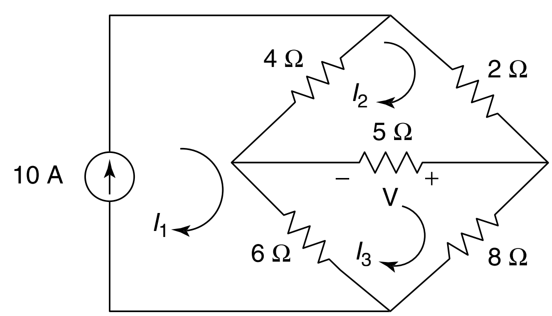


Fig. 3.452

Applying KVL to Mesh 2,

$$-4(I_2 - I_1) - 2I_2 - 5(I_2 - I_3) = 0$$

$$-4I_1 + 11I_2 - 5I_3 = 0 \quad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-6(I_3 - I_1) - 5(I_3 - I_2) - 8I_3 = 0$$

$$-6I_1 - 5I_2 + 19I_3 = 0 \quad \ldots\text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1 = 10 \text{ A}$$

$$I_2 = 5.76 \text{ A}$$

$$I_3 = 4.67 \text{ A}$$

$$V = 5(I_2 - I_3) = 5(5.76 - 4.67) = 5.45 \text{ V}$$

Case II Calculation of voltage V when excitation and response are interchanged (Fig. 3.453).

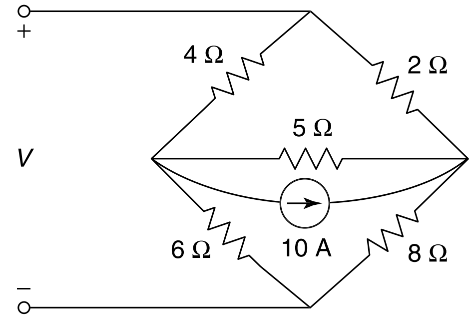


Fig. 3.453

By source transformation (Fig. 3.454),

Applying KVL to Mesh 1,

$$-4I_1 - 2I_1 - 50 - 5(I_1 - I_2) = 0$$

$$11I_1 - 5I_2 = -50 \quad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-6I_2 - 5(I_2 - I_1) + 50 - 8I_2 = 0$$

$$-5I_1 + 19I_2 = 50 \quad \ldots\text{(ii)}$$

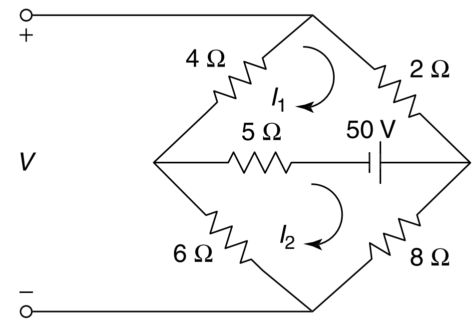


Fig. 3.454

Solving Eqs (i) and (ii),

$$I_1 = -3.8 \text{ A}$$

$$I_2 = 1.63 \text{ A}$$

From Fig. 3.454,

$$V + 4I_1 + 6I_2 = 0$$

$$V + 4(-3.8) + 6(1.63) = 0$$

$$V = 5.42 \text{ V}$$

Since the voltage V is same in both the cases, the reciprocity theorem is verified.

3.7 MILLMAN'S THEOREM

It states that '*if there are n voltage sources $V_1, V_2, \ldots, V_n$ with internal resistances $R_1, R_2, \ldots, R_n$ respectively connected in parallel then these voltage sources can be replaced by a single voltage source V_m and a single series resistance R_m*, '(Fig. 3.455).

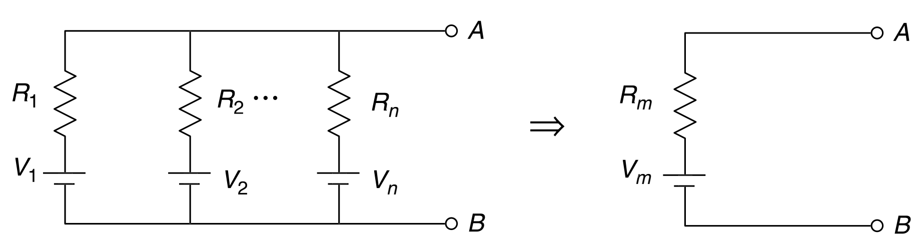


Fig. 3.455 *Millman's network*

where
$$V_m = \frac{V_1G_1 + V_2G_2 + \ldots + V_nG_n}{G_1 + G_2 + \ldots + G_n}$$

Exercises

Superposition Theorem

3.1 Find the current through the 10 Ω resistor in Fig. 3.490.

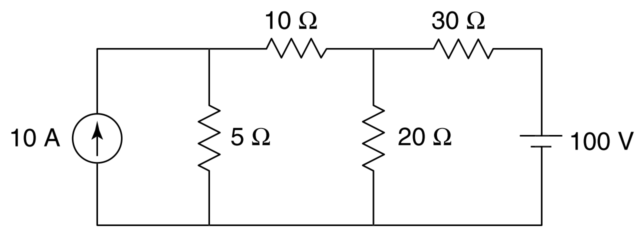


Fig. 3.490

[0.37 A]

3.2 Find the current through the 8 Ω resistor in Fig. 3.491.

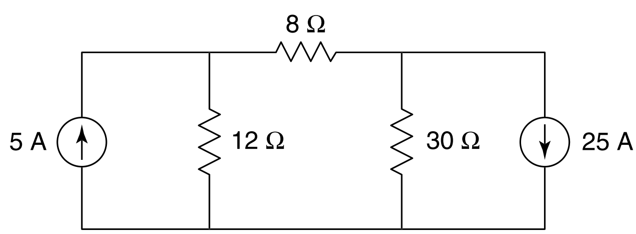


Fig. 3.491

[16.2 A]

3.3 Find the potential across the 3 Ω resistor in Fig. 3.492.

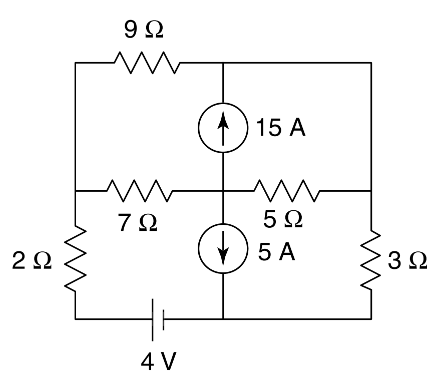


Fig. 3.492

[3.3 V]

3.4 Calculate the current through the 10 Ω resistor in Fig. 3.493.

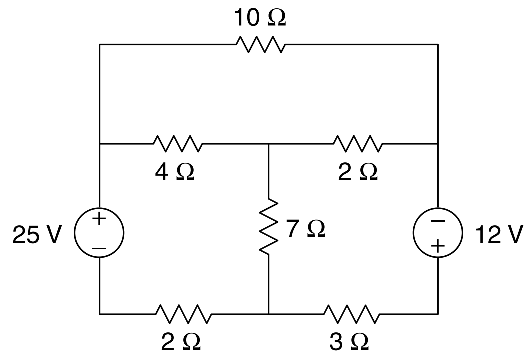


Fig. 3.493

[1.62 A]

3.5 Find the current through the 1Ω resistor in Fig. 3.494.

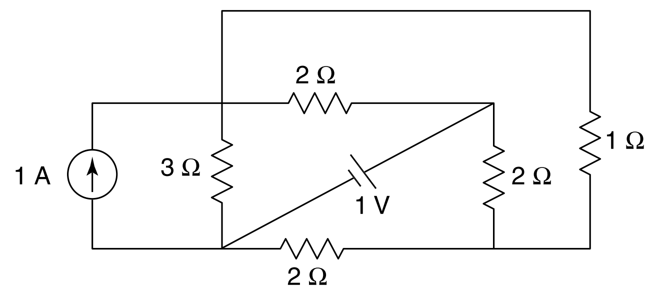


Fig. 3.494

[0.41 A]

3.6 Find the current through the 4 Ω resistor in Fig. 3.495.

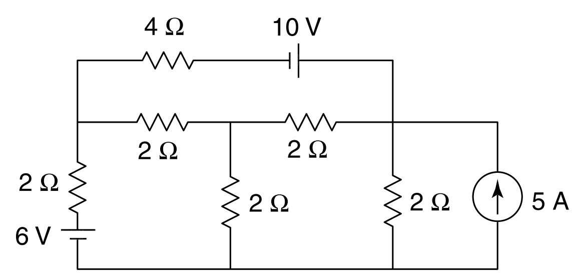


Fig. 3.495

[1.33 A]

3.7 Find the current I_x in Fig. 3.496.

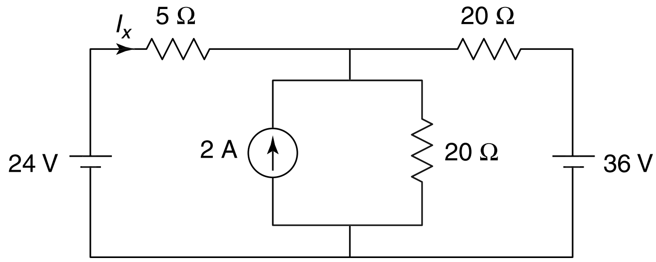


Fig. 3.496

[−1.143 A]

3.8 Find the voltage V_x in Fig. 3.497.

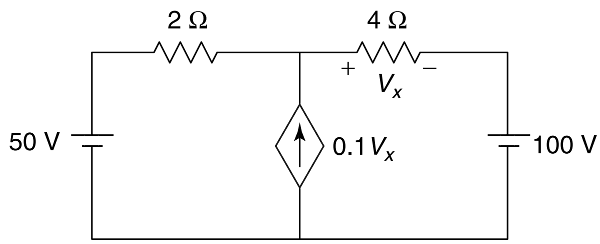


Fig. 3.497

[−38.5 V]

3.9 Determine the voltages V_1 and V_2 in Fig. 3.498.

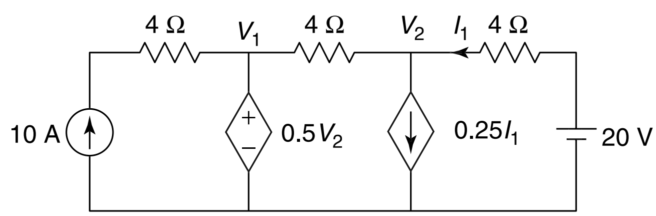


Fig. 3.498

[6 V,12 V]

3.10 Find the voltage V_x in Fig. 3.499.

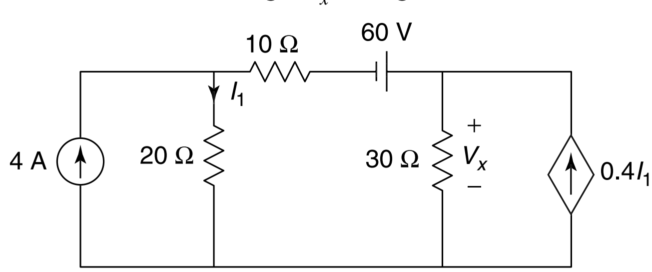


Fig. 3.499

[7.5 V]

Thevenin's Theorem

3.11 Find the current through the 5 Ω resistor in Fig. 3.500.

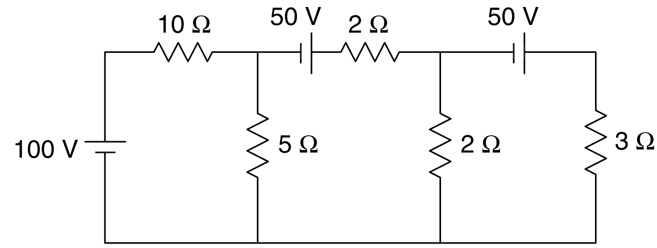


Fig. 3.500

[3.87 A]

3.12 Find the current through the 6 Ω resistor in Fig. 3.501.

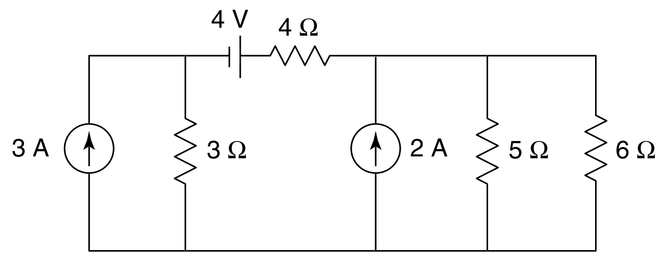


Fig. 3.501

[1.26 A]

3.13 Find the current through the 6 Ω resistor in Fig. 3.502.

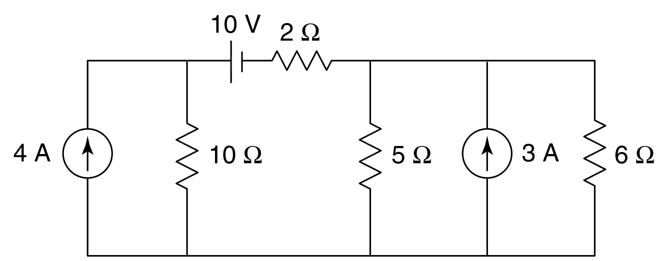


Fig. 3.502

[2.04 A]

3.14 Find the current through the 2 Ω resistor connected between terminals *A* and *B* in Fig. 3.503.

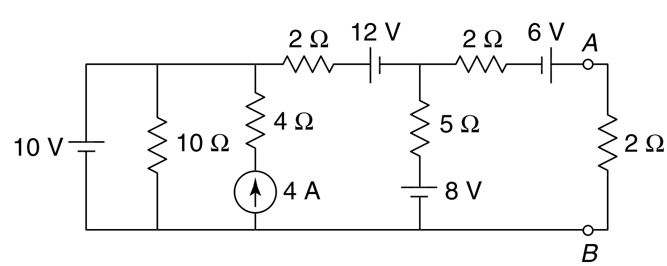


Fig. 3.503

[1.26 A]

3.15 Find the current through the 5 Ω resistor in Fig. 3.504.

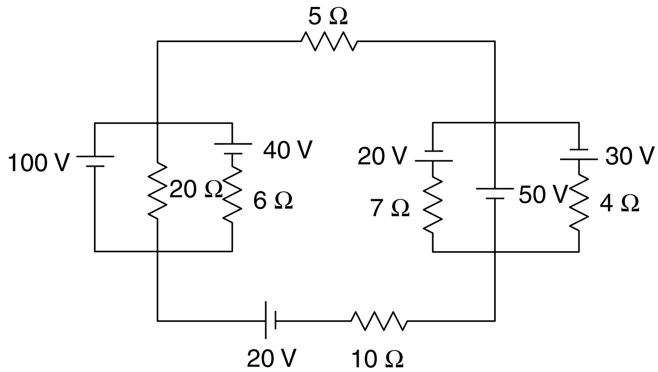


Fig. 3.504

[4.67 A]

3.16 Find the current through the 20 Ω resistor in Fig. 3.505.

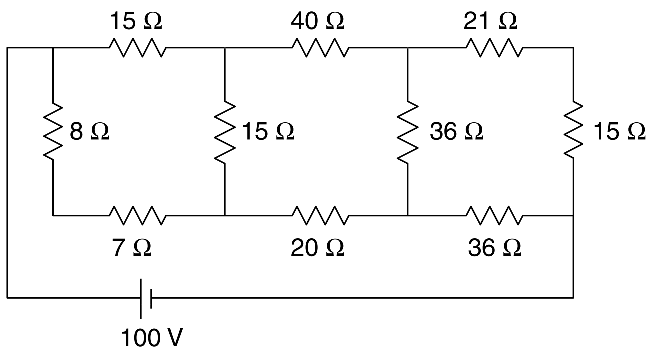


Fig. 3.505

[1.54 A]

3.17 Calculate the current through the 10 Ω resistor in Fig. 3.506.

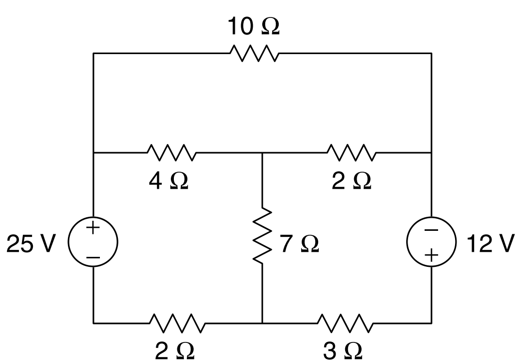


Fig. 3.506

[1.62 A]

3.18 Determine Thevenin's equivalent network for figures 3.507 to 3.510 shown below.

(i)

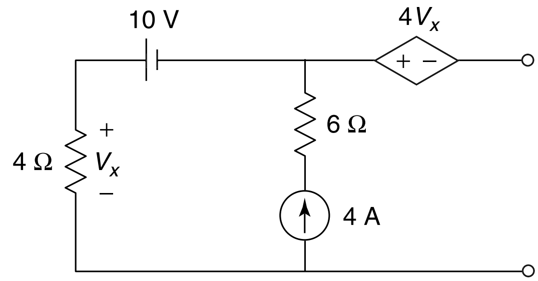


Fig. 3.507

[−58 V, 12 Ω]

(ii)

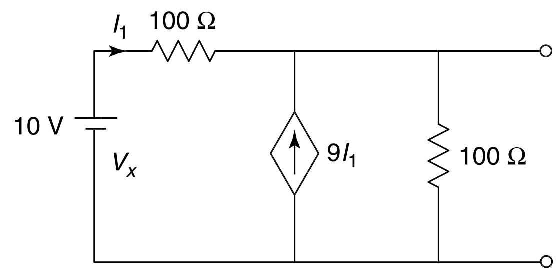


Fig. 3.508

[9.09 V, 9.09 Ω]

(iii)

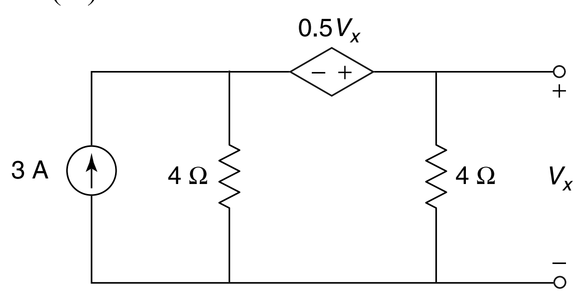


Fig. 3.509

[8 V, 2.66 Ω]

(iv)

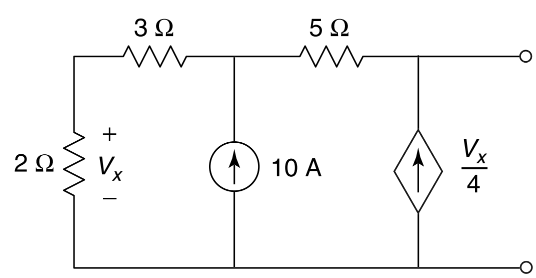


Fig. 3.510

[150 V, 20 Ω]

3.19 Find the current I_x in Fig. 3.511.

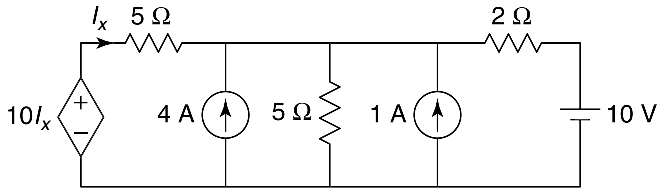


Fig. 3.511

[4 A]

3.20 Find the current in the 24 Ω resistor in Fig. 3.512.

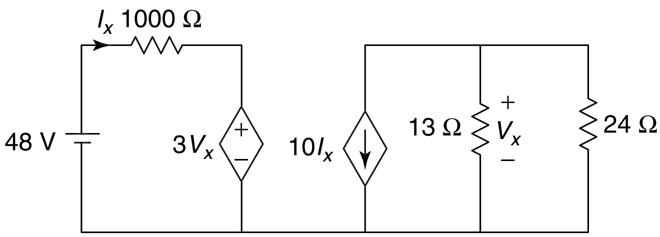


Fig. 3.512

[0.225 A]

Norton's Theorem

3.21 Find the current through the 10 Ω resistor in Fig. 3.513.

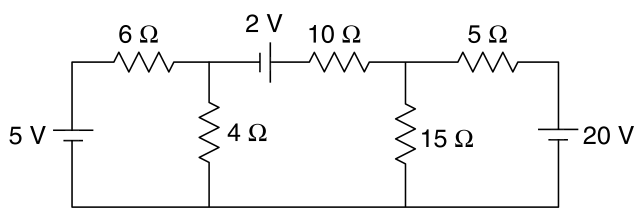


Fig. 3.513

[0.68 A]

3.22 Find the current through the 20 Ω resistor in Fig. 3.514.

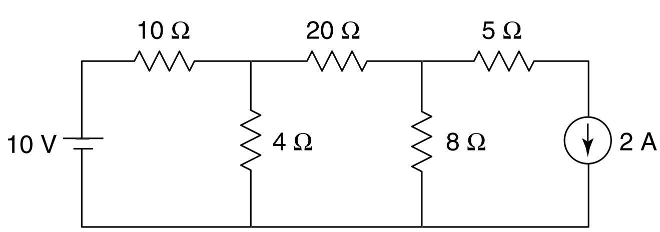


Fig. 3.514

[0.61 A]

3.23 Find the current through the 2 Ω resistor in Fig. 3.515.

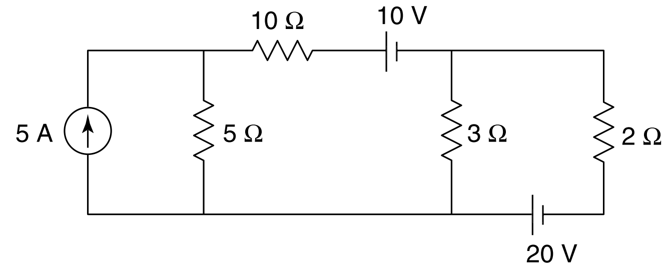


Fig. 3.515

[5 A]

3.24 Find the current through the 5 Ω resistor in Fig. 3.516.

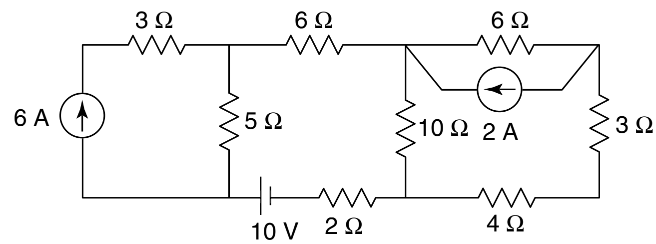


Fig. 3.516

[4.13 A]

3.25 Find Norton's equivalent circuit for the portion of network shown in Fig. 3.517 to the left of *ab*. Hence obtain the current in the 10 Ω resistor.

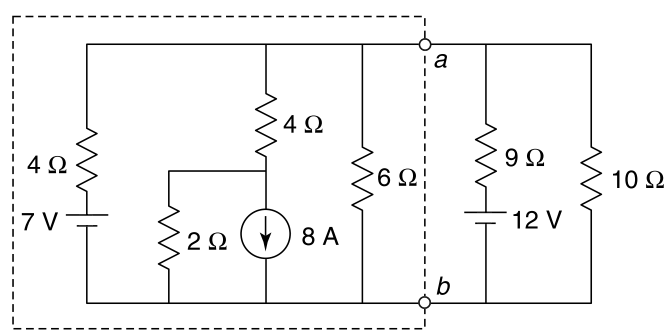


Fig. 3.517

[0.053 A]

3.26 Find Norton's equivalent network and hence find the current in the 10 Ω resistor in Fig. 3.518.

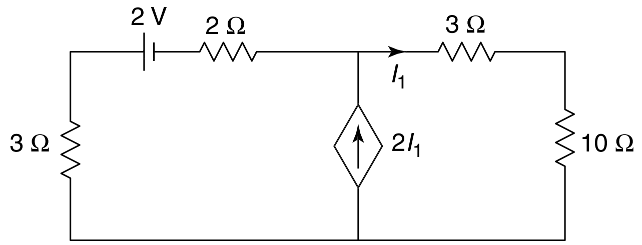


Fig. 3.518

$[0.25\ \text{A}]$

3.27 Find Norton's equivalent network In Fig. 3.519.

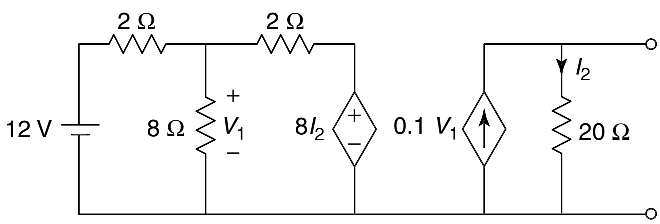


Fig. 3.519

$[0.533\ \text{A}, 31\ \Omega]$

Maximum Power Transfer Theorem

3.28 Find the value of the resistance R_L in Fig. 3.520 for maximum power transfer and calculate the maximum power.

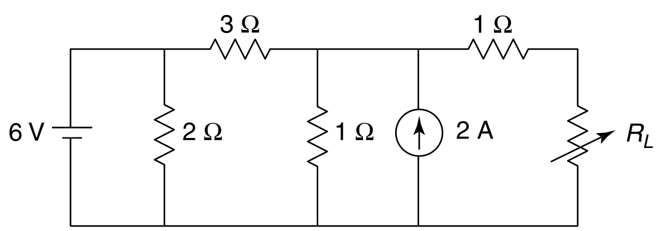


Fig. 3.520

$[1.75\ \Omega, 1.29\ \text{W}]$

3.29 Find the value of the resistance R_L in Fig. 3.521 for maximum power transfer and calculate the maximum power.

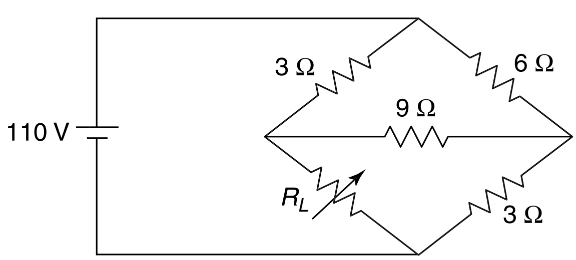


Fig. 3.521

$[2.36\ \Omega, 940\ \text{W}]$

3.30 Find the value of the resistance R_L in Fig. 3.522 for maximum power transfer and calculate the maximum power.

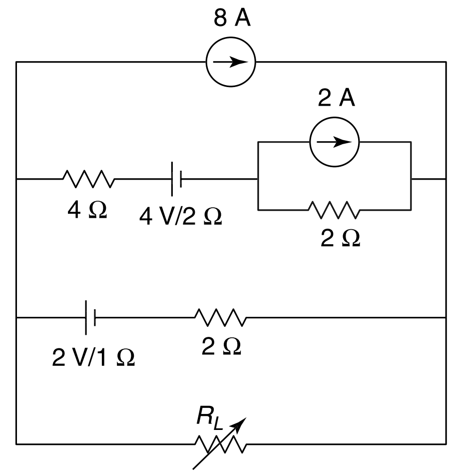


Fig. 3.522

$[2.18\ \Omega, 29.35\ \text{W}]$

3.31 Find the value of the resistance R_L in Fig. 3.523 for maximum power transfer and calculate the maximum power.

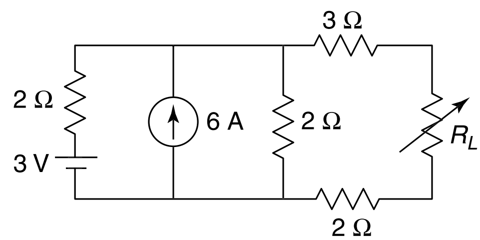


Fig. 3.523

$[3\ \Omega, 2.52\ \text{W}]$

3.32 Find the value of the resistance R_L in Fig. 3.524 for maximum power transfer and calculate the maximum power.

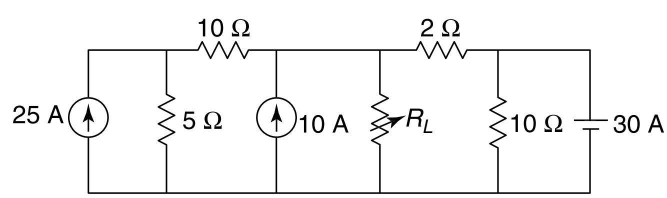


Fig. 3.524

$[1.76\ \Omega, 490.187\ \text{W}]$

Objective-Type Questions

3.1 The value of the resistance R connected across the terminals A and B in Fig. 3.525, which will absorb the maximum power is

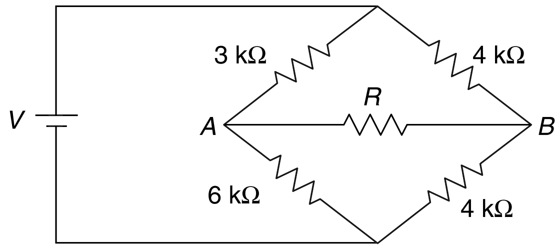


Fig. 3.525

(a) 4 kΩ (b) 4.11 kΩ
(c) 8 kΩ (d) 9 k Ω

3.2 Superposition theorem is not applicable to networks containing

(a) nonlinear elements
(b) dependent voltage source
(c) dependent current source
(d) transformers

3.3 The value of R required for maximum power transfer in the network shown in Fig. 3.526 is

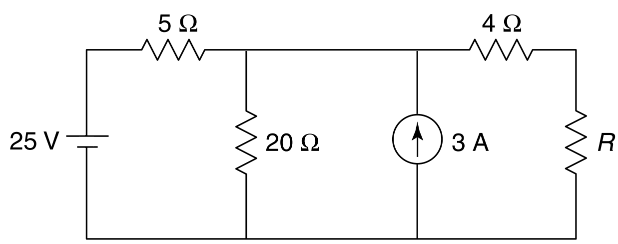


Fig. 3.526

(a) 2 Ω (b) 4 Ω (c) 8 Ω (d) 16 Ω

3.4 In the network of Fig. 3.527, the maximum power is delivered to R_L if its value is

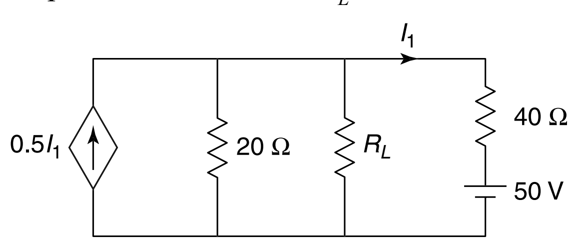


Fig. 3.527

(a) 16 Ω (b) $\frac{40}{3}\,\Omega$

(c) 60 Ω (d) 20 Ω

3.5 The maximum power that can be transferred to the load R_L from the voltage source in Fig. 3.528 is

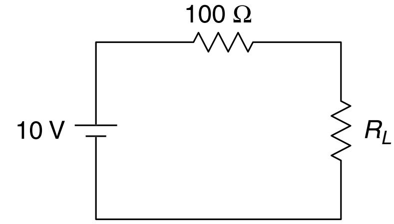


Fig. 3.528

(a) 1 W (b) 10 W
(c) 0.25 W (d) 0.5 W

3.6 For the circuit shown in Fig. 3.529, Thevenin's voltage and Thevenin's equivalent resistance at terminals *a*-*b* is

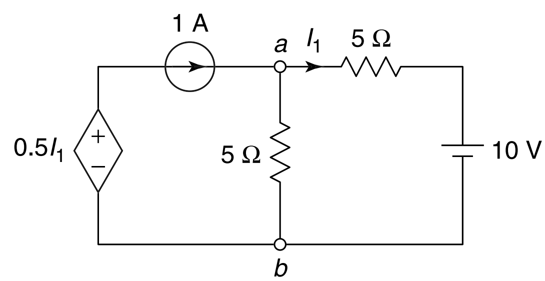


Fig. 3.529

(a) 5 V and 2 Ω (b) 7.5 V and 2.5 Ω
(c) 4 V and 2 Ω (d) 3 V and 2.5 Ω

3.7 The value of R_L in Fig. 3.530 for maximum power transfer is

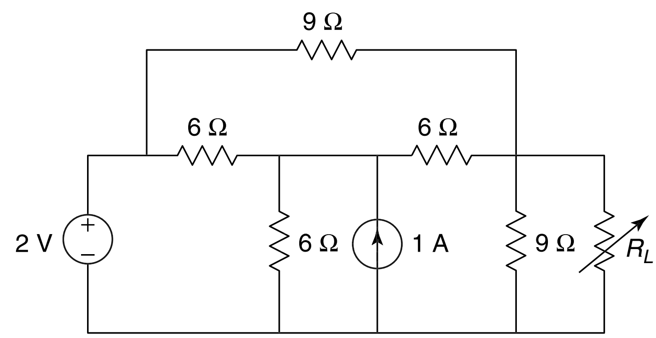


Fig. 3.530

(a) 3 Ω (b) 1.125 Ω
(c) 4.1785 Ω (d) none of these

6 Network Theorems (Application to ac Circuits)

6.1 INTRODUCTION

We have discussed the network theorems with reference to resistive load and dc sources. Now, all the theorems will be discussed when a network consists of ac sources, resistors, inductors and capacitors. All the theorems are also valid for ac sources.

6.2 MESH ANALYSIS

Mesh analysis is useful if a network has a large number of voltage sources. In this method, currents are assigned in each mesh. We can write mesh equations by Kirchhoff's voltage law in terms of unknown mesh currents,

Example 6.1 *Find mesh currents $\mathbf{I}_1$ and $\mathbf{I}_2$ in the network of Fig. 6.1.*

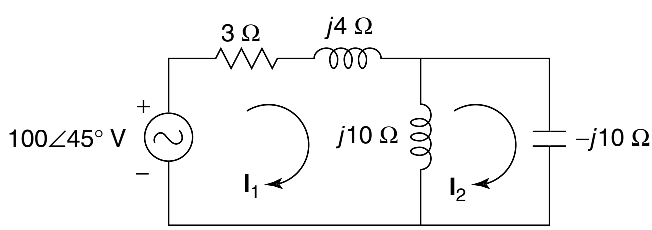


Fig. 6.1

Solution Applying KVL to Mesh 1,

$$100\angle 45^\circ - (3 + j4)\mathbf{I}_1 - j10(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(3 + j14)\mathbf{I}_1 - j10\mathbf{I}_2 = 100\ \angle 45^\circ \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-j10(\mathbf{I}_2 - \mathbf{I}_1) + j10(\mathbf{I}_2) = 0$$

$$j10\mathbf{I}_1 = 0 \qquad \ldots\text{(ii)}$$

$$\mathbf{I}_1 = 0$$

Substituting $\mathbf{I}_1$ in Eq. (i),

$$-j10\mathbf{I}_2 = 100\angle 45^\circ$$

$$\mathbf{I}_2 = \frac{100\angle 45^\circ}{-j10} = 10\angle 135^\circ\ \text{A}$$

Example 6.2 *Find mesh current I_1, I_2 and I_3 in the network of Fig. 6.2.*

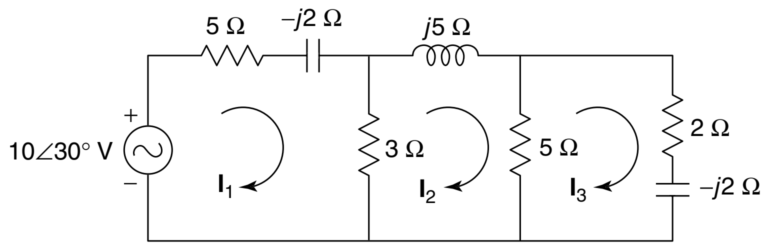


Fig. 6.2

Solution Applying KVL to Mesh 1,

$$10\angle 30° - (5 - j2)\,\mathbf{I}_1 - 3\,(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(8 - j2)\,\mathbf{I}_1 - 3\mathbf{I}_2 = 10\angle 30° \qquad \ldots \text{(i)}$$

Applying KVL to Mesh 2,

$$-3\,(\mathbf{I}_2 - \mathbf{I}_1) - j5\mathbf{I}_2 - 5\,(\mathbf{I}_2 - \mathbf{I}_3) = 0$$

$$-3\mathbf{I}_1 + (8 + j5)\,\mathbf{I}_2 - 5\mathbf{I}_3 = 0 \qquad \ldots \text{(ii)}$$

Applying KVL to Mesh 3,

$$-5\,(\mathbf{I}_3 - \mathbf{I}_2) - (2 - j2)\,\mathbf{I}_3 = 0$$

$$-5\mathbf{I}_2 + (7 - j2)\,\mathbf{I}_3 = 0 \qquad \ldots \text{(iii)}$$

Writing Eqs (i), (ii) and (iii),

$$\begin{bmatrix} 8-j2 & -3 & 0 \\ -3 & 8+j5 & -5 \\ 0 & -5 & 7-j2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 10\angle 30° \\ 0 \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_1 = \frac{\begin{vmatrix} 10\angle 30° & -3 & 0 \\ 0 & 8+j5 & -5 \\ 0 & -5 & 7-j2 \end{vmatrix}}{\begin{vmatrix} 8-j2 & -3 & 0 \\ -3 & 8+j5 & -5 \\ 0 & -5 & 7-j2 \end{vmatrix}} = 1.43\angle 38.7° \text{ A}$$

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 8-j2 & 10\angle 30° & 0 \\ -3 & 0 & -5 \\ 0 & 0 & 7-j2 \end{vmatrix}}{\begin{vmatrix} 8-j2 & -3 & 0 \\ -3 & 8+j5 & -5 \\ 0 & -5 & 7-j2 \end{vmatrix}} = 0.693\angle -2.2° \text{ A}$$

$$\mathbf{I}_3 = \frac{\begin{vmatrix} 8-j2 & -3 & 10\angle 30° \\ -3 & 8+j5 & 0 \\ 0 & -5 & 0 \end{vmatrix}}{\begin{vmatrix} 8-j2 & -3 & 0 \\ -3 & 8+j5 & -5 \\ 0 & -5 & 7-j2 \end{vmatrix}} = 0.476\angle 13.8° \text{ A}$$

Example 6.3 *In the network of Fig. 6.3, find the value of $\mathbf{V}_2$ so that the current through (2 + j3) ohm impedance is zero.*

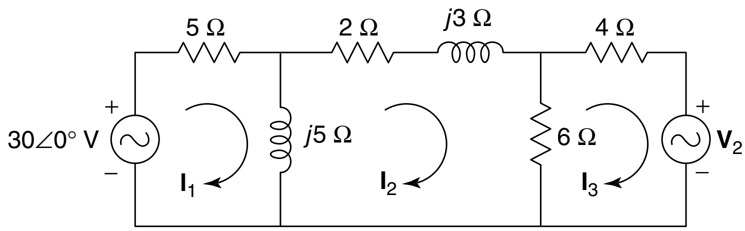


Fig. 6.3

Solution Applying KVL to Mesh 1,

$$30\angle 0° - 5\mathbf{I}_1 - j5(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(5 + j5)\mathbf{I}_1 - j5\mathbf{I}_2 = 30\angle 0° \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-j5(\mathbf{I}_2 - \mathbf{I}_1) - (2 + j3)\mathbf{I}_2 - 6(\mathbf{I}_2 - \mathbf{I}_3) = 0$$

$$-j5\mathbf{I}_1 + (8 + j8)\mathbf{I}_2 - 6\mathbf{I}_3 = 0 \qquad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-6(\mathbf{I}_3 - \mathbf{I}_2) - 4\mathbf{I}_3 - \mathbf{V}_2 = 0$$

$$-6\mathbf{I}_2 + 10\mathbf{I}_3 = -\mathbf{V}_2 \qquad \text{...(iii)}$$

Writing Eqs (i), (ii) and (iii) in matrix form,

$$\begin{bmatrix} 5+j5 & -j5 & 0 \\ -j5 & 8+j8 & -6 \\ 0 & -6 & 10 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 30\angle 0° \\ 0 \\ -\mathbf{V}_2 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 5+j5 & 30\angle 0° & 0 \\ -j5 & 0 & -6 \\ 0 & -\mathbf{V}_2 & 10 \end{vmatrix}}{\begin{vmatrix} 5+j5 & -j5 & 0 \\ -j5 & 8+j8 & -6 \\ 0 & -6 & 10 \end{vmatrix}} = 0$$

$$(5 + j5)(-6\mathbf{V}_2) - (30)(-j50) = 0$$

$$\mathbf{V}_2 = \frac{j1500}{30 + j30} = 35.36\angle 45° \text{ V}$$

Example 6.4 *Find the value of the current $\mathbf{I}_3$ in the network shown in Fig. 6.4.*

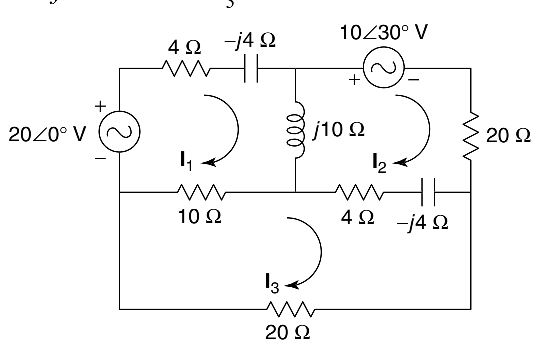


Fig. 6.4

Solution Applying KVL to Mesh 1,

$$20\angle 0° - (4 - j4)\mathbf{I}_1 - j10(\mathbf{I}_1 - \mathbf{I}_2) - 10(\mathbf{I}_1 - \mathbf{I}_3) = 0$$

$$(14 + j6)\mathbf{I}_1 - j10\mathbf{I}_2 - 10\mathbf{I}_3 = 20\angle 0° \quad \ldots \text{(i)}$$

Applying KVL to Mesh 2,

$$-j10(\mathbf{I}_2 - \mathbf{I}_1) - 10\angle 30° - 20\mathbf{I}_2 - (4 - j4)(\mathbf{I}_2 - \mathbf{I}_3) = 0$$

$$-j10\mathbf{I}_1 + (24 + j6)\mathbf{I}_2 - (4 - j4)\mathbf{I}_3 = -10\angle 30° \quad \ldots \text{(ii)}$$

Applying KVL to Mesh 3,

$$-10(\mathbf{I}_3 - \mathbf{I}_1) - (4 - j4)(\mathbf{I}_3 - \mathbf{I}_2) - 20\mathbf{I}_3 = 0$$

$$-10\mathbf{I}_1 - (4 - j4)\mathbf{I}_2 + (34 - j4)\mathbf{I}_3 = 0 \quad \ldots \text{(iii)}$$

Writing Eqs (i), (ii) and (iii) in matrix form,

$$\begin{bmatrix} 14+j6 & -j10 & -10 \\ -j10 & 24+j6 & -(4-j4) \\ -10 & -(4-j4) & 34-j4 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 20\angle 0° \\ -10\angle 30° \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_3 = \frac{\begin{vmatrix} 14+j6 & -j10 & 20\angle 0° \\ -j10 & 24+j6 & -10\angle 30° \\ -10 & -(4-j4) & 0 \end{vmatrix}}{\begin{vmatrix} 14+j6 & -10 & -10 \\ -j10 & 24+j6 & -(4-j4) \\ -10 & -(4-j4) & 34-j4 \end{vmatrix}} = 0.44\angle -14° \text{ A}$$

Example 6.5 *Find the voltage V_{AB} in the network of Fig. 6.5.*

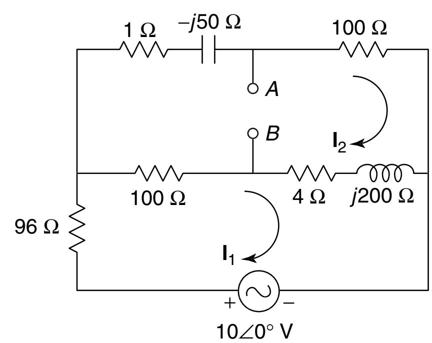


Fig. 6.5

Solution Applying KVL to Mesh 1,

$$-96\,\mathbf{I}_1 - (100 + 4 + j200)\,(\mathbf{I}_1 - \mathbf{I}_2) + 10\angle 0° = 0$$

$$(200 + j200)\,\mathbf{I}_1 - (104 + j200)\,\mathbf{I}_2 = 10\angle 0° \quad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-(1 - j50 - 100)\,\mathbf{I}_2 - (100 + 4 + j200)\,(\mathbf{I}_2 - \mathbf{I}_1) = 0$$

$$-(104 + j200)\,\mathbf{I}_1 + (205 + j150)\,\mathbf{I}_2 = 0 \quad \ldots\text{(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 200+j200 & -(104+j200) \\ -(104+j200) & 205+j150 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 10\angle 0° \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_1 = \frac{\begin{vmatrix} 10\angle 0^\circ & -(104+j200) \\ 0 & 205+j150 \end{vmatrix}}{\begin{vmatrix} 200+j200 & -(104+j200) \\ -(104+j200) & 205+j150 \end{vmatrix}} = 0.051\angle 2.72\times 10^{-3}\,^\circ\text{A}$$

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 200+j200 & 10\angle 0^\circ \\ -(104+j200) & 0 \end{vmatrix}}{\begin{vmatrix} 200+j200 & -(104+j200) \\ -(104+j200) & 205+j150 \end{vmatrix}} = 0.045\angle 26.34^\circ \text{ A}$$

$$\begin{aligned}\mathbf{V}_{AB} &= 100\mathbf{I}_2 - (4+j200)(\mathbf{I}_1 - \mathbf{I}_2) \\ &= 100(0.045\angle 26.34^\circ) - (4+j200)(0.051\angle 2.72\times 10^{-3}\,^\circ - 0.045\angle 26.34^\circ) \\ &= 0.058\angle -92.65^\circ \text{ V}\end{aligned}$$

Example 6.6 *For the network shown in Fig. 6.6, find the voltage across the capacitor.*

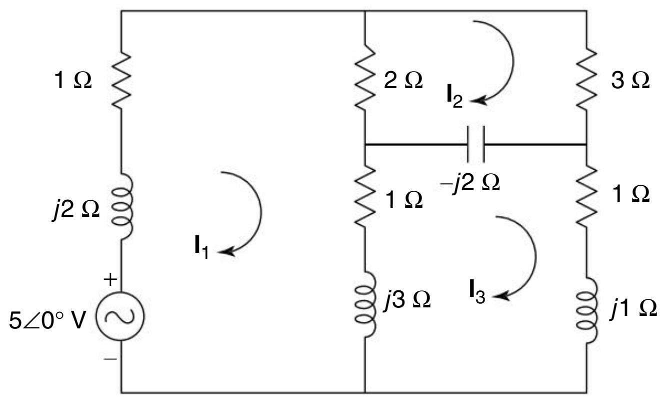


Fig. 6.6

Solution Applying KVL to Mesh 1,

$$5\angle 0^\circ - (1+j2)\mathbf{I}_1 - 2(\mathbf{I}_1 - \mathbf{I}_2) - (1+j3)(\mathbf{I}_1 - \mathbf{I}_3) = 0$$

$$(4+j5)\mathbf{I}_1 - 2\mathbf{I}_2 - (1+j3)\mathbf{I}_3 = 5\angle 0^\circ \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-2(\mathbf{I}_2 - \mathbf{I}_1) - 3\mathbf{I}_2 + j2(\mathbf{I}_2 - \mathrm{I}_3) = 0$$

$$-2\mathbf{I}_1 + (5-j2)\mathbf{I}_2 + j2\mathbf{I}_3 = 0 \qquad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-(1+j3)(\mathbf{I}_3 - \mathbf{I}_1) + j2(\mathbf{I}_3 - \mathbf{I}_2) - (1+j1)\mathbf{I}_3 = 0$$

$$-(1+j3)\mathbf{I}_1 + j2\mathbf{I}_2 + (2+j2)\mathbf{I}_3 = 0 \qquad \text{...(iii)}$$

Writing Eqs (i), (ii) and (iii) in matrix form,

$$\begin{bmatrix} 4+j5 & -2 & -(1+j3) \\ -2 & 5-j2 & j2 \\ -(1+j3) & j2 & 2+j2 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 5\angle 0^\circ \\ 0 \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 4+j5 & 5\angle 0° & -(1+j3) \\ -2 & 0 & j2 \\ -(1+j3) & 0 & 2+j2 \end{vmatrix}}{\begin{vmatrix} 4+j5 & -2 & -(1+j3) \\ -2 & 5-j2 & j2 \\ -(1+j3) & j2 & 2+j2 \end{vmatrix}} = 0.65\angle 130.51° \text{ A}$$

$$\mathbf{I}_3 = \frac{\begin{vmatrix} 4+j5 & -2 & 5\angle 0° \\ -2 & 5-j2 & 0 \\ -(1+j3) & j2 & 0 \end{vmatrix}}{\begin{vmatrix} 4+j5 & -2 & -(1+j3) \\ -2 & 5-j2 & j2 \\ -(1+j3) & j2 & 2+j2 \end{vmatrix}} = 0.91\angle -21.51° \text{ A}$$

$$\mathbf{V}_c = (-j2)(\mathbf{I}_3 - \mathbf{I}_2) = (-j2)(0.91\angle -21.51° - 0.65\angle 130.51°) = 3.03\angle -123.12° \text{ V}$$

Example 6.7 *Find the voltage across the 2 Ω resistor in the network of Fig. 6.7.*

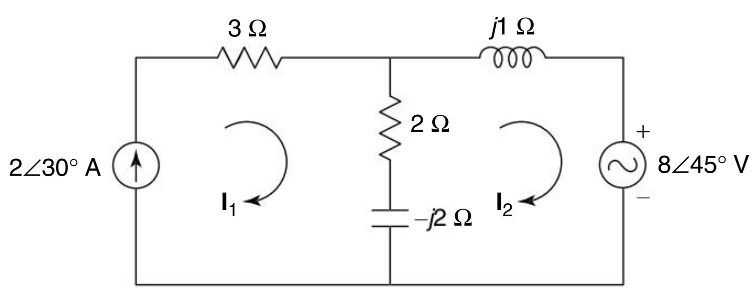


Fig. 6.7

Solution For Mesh 1,

$$\mathbf{I}_1 = 2\angle 30° \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-(2-j2)(\mathbf{I}_2 - \mathbf{I}_1) - j1\mathbf{I}_2 - 8\angle 45° = 0$$

$$(2-j2)\mathbf{I}_1 - (2-j1)\mathbf{I}_2 = 8\angle 45° \qquad \text{...(ii)}$$

Substituting $\mathbf{I}_1$ in Eq. (i),

$$(2-j2)(2\angle 30°) - (2-j1)\mathbf{I}_2 = 8\angle 45°$$

$$\mathbf{I}_2 = \frac{-(8\angle 45°) + (2-j2)(2\angle 30°)}{2-j1} = 3.19\angle -65° \text{ A}$$

$$\mathbf{V}_{2\,\Omega} = 2(\mathbf{I}_1 - \mathbf{I}_2) = 2(2\angle 30° - 3.19\angle -65°) = 7.82\angle 84.37° \text{ V}$$

Example 6.8 *Find the current through 3 Ω resistor in the network of Fig. 6.8.*

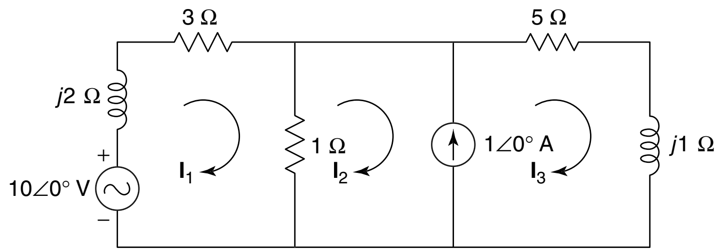


Fig. 6.8

Solution Applying KVL to Mesh 1,

$$10\angle 0° - j2\mathbf{I}_1 - 3\mathbf{I}_1 - 1(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(4 + j2)\mathbf{I}_1 - \mathbf{I}_2 = 10\angle 0° \qquad \ldots\text{(i)}$$

Meshes 2 and 3 will form a supermesh.
Writing current equation for the supermesh,

$$\mathbf{I}_3 - \mathbf{I}_2 = 1\angle 0° \qquad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$-1(\mathbf{I}_2 - \mathbf{I}_1) - 5\mathbf{I}_3 - j1\mathbf{I}_3 = 0$$

$$\mathbf{I}_1 - \mathbf{I}_2 - (5 + j1)\mathbf{I}_3 = 0 \qquad \ldots\text{(iii)}$$

Writing Eqs (i), (ii) and (iii) in matrix form,

$$\begin{bmatrix} 4+j2 & -1 & 0 \\ 0 & -1 & 1 \\ 1 & -1 & -(5+j1) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 10\angle 0° \\ 1\angle 0° \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_1 = \frac{\begin{vmatrix} 10\angle 0° & -1 & 0 \\ 1\angle 0° & -1 & 1 \\ 0 & -1 & -(5+j1) \end{vmatrix}}{\begin{vmatrix} 4+j2 & -1 & 0 \\ 0 & -1 & 1 \\ 1 & -1 & -(5+j1) \end{vmatrix}} = 2.11\angle -28.01° \text{ A}$$

$$\mathbf{I}_{3\,\Omega} = \mathbf{I}_1 = 2.11\angle -28.01° \text{ A}$$

Example 6.9 *Find the currents $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$ in the network of Fig. 6.9.*

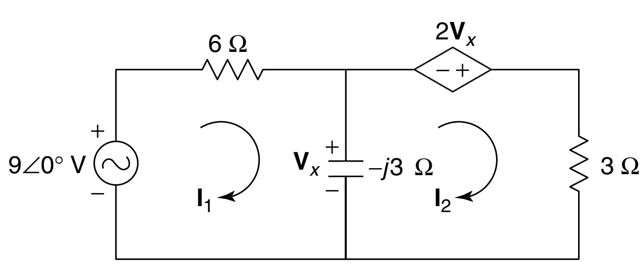


Fig. 6.9

Solution From Fig. 6.9,

$$\mathbf{V}_x = -j3(\mathbf{I}_1 - \mathbf{I}_2) \quad \text{...(i)}$$

Applying KVL to Mesh 1,

$$9\angle 0° - 6\mathbf{I}_1 + j3(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(6 - j3)\mathbf{I}_1 + j3\mathbf{I}_2 = 9\angle 0° \quad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$j3(\mathbf{I}_2 - \mathbf{I}_1) + 2\mathbf{V}_x - 3\mathbf{I}_2 = 0$$

$$j3\mathbf{I}_2 - j3\mathbf{I}_1 + 2[-j3(\mathbf{I}_1 - \mathbf{I}_2)] - 3\mathbf{I}_2 = 0$$

$$j9\mathbf{I}_1 + (3 - j9)\mathbf{I}_2 = 0 \quad \text{...(iii)}$$

Writing Eqs (ii) and (iii) in matrix form,

$$\begin{bmatrix} 6 - j3 & j3 \\ j9 & 3 - j9 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 9\angle 0° \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_1 = \frac{\begin{vmatrix} 9\angle 0° & j3 \\ 0 & 3 - j9 \end{vmatrix}}{\begin{vmatrix} 6 - j3 & j3 \\ j9 & 3 - j9 \end{vmatrix}} = 1.3\angle 2.49° \text{ A}$$

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 6 - j3 & 9\angle 0° \\ j9 & 0 \end{vmatrix}}{\begin{vmatrix} 6 - j3 & j3 \\ j9 & 3 - j9 \end{vmatrix}} = 1.24\angle -15.95° \text{ A}$$

Example 6.10 *Find the voltage across the 4 Ω resistor in the network of Fig. 6.10.*

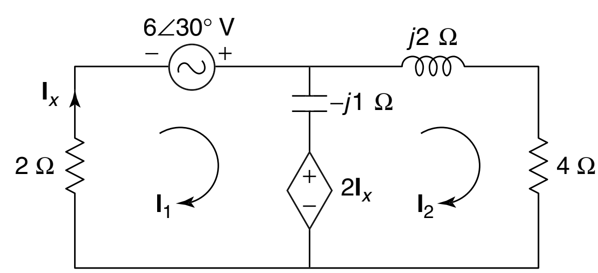


Fig. 6.10

Solution From Fig. 6.10,

$$\mathbf{I}_x = \mathbf{I}_1 \quad \text{...(i)}$$

Applying KVL to Mesh 1,

$$-2\mathbf{I}_1 + 6\angle 30° + j1(\mathbf{I}_1 - \mathbf{I}_2) - 2\mathbf{I}_x = 0$$

$$-2\mathbf{I}_1 + 6\angle 30° + j1\mathbf{I}_1 - j1\mathbf{I}_2 - 2\mathbf{I}_1 = 0$$

$$(4 - j1)\mathbf{I}_1 + j1\mathbf{I}_2 = 6\angle 30° \quad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$2\mathbf{I}_x + j1(\mathbf{I}_2 - \mathbf{I}_1) - j2\mathbf{I}_2 - 4\mathbf{I}_2 = 0$$

$$2\mathbf{I}_1 + j1\mathbf{I}_2 - j1\,\mathbf{I}_1 - j2\mathbf{I}_2 - 4\mathbf{I}_2 = 0$$

$$(2 - j1)\mathbf{I}_1 - (4 + j1)\mathbf{I}_2 = 0 \qquad \ldots\text{(iii)}$$

Writing Eqs (ii) and (iii) in matrix form,

$$\begin{bmatrix} 4 - j1 & j1 \\ 2 - j1 & -(4 + j1) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 6\angle 30^\circ \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 4 - j1 & 6\angle 30^\circ \\ 2 - j1 & 0 \end{vmatrix}}{\begin{vmatrix} 4 - j1 & j1 \\ 2 - j1 & -(4 + j1) \end{vmatrix}} = 0.74\angle -2.91^\circ \text{ A}$$

$$V_{4\Omega} = 4\mathbf{I}_2 = 4(0.74\angle -2.91^\circ) = 2.96\angle -2.91^\circ \text{ V}$$

6.3 NODE ANALYSIS

Node analysis uses Kirchhoff's current law for finding currents and voltages in a network. For ac networks, Kirchhoff's current law states that the phasor sum of currents meeting at a point is equal to zero.

Example 6.11 *In the network shown in Fig. 6.11, determine $\mathbf{V}_a$ and $\mathbf{V}_b$.*

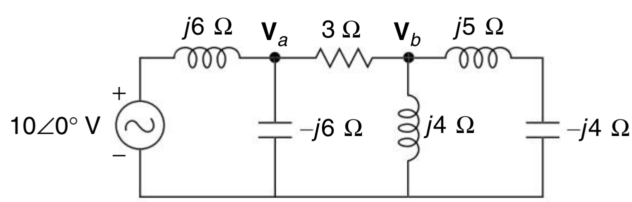


Fig. 6.11

Solution Applying KCL at Node *a*,

$$\frac{\mathbf{V}_a - 10\angle 0^\circ}{j6} + \frac{\mathbf{V}_a}{-j6} + \frac{\mathbf{V}_a - \mathbf{V}_b}{3} = 0$$

$$\left(\frac{1}{j6} - \frac{1}{j6} + \frac{1}{3}\right)\mathbf{V}_a - \frac{1}{3}\mathbf{V}_b = \frac{10\angle 0^\circ}{j6}$$

$$0.33\mathbf{V}_a - 0.33\mathbf{V}_b = 1.67\angle -90^\circ \qquad \ldots\text{(i)}$$

Applying KCL at Node *b*,

$$\frac{\mathbf{V}_b - \mathbf{V}_a}{3} + \frac{\mathbf{V}_b}{j4} + \frac{\mathbf{V}_b}{j1} = 0$$

$$-\frac{1}{3}\mathbf{V}_a + \left(\frac{1}{3} + \frac{1}{j4} + \frac{1}{j1}\right)\mathbf{V}_b = 0$$

$$-0.33\mathbf{V}_a + (0.33 - j1.25)\mathbf{V}_b = 0 \qquad \ldots\text{(ii)}$$

Adding Eqs (i) and (ii),

$$-j1.25\mathbf{V}_b = 1.67\angle -90^\circ$$

$$\mathbf{V}_b = \frac{1.67\angle -90^\circ}{-j1.25} = 1.34\angle 0^\circ \text{ V}$$

Substituting $\mathbf{V}_b$ in Eq. (i),

$$0.33\mathbf{V}_a - 0.33(1.34\angle 0^\circ) = 1.67\angle -90^\circ$$

$$\mathbf{V}_a = \frac{1.73\angle 75.17^\circ}{0.33} = 5.24\angle -75.17^\circ \text{ V}$$

Example 6.12 *For the network shown in Fig. 6.12, find the voltages $\mathbf{V}_1$ and $\mathbf{V}_2$.*

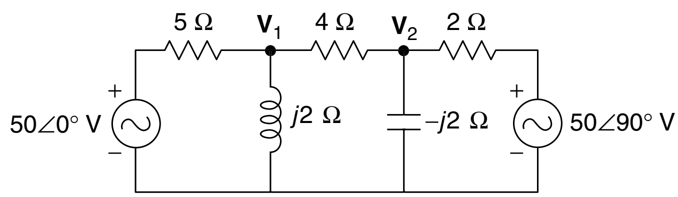


Fig. 6.12

Applying KCL at Node 1,

$$\frac{\mathbf{V}_1 - 50\angle 0^\circ}{5} + \frac{\mathbf{V}_1}{j2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{4} = 0$$

$$\left(\frac{1}{5} + \frac{1}{j2} + \frac{1}{4}\right)\mathbf{V}_1 - \frac{1}{4}\mathbf{V}_2 = 10\angle 0^\circ$$

$$(0.45 - j0.5)\mathbf{V}_1 - 0.25\mathbf{V}_2 = 10\angle 0^\circ \qquad \text{...(i)}$$

Applying KCL at Node 2,

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{4} + \frac{\mathbf{V}_2}{-j2} + \frac{\mathbf{V}_2 - 50\angle 90^\circ}{2} = 0$$

$$-\frac{1}{4}\mathbf{V}_1 + \left(\frac{1}{4} + \frac{1}{-j2} + \frac{1}{2}\right)\mathbf{V}_2 = 25\angle 90^\circ$$

$$-0.25\mathbf{V}_1 + (0.75 + j0.5)\mathbf{V}_2 = 25\angle 90^\circ \qquad \text{...(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 0.45 - j0.5 & -0.25 \\ -0.25 & 0.75 + j0.5 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 10\angle 0^\circ \\ 25\angle 90^\circ \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{V}_1 = \frac{\begin{vmatrix} 10\angle 0^\circ & -0.25 \\ j25 & 0.75 + j0.5 \end{vmatrix}}{\begin{vmatrix} 0.45 - j0.5 & -0.25 \\ -0.25 & 0.75 + j0.5 \end{vmatrix}} = 24.7\angle 72.25^\circ \text{ V}$$

$$\mathbf{V}_2 = \frac{\begin{vmatrix} 0.45 - j0.5 & 10\angle 0^\circ \\ -0.25 & 25\angle 90^\circ \end{vmatrix}}{\begin{vmatrix} 0.45 - j0.5 & -0.25 \\ -0.25 & 0.75 + j0.5 \end{vmatrix}} = 34.34\angle 52.82^\circ \text{ V}$$

Example 6.13 *Find the voltage $\mathbf{V}_{AB}$ in the network of Fig. 6.13.*

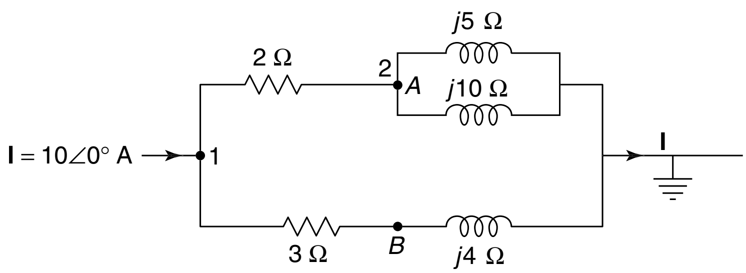


Fig. 6.13

Solution Applying KCL at Node 1,

$$10\angle 0^\circ = \frac{\mathbf{V}_1 - \mathbf{V}_2}{2} + \frac{\mathbf{V}_1}{3 + j4}$$

$$\left(\frac{1}{2} + \frac{1}{3 + j4}\right)\mathbf{V}_1 - \frac{1}{2}\mathbf{V}_2 = 10\angle 0^\circ$$

$$(0.62 - j0.16)\mathbf{V}_1 - 0.5\mathbf{V}_2 = 10\angle 0^\circ \qquad \ldots\text{(i)}$$

Applying KCL at Node 2,

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{2} + \frac{\mathbf{V}_2}{j5} + \frac{\mathbf{V}_2}{j10} = 0$$

$$-\frac{1}{2}\mathbf{V}_1 + \left(\frac{1}{2} + \frac{1}{j5} + \frac{1}{j10}\right)\mathbf{V}_2 = 0$$

$$-0.5\mathbf{V}_1 + (0.5 - j0.3)\mathbf{V}_2 = 0 \qquad \ldots\text{(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 0.62 - j0.16 & -0.5 \\ -0.5 & 0.5 - j0.3 \end{bmatrix}\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 10\angle 0^\circ \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{V}_1 = \frac{\begin{vmatrix} 10\angle 0^\circ & -0.5 \\ 0 & 0.5 - j0.3 \end{vmatrix}}{\begin{vmatrix} 0.62 - j0.16 & -0.5 \\ -0.5 & 0.5 - j0.3 \end{vmatrix}} = 21.8\angle 56.42^\circ \text{ V}$$

$$\mathbf{V}_2 = \frac{\begin{vmatrix} 0.62 - j0.16 & 10\angle 0^\circ \\ -0.5 & 0 \end{vmatrix}}{\begin{vmatrix} 0.62 - j0.16 & -0.5 \\ -0.5 & 0.5 - j0.3 \end{vmatrix}} = 18.7\angle 87.42^\circ \text{ V}$$

$$\mathbf{V}_A = \mathbf{V}_2 = 18.7\angle 87.42^\circ \text{ V}$$

$$\mathbf{V}_B = \frac{\mathbf{V}_1}{3 + j4}(j4) = \frac{21.8\angle 56.42^\circ}{3 + j4}(j4) = 17.45\angle 93.32^\circ \text{ V}$$

$$\mathbf{V}_{AB} = \mathbf{V}_A - \mathbf{V}_B = (18.7\angle 87.42^\circ) - (17.45\angle 93.32^\circ) = 2.23\angle 34.1^\circ \text{ V}$$

Example 6.14 *Find the node voltages V_1 and V_2 in the network of Fig. 6.14.*

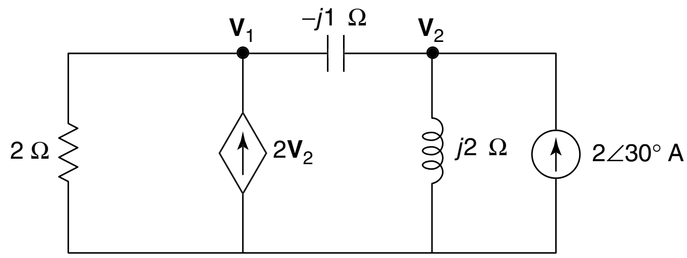


Fig. 6.14

Solution Applying KCL at Node 1,

$$\frac{\mathbf{V}_1}{2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j1} = 2\mathbf{V}_2$$

$$\left(\frac{1}{2} + \frac{1}{-j1}\right)\mathbf{V}_1 - \left(2 - \frac{1}{j1}\right)\mathbf{V}_2 = 0$$

$$(0.5 + j1)\,\mathbf{V}_1 - (2 + j1)\mathbf{V}_2 = 0 \qquad \ldots\text{(i)}$$

Applying KCL at Node 2,

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{-j1} + \frac{\mathbf{V}_2}{j2} = 2\angle 30°$$

$$\frac{1}{j1}\mathbf{V}_1 + \left(\frac{1}{-j1} + \frac{1}{j2}\right)\mathbf{V}_2 = 2\angle 30°$$

$$-j1\,\mathbf{V}_1 + j\,0.5\,\mathbf{V}_2 = 2\angle 30° \qquad \ldots\text{(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 0.5 + j1 & -(2 + j1) \\ -j1 & j0.5 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2\angle 30° \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{V}_1 = \frac{\begin{vmatrix} 0 & -(2 + j1) \\ 2\angle 30° & j0.5 \end{vmatrix}}{\begin{vmatrix} 0.5 + j1 & -(2 + j1) \\ -j1 & j0.5 \end{vmatrix}} = 2.46\angle 130.62° \text{ V}$$

$$\mathbf{V}_2 = \frac{\begin{vmatrix} 0.5 + j1 & 0 \\ -j1 & 2\angle 30° \end{vmatrix}}{\begin{vmatrix} 0.5 + j1 & -(2 + j1) \\ -j1 & j0.5 \end{vmatrix}} = 1.23\angle 167.49° \text{ V}$$

Example 6.15 *In the network of Fig 6.15, find the voltage V_2 which results in zero current through 4 Ω resistor.*

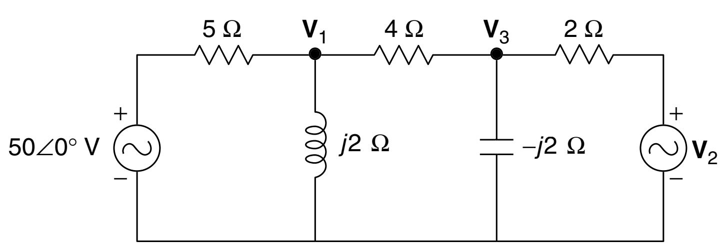


Fig. 6.15

Solution Applying KCL at Node 1,

$$\frac{\mathbf{V}_1 - 50\angle 0^\circ}{5} + \frac{\mathbf{V}_1}{j2} + \frac{\mathbf{V}_1 - \mathbf{V}_3}{4} = 0$$

$$\left(\frac{1}{5} + \frac{1}{j2} + \frac{1}{4}\right)\mathbf{V}_1 - \frac{1}{4}\mathbf{V}_3 = 10\angle 0^\circ$$

$$(0.45 - j0.5)\mathbf{V}_1 - 0.25\mathbf{V}_3 = 10\angle 0^\circ \qquad \text{...(i)}$$

Applying KCL at Node 3,

$$\frac{\mathbf{V}_3 - \mathbf{V}_1}{4} + \frac{\mathbf{V}_3}{-j2} + \frac{\mathbf{V}_3 - \mathbf{V}_2}{2} = 0$$

$$-\frac{1}{4}\mathbf{V}_1 + \left(\frac{1}{4} + \frac{1}{-j2} + \frac{1}{2}\right)\mathbf{V}_3 = 0.5\mathbf{V}_2$$

$$-0.25\mathbf{V}_1 + (0.75 + j0.5)\,\mathbf{V}_3 = 0.5\mathbf{V}_2 \qquad \text{...(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 0.45 - j0.5 & -0.25 \\ -0.25 & 0.75 + j0.5 \end{bmatrix}\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_3 \end{bmatrix} = \begin{bmatrix} 10\angle 0^\circ \\ 0.5\mathbf{V}_2 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{V}_1 = \frac{\begin{vmatrix} 10\angle 0^\circ & -0.25 \\ 0.5\mathbf{V}_2 & 0.75 + j0.5 \end{vmatrix}}{\begin{vmatrix} 0.45 - j0.5 & -0.25 \\ -0.25 & 0.75 + j0.5 \end{vmatrix}} = \frac{10(0.75 + j0.5) + 0.125\,\mathbf{V}_2}{0.55\angle -15.95^\circ}$$

$$\mathbf{V}_3 = \frac{\begin{vmatrix} 0.45 - j\,0.5 & 10\angle 0^\circ \\ -0.25 & 0.5\mathbf{V}_2 \end{vmatrix}}{\begin{vmatrix} 0.45 - j0.5 & -0.25 \\ -0.25 & 0.75 + j0.5 \end{vmatrix}} = \frac{0.5\,\mathbf{V}_2(0.45 - j0.5) + 2.5}{0.55\angle -15.95^\circ}$$

$$\mathbf{I}_{4\,\Omega} = \frac{\mathbf{V}_1 - \mathbf{V}_3}{4} = 0$$

$$\mathbf{V}_1 = \mathbf{V}_3$$

$$\frac{10(0.75 + j0.5) + 0.125\mathbf{V}_2}{0.55\angle -15.95^\circ} = \frac{0.5\mathbf{V}_2(0.45 - j0.5) + 2.5}{0.55\angle -15.95^\circ}$$

$$7.5 + 0.125\mathbf{V}_2 - j\,5 = 2.5 + 0.225\mathbf{V}_2 - j0.25\mathbf{V}_2$$

$$5 + j\,5 = \mathbf{V}_2(0.1 - j0.25)$$

$$\mathbf{V}_2 = \frac{5 + j5}{0.1 - j0.25} = 26.26\angle 113.2^\circ \text{ V}$$

Example 6.16 *Find the voltage across the capacitor in the network of Fig. 6.16.*

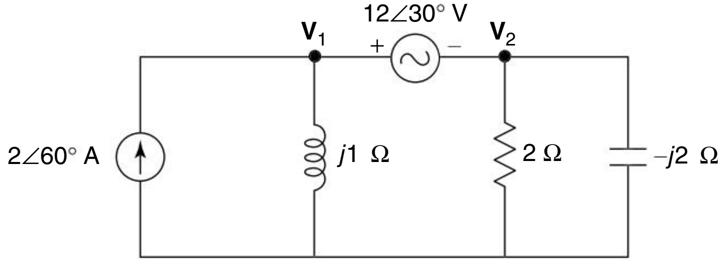


Fig. 6.16

Solution Nodes 1 and 2 will form a supernode.
Writing the voltage equation for the supernode,

$$\mathbf{V}_1 - \mathbf{V}_2 = 12\angle 30^\circ \qquad \ldots\text{(i)}$$

Applying KCL to the supernode,

$$\frac{\mathbf{V}_1}{j1} + \frac{\mathbf{V}_2}{2} + \frac{\mathbf{V}_2}{-j2} = 2\angle 60^\circ$$

$$(-j1)\,\mathbf{V}_1 + (0.5 + j0.5)\,\mathbf{V}_2 = 2\angle 60^\circ \qquad \ldots\text{(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 1 & -1 \\ -j1 & 0.5 + j0.5 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 12\angle 30^\circ \\ 2\angle 60^\circ \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{V}_2 = \frac{\begin{vmatrix} 1 & 12\angle 30^\circ \\ -j1 & 2\angle 60^\circ \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ -j1 & 0.5 + j0.5 \end{vmatrix}} = 18.55\angle 157.42^\circ \text{ V}$$

$$\mathbf{V}_c = \mathbf{V}_2 = 18.55\angle 157.42^\circ \text{ V}$$

6.4 SUPERPOSITION THEOREM

The superposition theorem can be used to analyse an ac network containing more than one source. The superposition theorem states that *in a network containing more than one voltage source or current source, the total current or voltage in any branch of the network is the phasor sum of currents or voltages produced in that branch by each source acting separately*. As each source is considered, all of the other sources are replaced by their internal impedances. This theorem is valid only for linear systems.

Example 6.17 *Find the current through the 3 + j4 ohm impedance.*

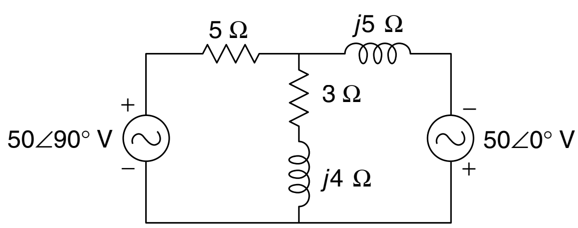


Fig. 6.17

Solution

Step I When the 50 ∠90° V source is acting alone (Fig. 6.18)

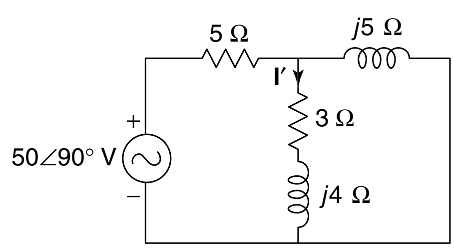


Fig. 6.18

$$\mathbf{Z}_T = 5 + \frac{(3 + j4)(j5)}{3 + j9} = 6.35\angle 23.2^\circ\ \Omega$$

$$\mathbf{I}_T = \frac{50\angle 90^\circ}{6.35\angle 23.2^\circ} = 7.87\angle 66.8^\circ \text{ A}$$

By current division rule,

$$\mathbf{I}' = (7.87\angle 66.8°)\left(\frac{j5}{3+j9}\right) = 4.15\angle 85.3° \text{ A}(\downarrow)$$

Step II When the $50\angle 0°$ V source is acting alone (Fig. 6.19)

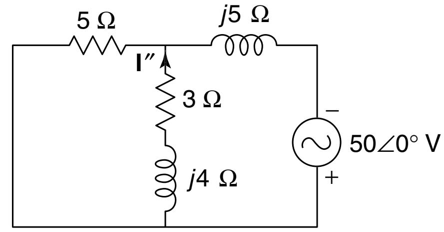


Fig. 6.19

$$\mathbf{Z}_T = j5 + \frac{5(3+j4)}{8+j4} = 6.74\angle 68.2° \ \Omega$$

$$\mathbf{I}_T = \frac{50\angle 0°}{6.74\angle 68.2°} = 7.42\angle -68.2° \text{ A}$$

By current division rule,

$$\mathbf{I}'' = (7.42\angle -68.2°)\left(\frac{5}{8+j4}\right) = 4.15\angle -94.77° \text{ A}(\uparrow) = 4.15\angle 85.3° \text{ A}(\downarrow)$$

Step III By superposition theorem,

$$\mathbf{I} = \mathbf{I}' + \mathbf{I}'' = 4.15\ \angle 85.3° + 4.15\ \angle 85.3° = 8.31\ \angle 85.3° \text{A}\ (\downarrow)$$

Example 6.18 *Determine the voltage across the (2 + j5) ohm impedance for the network shown in Fig.6.20.*

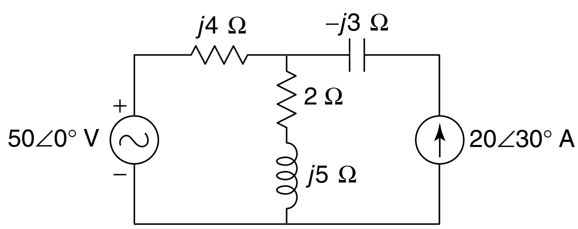


Fig. 6.20

Solution

Step I When the $50\angle 0°$ V source is acting alone (Fig. 6.21)

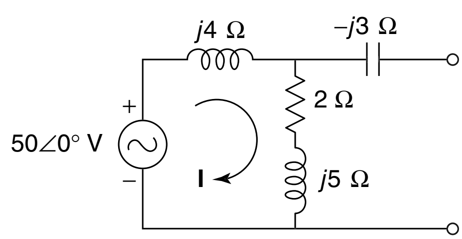


Fig. 6.21

$$\mathbf{I} = \frac{50\angle 0°}{2+j4+j5} = 5.42\angle -77.47° \text{ A}$$

Voltage cross $(2+j5)\ \Omega$ impedance

$$\mathbf{V}' = (2+j5)\ (5.42\ \angle -77.47°) = 29.16\ \angle -9.28° \text{ V}$$

Step II When the $20\angle 30°$ A source is acting alone (Fig. 6.22)
By current division rule,

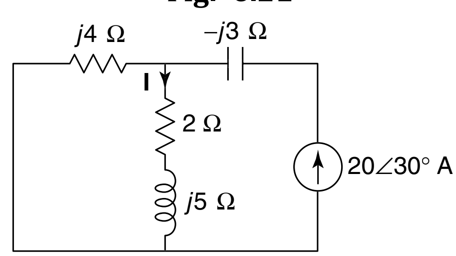


Fig. 6.22

$$\mathbf{I} = (20\angle 30°)\left(\frac{j4}{2+j9}\right) = 8.68\angle 42.53° \text{ A}$$

Voltage across $(2+j5)\ \Omega$ impedance

$$\mathbf{V}'' = (2+j5)\ (8.68\ \angle 42.53°) = 46.69\ \angle 110.72° \text{ V}$$

Step III By superposition theorem,

$$\mathbf{V} = \mathbf{V}' + \mathbf{V}'' = 29.16\ \angle{-9.28°} + 46.69\ \angle 110.72° = 40.85\ \angle 72.53°\ \text{V}$$

Example 6.19 *Determine the voltage $\mathbf{V}_{AB}$ for the network shown in Fig. 6.23.*

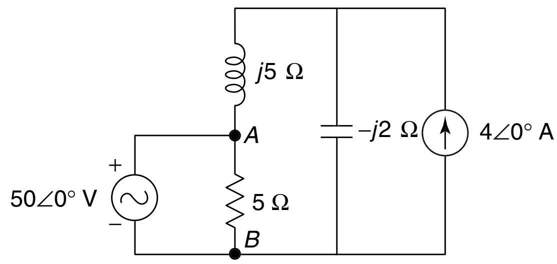


Fig. 6.23

Solution

Step I When the 50∠0° V source is acting alone (Fig. 6.24)

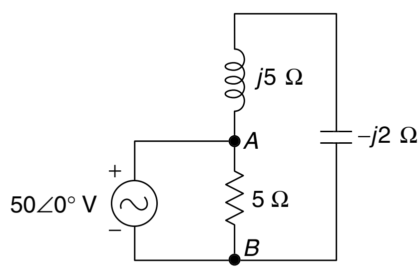


Fig. 6.24

$$\mathbf{V}'_{AB} = 50\angle 0°\ \text{V}$$

Step II When the 4∠0° A source is acting alone (Fig. 6.25)

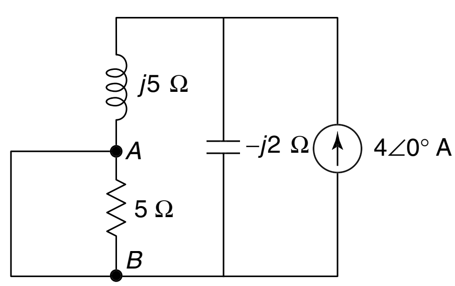


Fig. 6.25

$$\mathbf{V}''_{AB} = 0$$

Step III By superposition theorem,

$$\mathbf{V}_{AB} = \mathbf{V}'_{AB} + \mathbf{V}''_{AB} = 50\angle 0° + 0 = 50\angle 0°\ \text{V}$$

Example 6.20 *Find the current **I** in the network shown in Fig. 6.26.*

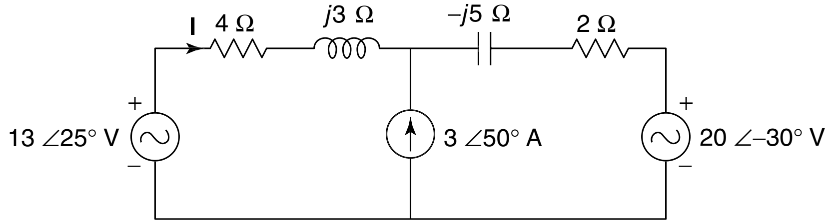


Fig. 6.26

Solution

Step I When the 13∠25° V source is acting alone (Fig. 6.27)

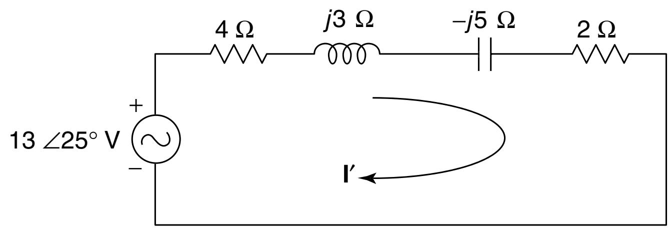


Fig. 6.27

$$\mathbf{I}' = \frac{13\angle 25^\circ}{6 - j2} = 2.057\angle 43.43^\circ \text{ A} \quad (\rightarrow)$$

Step II When the 20∠−30° V source is acting alone (Fig. 6.28)

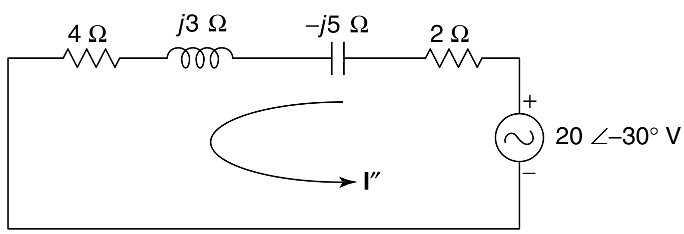


Fig. 6.28

$$\mathbf{I}'' = \frac{20\angle -30^\circ \text{ V}}{6 - j2} = 3.16\angle -11.57^\circ \text{ A}(\leftarrow) = 3.16\angle 168.43^\circ \text{ A}(\rightarrow)$$

Step III When the 3∠50° A source is acting alone (Fig. 6.29)

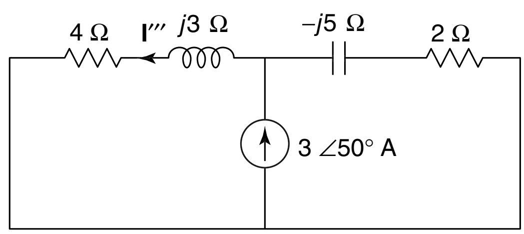


Fig. 6.29

By current division rule,

$$\mathbf{I}''' = 3\angle 50° \times \frac{2 - j5}{6 - j2} = 2.56\angle 0.23° \text{ A}(\leftarrow) = 2.56\angle -179.77° \text{ A}(\rightarrow)$$

Step IV By superposition theorem,

$$\mathbf{I} = \mathbf{I}' + \mathbf{I}'' + \mathbf{I}''' = 2.057\angle 43.13° + 3.16\angle 168.43° + 2.56\angle -179.77° \text{ A} = 4.62\angle 153.99° \text{ A} (\rightarrow)$$

Example 6.21 *Find the current through the j3 Ω reactance in the network of Fig 6.30.*

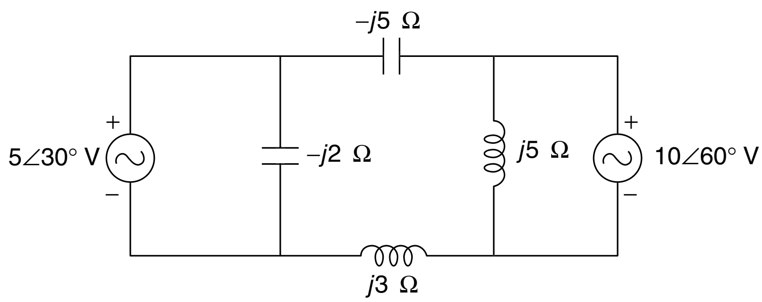


Fig. 6.30

Solution

Step I When the $5\angle 30°$ V source is acting alone (Fig. 6.31)

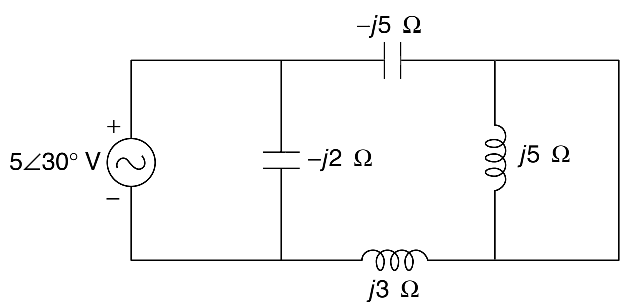


Fig. 6.31

When a short circuit is placed across $j15\ \Omega$ reactance, it gets shorted as shown in Fig 6.32.

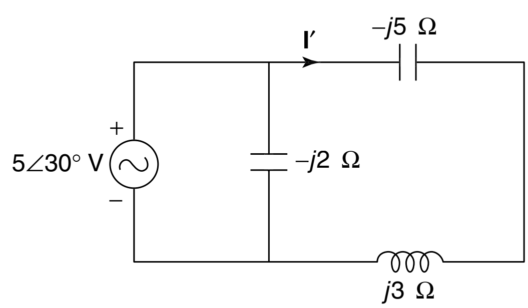


Fig. 6.32

$$\mathbf{I}' = \frac{5\angle 30°}{-j5 + j3} = 2.5\angle 120° \text{ A}(\leftarrow)$$

Step II When the 10∠60° V source is acting alone (Fig. 6.33)

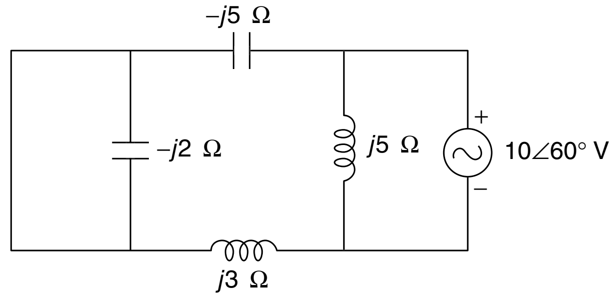


Fig. 6.33

When a short circuit is placed across the $-j2\ \Omega$ reactance, it gets shorted as shown in Fig. 6.34

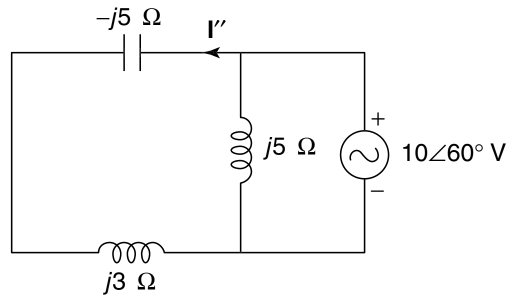


Fig. 6.34

$$\mathbf{I}'' = \frac{10\angle 60^\circ}{-j5 + j3} = 5\angle 150^\circ \text{ A } (\rightarrow) = 5\angle -30^\circ \text{ A } (\leftarrow)$$

Step III By superposition theorem,

$$\mathbf{I} = \mathbf{I}' + \mathbf{I}'' = 2.5\angle 120^\circ + 5\angle -30^\circ = 3.1\angle -6.21^\circ \text{ A} \quad (\leftarrow)$$

Example 6.22 *Find the current $\mathbf{I}_0$ in the network of Fig. 6.35.*

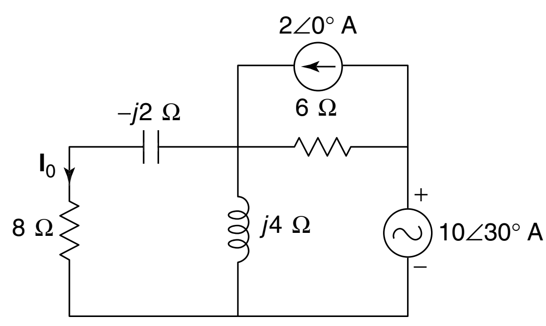


Fig. 6.35

Solution

Step I When the 10∠30° V source is acting alone (Fig. 6.36)

$$\mathbf{Z}_T = 6 + \frac{j4(8 - j2)}{j4 + 8 - j2} = 8.64\angle 24.12^\circ\ \Omega$$

$$\mathbf{I}_T = \frac{10\angle 30^\circ}{8.64\angle 24.12^\circ} = 1.16\angle 5.88^\circ \text{ A}$$

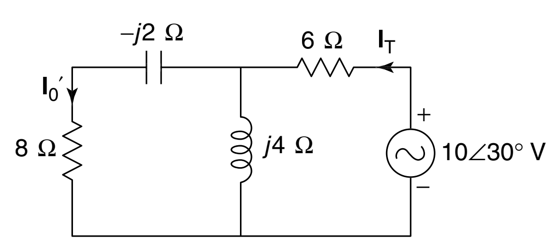


Fig. 6.36

By current division rule,

$$\mathbf{I}_0' = 1.16\angle 5.88° \times \frac{j4}{8 - j2 + j4} = 0.56\angle 81.84° \text{ A} \quad (\downarrow)$$

Step II When the $2\angle 0°$ A source is acting alone (Fig. 6.37)

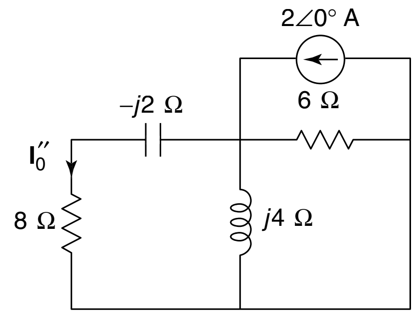


Fig. 6.37

The network can be redrawn as shown in Fig. 6.38.

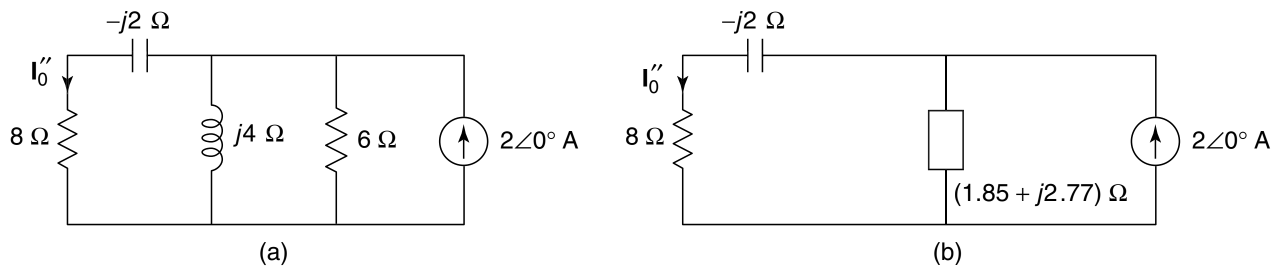


Fig. 6.38

By current division rule,

$$\mathbf{I}_0'' = 2\angle 0° \times \frac{1.85 + j2.77}{1.85 + j2.77 + 8 - j2} = 0.67\angle 51.83° \text{ A } (\downarrow)$$

Step III By superposition theorem,

$$\mathbf{I}_0 = \mathbf{I}_0' + \mathbf{I}_0'' = 0.56\angle 81.84° + 0.67\angle 51.83° = 1.19\angle 65.46° \text{ A } (\downarrow)$$

Example 6.23 *Find the current through the j5 Ω branch for the network shown in Fig. 6.39.*

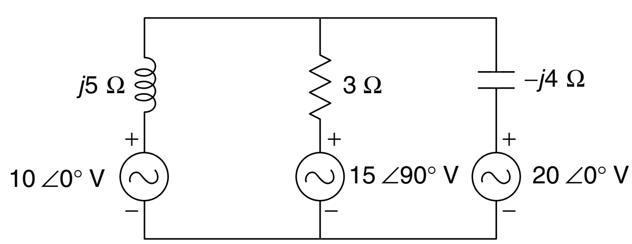


Fig. 6.39

Solution

Step I When the $10\angle 0°$ V source is acting alone (Fig. 6.40)

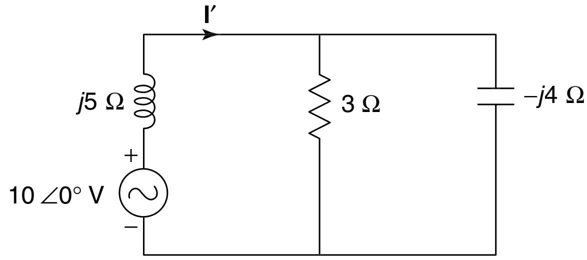


Fig. 6.40

$$\mathbf{Z}_T = j5 + \frac{3(-j4)}{3-j4} = 4.04\angle 61.66°\ \Omega$$

$$\mathbf{I}' = \frac{10\angle 0°}{4.04\angle 61.66°} = 2.48\angle -61.66°\text{A} \quad (\rightarrow)$$

Step II When the $15\angle 90°$ V source is acting alone (Fig. 6.41)

$$\mathbf{Z}_T = 3 + \frac{(j5)(-j4)}{j5-j4} = 20.22\angle -81.47°\ \Omega$$

$$\mathbf{I}_T = \frac{15\angle 90°}{20.22\angle -81.47°} = 0.74\angle 171.47°\ \text{A}$$

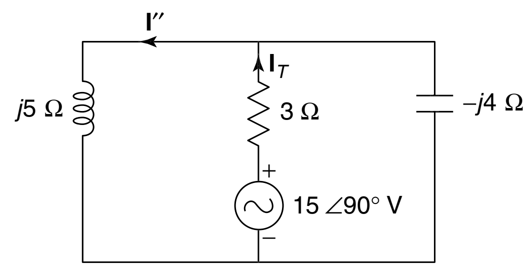


Fig. 6.41

By current division rule,

$$\mathbf{I}'' = 0.74\angle 171.47° \times \frac{-j4}{-j4+j5} = 2.96\angle -8.53°\ \text{A}\ (\leftarrow) = 2.96\angle 171.47°\ \text{A} \quad (\rightarrow)$$

Step III When the $20\ \angle 0°$ V source is acting along (Fig. 6.42)

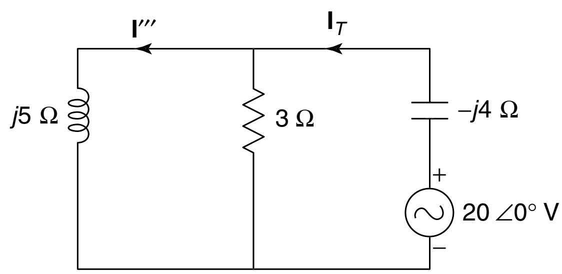


Fig. 6.42

$$\mathbf{Z}_T = -j4 + \frac{3(j5)}{3+j5} = 3.47\angle -50.51°\ \Omega$$

$$\mathbf{I}_T = \frac{20\angle 0°}{3.47\angle -50.51°} = 5.76\angle 50.51°\ \text{A}$$

By current division rule,

$$\mathbf{I}''' = 5.76\angle 50.51° \times \frac{3}{3+j5} = 2.96\angle -8.53°\ \text{A}\ (\leftarrow) = 2.96\angle 171.47°\ \text{A}\ (\rightarrow)$$

Step IV By superposition theorem,

$$\mathbf{I} = \mathbf{I}' + \mathbf{I}'' + \mathbf{I}''' = 2.48\angle -61.66° + 2.96\angle 171.47° + 2.96\angle 171.47° = 4.86\angle -164.41° \text{ A}$$

Example 6.24 *Find the voltage drop across the capacitor for the network shown in Fig. 6.43.*

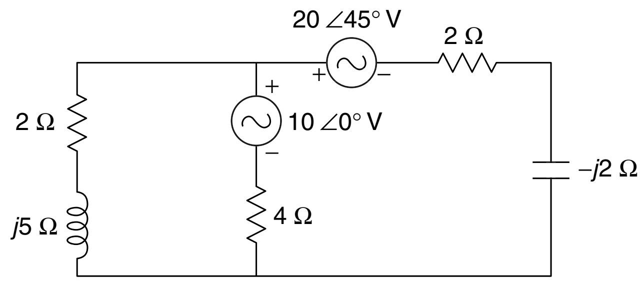


Fig. 6.43

Solution

Step I When the $10\angle 0°$ V source is acting alone (Fig. 6.44)

$$\mathbf{Z}_T = 4 + \frac{(2+j5)(2-j2)}{2+j5+2-j2}$$

$$= 7\angle -5.91° \ \Omega$$

$$\mathbf{I}_T = \frac{10\angle 0°}{7\angle -5.91°} = 1.43\angle 5.91° \text{ A}$$

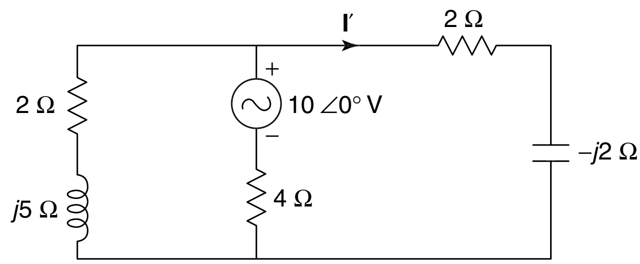


Fig. 6.44

By current division rule,

$$\mathbf{I}' = (1.43\angle 5.91°)\left(\frac{2+j5}{2+j5+2-j2}\right) = 1.54\angle 37.24° \text{ A} \quad (\rightarrow)$$

Step II When the $20\angle 45°$ V source is acting alone (Fig. 6.45)

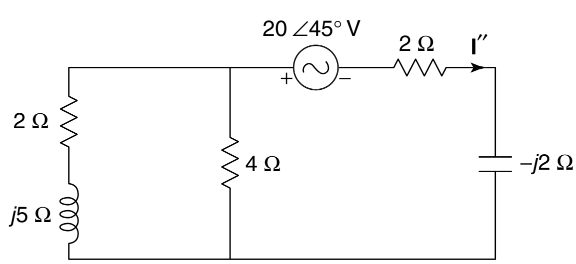


Fig. 6.45

$$\mathbf{Z}_T = (2-j2) + \frac{4(2+j5)}{4+2+j5} = 4.48\angle -8.84° \ \Omega$$

$$\mathbf{I}'' = \frac{20\angle 45°}{4.48\angle -8.84°} = 4.46\angle 53.84° \text{ A} \quad (\leftarrow) = -4.46\angle 53.84° \text{ A} \quad (\rightarrow)$$

Step III By superposition theorem,

$$\mathbf{I} = \mathbf{I}' + \mathbf{I}'' = 1.54\angle 37.24 - 4.46\angle 53.84° = 3.01\angle -117.78° \text{ A}$$

$$\mathbf{V}_c = (-j2)\mathbf{I} = (-j2)\ (3.01\angle -117.78°) = 6.02\angle 152.22° \text{ V}$$

Example 6.25 *Find the node voltage V_2 in the network of Fig. 6.46.*

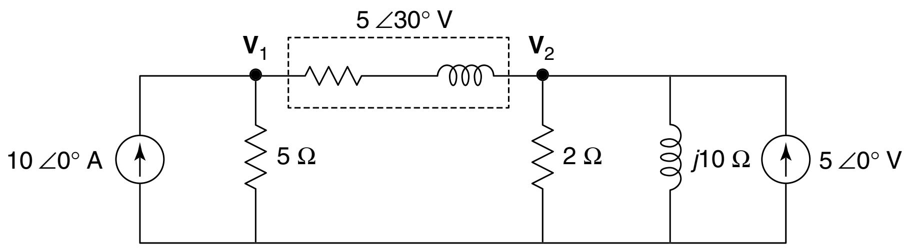


Fig. 6.46

Solution

Step I When the $10\angle 0°$ A source is acting alone (Fig. 6.47)

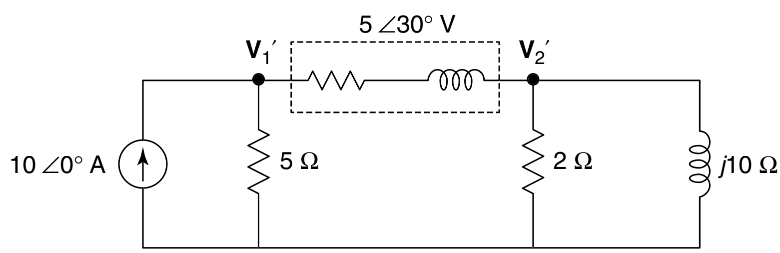


Fig. 6.47

Applying KCL at Node 1,

$$\frac{\mathbf{V}_1'}{5} + \frac{\mathbf{V}_1' - \mathbf{V}_2'}{5\angle 30°} = 10\angle 0°$$

$$\left(\frac{1}{5} + \frac{1}{5\angle 30°}\right)\mathbf{V}_1' - \frac{1}{5\angle 30°}\mathbf{V}_2' = 10\angle 0°$$

$$(0.37 - j0.1)\mathbf{V}_1' - (0.17 - j0.1)\mathbf{V}_2' = 10\angle 0° \qquad \text{...(i)}$$

Applying KCL at Node 2,

$$\frac{\mathbf{V}_2' - \mathbf{V}_1'}{5\angle 30°} + \frac{\mathbf{V}_2'}{2} + \frac{\mathbf{V}_2'}{j10} = 0$$

$$-\frac{1}{5\angle 30°}\mathbf{V}_1' + \left(\frac{1}{5\angle 30°} + \frac{1}{2} + \frac{1}{j10}\right)\mathbf{V}_2' = 0$$

$$-(0.17 - j0.1)\mathbf{V}_1' + (0.67 - j0.2)\mathbf{V}_2' = 0 \qquad \text{...(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 0.37 - j0.1 & -(0.17 - j0.1) \\ -(0.17 - j0.1) & 0.67 - j0.2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1' \\ \mathbf{V}_2' \end{bmatrix} = \begin{bmatrix} 10\angle 0^\circ \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{V}_2' = \frac{\begin{vmatrix} 0.37 - j0.1 & 10\angle 0^\circ \\ -(0.17 - j0.1) & 0 \end{vmatrix}}{\begin{vmatrix} 0.37 - j0.1 & -(0.17 - j0.1) \\ -(0.17 - j0.1) & 0.67 - j0.2 \end{vmatrix}} = 8.57\angle -3.36^\circ \text{ V}$$

Step II When the $5\angle 0^\circ$ A source is acting alone (Fig. 6.48)

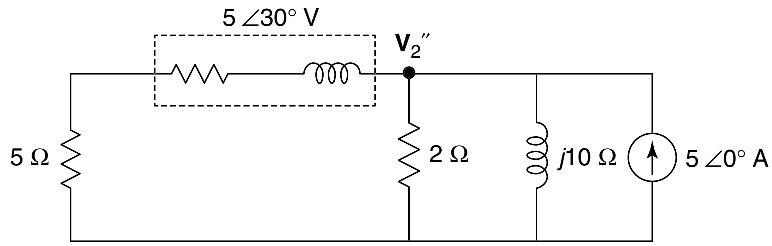


Fig. 6.48

$$\frac{\mathbf{V}_2''}{5\angle 30^\circ + 5} + \frac{\mathbf{V}_2''}{2} + \frac{\mathbf{V}_2''}{j10} = 5\angle 0^\circ$$

$$(0.61\angle -11.93^\circ)\mathbf{V}_2'' = 5\angle 0^\circ$$

$$\mathbf{V}_2'' = 8.2\angle 11.93^\circ \text{ V}$$

Step III By superposition theorem,

$$\mathbf{V}_2 = \mathbf{V}_2' + \mathbf{V}_2'' = 8.57\angle -3.36^\circ + 8.2\angle 11.93^\circ = 16.62\angle 4.12^\circ \text{ V}$$

Example 6.26 *Find current through inductor in the network of Fig. 6.49.*

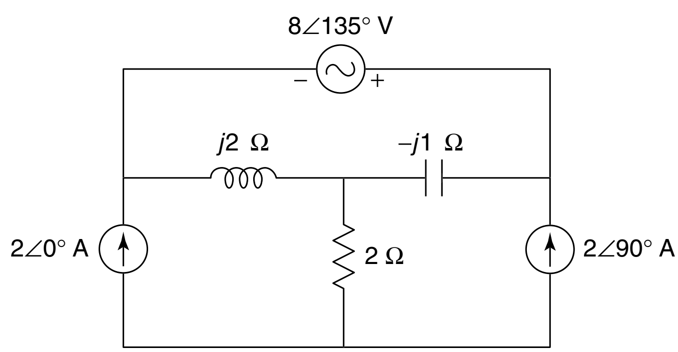


Fig. 6.49

Solution

Step I When the $8\angle 135^\circ$ V source is acting alone (Fig. 6.50)
Applying KVL to the mesh,

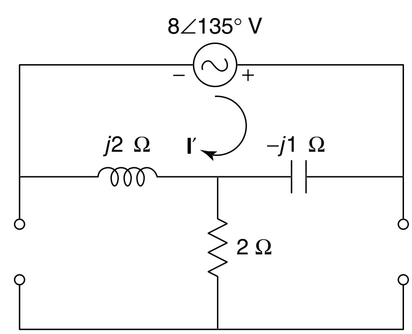


Fig. 6.50

$$8\angle 135^\circ - (-j1)\mathbf{I}' - j2\mathbf{I}' = 0$$

$$\mathbf{I}' = \frac{8\angle 135^\circ}{j1} = 8\angle 45^\circ \text{ A } (\leftarrow) = 8\angle -135^\circ \text{ A } (\rightarrow)$$

Step II When the $2\angle 0°$ A source is acting alone (Fig. 6.51)

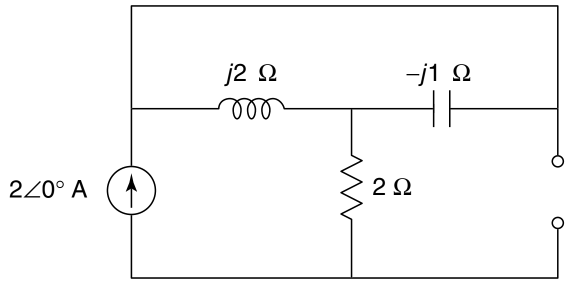


Fig. 6.51

The network can be redrawn as shown in Fig. 6.52.
By current division rule,

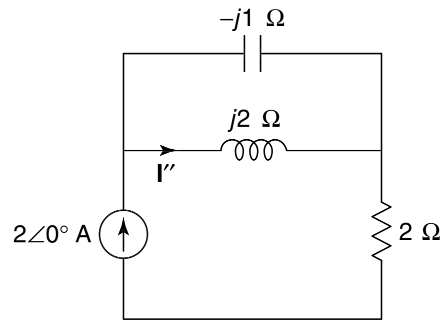


Fig. 6.52

$$\mathbf{I}'' = 2\angle 0°\left(\frac{-j1}{-j1+j2}\right) = 2\angle 0°\left(\frac{-j1}{j1}\right) = 2\angle 180°\ \text{A}(\rightarrow)$$

Step III When the $2\angle 90°$ A source is acting alone (Fig. 6.53)

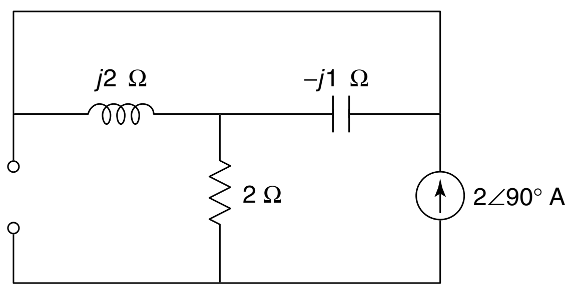


Fig. 6.53

The network can be redrawn as shown in Fig. 6.54.
By current division rule,

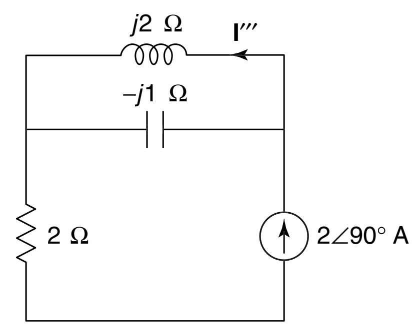


Fig. 6.54

$$\mathbf{I}''' = 2\angle 90°\left(\frac{-j1}{-j1+j2}\right) = 2\angle -90°\ \text{A}\ \ (\leftarrow) = 2\angle 90°\ \text{A}\ \ (\rightarrow)$$

Step III By superposition theorem,

$$\mathbf{I} = \mathbf{I}' + \mathbf{I}'' + \mathbf{I}''' = 8\ \angle{-135°} + 2\angle 180° + 2\angle 90° = 8.49\angle{-154.47°}\text{A}$$

Example 6.27 *Determine the source voltage V_S so that the current through 2 Ω resistor is zero in the network of Fig. 6.55.*

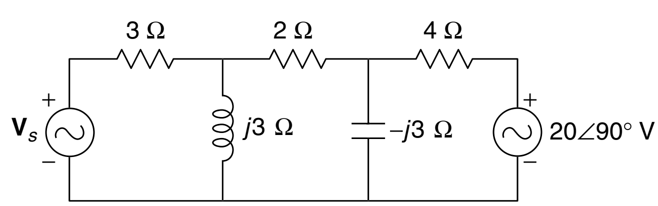


Fig. 6.55

Solution

Step I When the voltage source $\mathbf{V}_s$ is acting alone (Fig. 6.56)

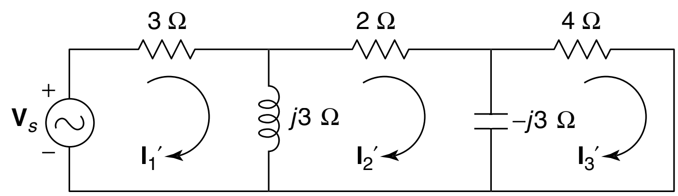


Fig. 6.56

Appling KVL to Mesh 1,

$$\mathbf{V}_s - 3\mathbf{I}_1' - j3(\mathbf{I}_1' - \mathbf{I}_2') = 0$$

$$(3 + j3)\mathbf{I}_1' - j3\mathbf{I}_2' = \mathbf{V}_s \qquad \ldots(\text{i})$$

Appling KVL to Mesh 2,

$$-j3(\mathbf{I}_2' - \mathbf{I}_1') - 2\mathbf{I}_2' + j3(\mathbf{I}_2' - \mathbf{I}_3') = 0$$

$$-j3\mathbf{I}_1' + 2\mathbf{I}_2' + j3\mathbf{I}_3' = 0 \qquad \ldots(\text{ii})$$

Appling KVL to Mesh 3,

$$-j3(\mathbf{I}_3' - \mathbf{I}_2') - 4\mathbf{I}_3' = 0$$

$$j3\mathbf{I}_2' + (4 - j3)\,\mathbf{I}_3' = 0 \qquad \ldots(\text{iii})$$

Writing Eqs (i), (ii) and (iii) in matrix form,

$$\begin{bmatrix} 3+j3 & -j3 & 0 \\ -j3 & 2 & j3 \\ 0 & j3 & 4-j3 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1' \\ \mathbf{I}_2' \\ \mathbf{I}_3' \end{bmatrix} = \begin{bmatrix} \mathbf{V}_s \\ 0 \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2' = \frac{\begin{vmatrix} 3+j3 & \mathbf{V}_s & 0 \\ -j3 & 0 & j3 \\ 0 & 0 & 4-j3 \end{vmatrix}}{\begin{vmatrix} 3+j3 & -j3 & 0 \\ -j3 & 2 & j3 \\ 0 & j3 & 4-j3 \end{vmatrix}} = \frac{(9 + j12)\,\mathbf{V}_s}{\Delta}$$

Step II When the 20 ∠90° V source is acting alone (Fig. 6.57)

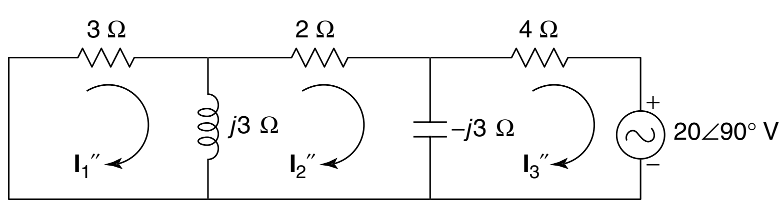


Fig. 6.57

Applying KVL to Mesh 1,

$$-3\mathbf{I}_1'' - j3(\mathbf{I}_1'' - \mathbf{I}_2'') = 0$$

$$(3 + j3)\,\mathbf{I}_1'' - j3\mathbf{I}_2'' = 0 \qquad \ldots(\text{i})$$

Applying KVL to Mesh 2,

$$-j3(\mathbf{I}_2'' - \mathbf{I}_1'') - 2\mathbf{I}_2'' + j3(\mathbf{I}_2'' - \mathbf{I}_3'') = 0$$

$$-j3\mathbf{I}_1'' + 2\mathbf{I}_2'' + j3\mathbf{I}_3'' = 0 \qquad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$j3(\mathbf{I}_3'' - \mathbf{I}_2'') - 4\mathbf{I}_3'' - 20\angle 90° = 0$$

$$j3\mathbf{I}_2'' + (4 - j3)\mathbf{I}_3'' = -20\angle 90° \qquad \text{...(iii)}$$

Writing Eqs (i), (ii) and (iii) in matrix form,

$$\begin{bmatrix} 3+j3 & -j3 & 0 \\ -j3 & 2 & j3 \\ 0 & j3 & 4-j3 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1'' \\ \mathbf{I}_2'' \\ \mathbf{I}_3'' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -20\angle 90° \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2'' = \frac{\begin{vmatrix} 3+j3 & 0 & 0 \\ -j3 & 0 & j3 \\ 0 & -20\angle 90° & 4-j3 \end{vmatrix}}{\begin{vmatrix} 3+j3 & -j3 & 0 \\ -j3 & 2 & j3 \\ 0 & j3 & 4-j3 \end{vmatrix}} = \frac{-180 - j180}{\Delta}$$

Step III By superposition theorem,

$$\mathbf{I}_2 = \mathbf{I}_2' + \mathbf{I}_2'' = \frac{(9 + j12)\mathbf{V}_s + (-180 - j180)}{\Delta} = 0$$

$$(9 + j12)\mathbf{V}_s + (-180 - j180) = 0$$

$$(9 + j12)\mathbf{V}_s = 180 + j180$$

$$\mathbf{V}_s = 16.97\angle -8.13° \text{ V}$$

6.5 THEVENIN'S THEOREM

Thevenin's theorem gives us a method for simplifying a network. In Thevenin's theorem, *any linear network can be replaced by a voltage source* $\mathbf{V}_{\text{Th}}$ *in series with an impedance* $\mathbf{Z}_{\text{Th}}$.

Example 6.28 *Obtain Thevenin's equivalent network for the terminals A and B in Fig. 6.58.*

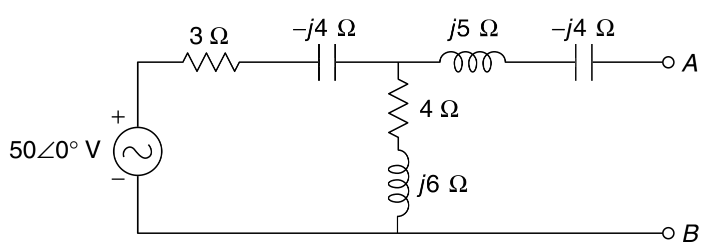


Fig. 6.58

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 6.59)

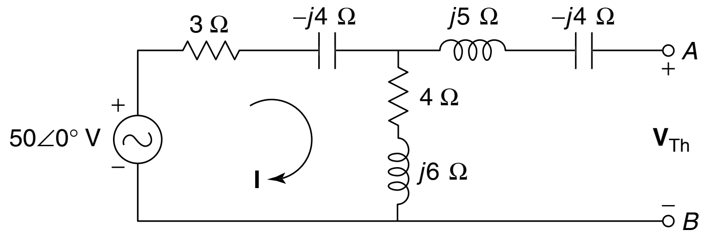


Fig. 6.59

Applying KVL to the mesh,

$50\ \angle 0° - (3 - j4)\ \mathbf{I} - (4 + j6)\ \mathbf{I} = 0$

$$\mathbf{I} = \frac{50\angle 0°}{(3-j4)+(4+j6)} = 6.87\angle -15.95°\ \text{A}$$

$$\mathbf{V}_{\text{Th}} = (4 + j6)\ \mathbf{I} = (4 + j6)\ (6.87\ \angle -15.95°) = 49.5\ \angle 40.35°\ \text{V}$$

Step II Calculation of $\mathbf{Z}_{\text{Th}}$ (Fig. 6.60)

$$\mathbf{Z}_{\text{Th}} = (j5 - j4) + \frac{(3-j4)(4+j6)}{(3-j4)+(4+j6)} = 4.83\angle -1.13°\ \Omega$$

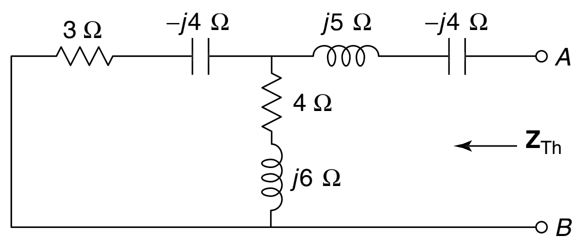


Fig. 6.60

Step III Thevenin's Equivalent Network (Fig. 6.61)

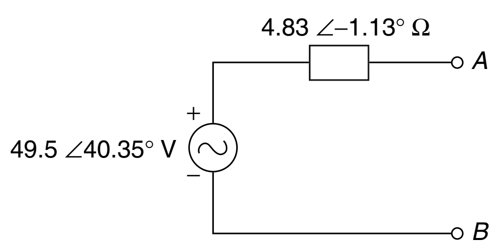


Fig. 6.61

Example 6.29 *Find Thevenin's equivalent network for Fig. 6.62.*

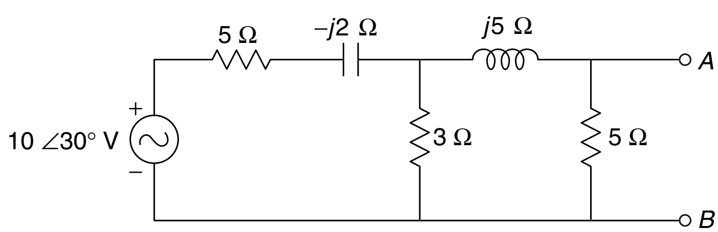


Fig. 6.62

Solution

Step I Calculation of $\mathbf{V}_{Th}$ (Fig. 6.63)

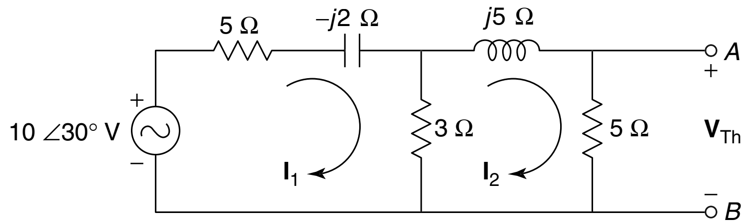


Fig. 6.63

Applying KVL to Mesh 1,

$$10\angle 30^\circ - (5 - j2)\,\mathbf{I}_1 - 3(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(8 - j2)\,\mathbf{I}_1 - 3\mathbf{I}_2 = 10\angle 30^\circ \quad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-3(\mathbf{I}_2 - \mathbf{I}_1) - j5\,\mathbf{I}_2 - 5\,\mathbf{I}_2 = 0$$

$$-3\mathbf{I}_1 + (8 + j5)\,\mathbf{I}_2 = 0 \quad \ldots\text{(ii)}$$

Writing Eqs (i) and (ii) in matrix form;

$$\begin{bmatrix} 8 - j2 & -3 \\ -3 & 8 + j5 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 10\angle 30^\circ \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 8 - j2 & 10\angle 30^\circ \\ -3 & 0 \end{vmatrix}}{\begin{vmatrix} 8 - j2 & -3 \\ -3 & 8 + j5 \end{vmatrix}} = 0.433\angle 9.7^\circ \text{ A}$$

$$\mathbf{V}_{Th} = 5\mathbf{I}_2 = 5(0.433\angle 9.7^\circ) = 2.16\angle 9.7^\circ \text{ V}$$

Step II Calculation of $\mathbf{Z}_{Th}$ (Fig. 6.64)

$$\mathbf{Z}_{Th} = \left[\left\{\frac{(5 - j2)3}{5 - j2 + 3}\right\} + j5\right] \| \, 5$$

$$= [1.94 - j0.265 + j5] \,\|\, 5 = (1.94 + j4.735) \,\|\, 5$$

$$= \frac{(1.94 + j4.735)5}{6.94 + j4.735} = 3.04\angle 33.4^\circ \ \Omega$$

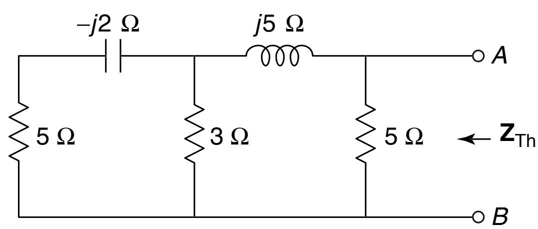


Fig. 6.64

Step III Thevenin's equivalent Network (Fig. 6.65)

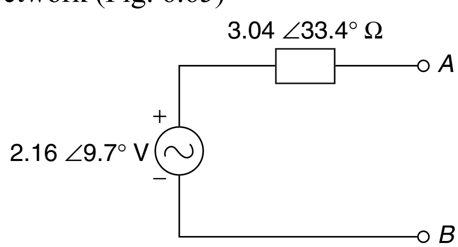


Fig. 6.65

Example 6.30 *Obtain Thevenin's equivalent network for Fig. 6.66.*

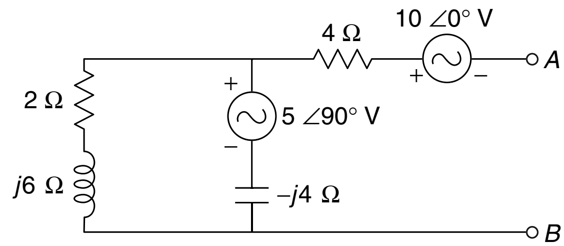


Fig. 6.66

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$

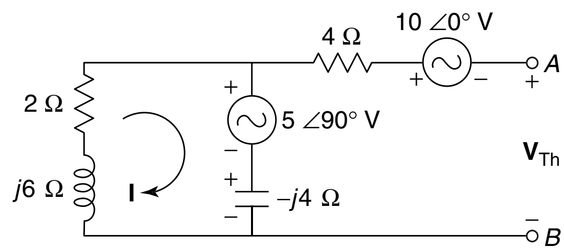


Fig. 6.67

Applying KVL to the mesh,

$$(2+j6-j4)\mathbf{I}-5\angle 90^\circ=0$$

$$\mathbf{I}=\frac{5\angle 90^\circ}{2+j2}=1.77\angle 45^\circ\text{ A}$$

$$\mathbf{V}_{\text{Th}}=(-j4)\,\mathbf{I}+5\angle 90^\circ-10\angle 0^\circ=(4\angle -90^\circ)\,(1.77\angle 45^\circ)+5\angle 90^\circ-10\angle 0^\circ=18\angle 146.31^\circ\text{ V}$$

Step II Calculation of $\mathbf{Z}_{\text{Th}}$ (Fig. 6.67)

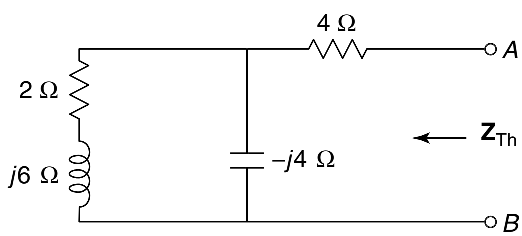


Fig. 6.68

$$\mathbf{Z}_{\text{Th}}=4+\frac{(2+j6)(-j4)}{2+j2}=11.3\angle -44.93^\circ\ \Omega$$

Step III Thevenin's Equivalent Network

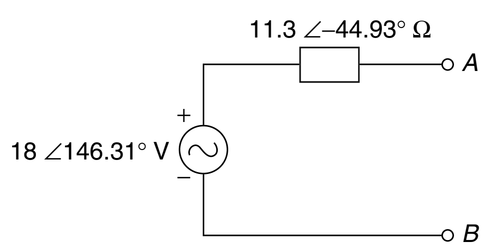


Fig. 6.69

Example 6.31 *Obtain Thevenin's equivalent network for Fig. 6.70.*

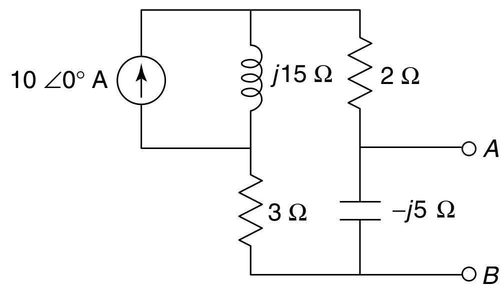


Fig. 6.70

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 6.71)

By current division rule,

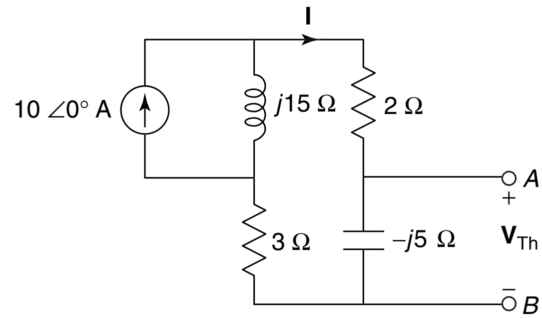


Fig. 6.71

$$\mathbf{I} = \frac{(10\angle 0°)(j15)}{5 - j5 + j15} = 13.42\angle 26.57° \text{ A}$$

$$\begin{aligned}\mathbf{V}_{\text{Th}} &= (-j5)\,\mathbf{I}\\ &= (5\angle -90°)\,(13.42\angle 26.57°) = 67.08\angle -63.43° \text{ V}\end{aligned}$$

Step II Calculation of $\mathbf{Z}_{\text{Th}}$ (Fig. 6.72)

$$\mathbf{Z}_{\text{Th}} = \frac{(-j5)(5 + j15)}{-j5 + 5 + j15} = 7.07\angle -81.86° \ \Omega$$

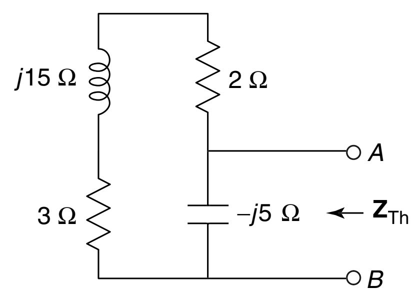


Fig. 6.72

Step III Thevenin's Equivalent Network

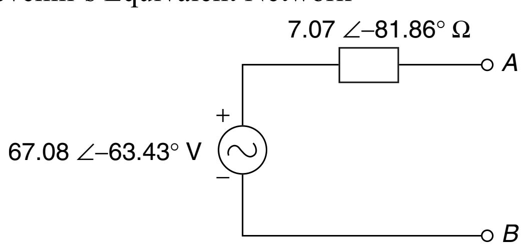


Fig. 6.73

Example 6.32 *Obtain Thevenin's equivalent network for Fig. 6.74.*

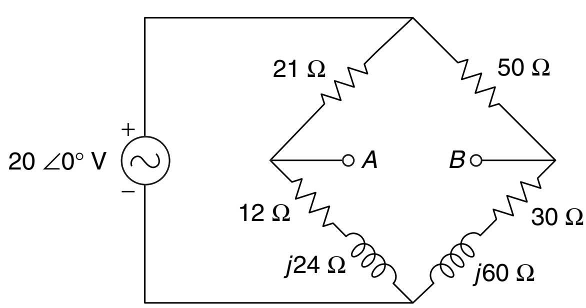


Fig. 6.74

Solution

Step I Calculation of $\mathbf{V}_{Th}$ (Fig 6.75)

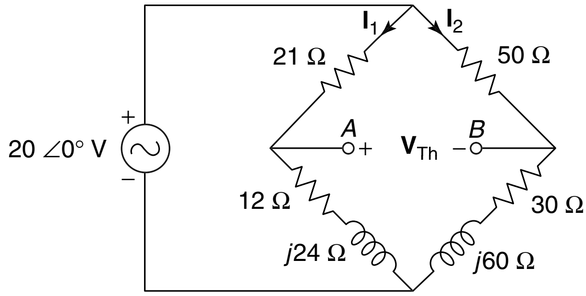


Fig. 6.75

$$\mathbf{I}_1 = \frac{20\angle 0^\circ}{21+12+j24} = 0.49\angle -36.02^\circ \text{ A}$$

$$\mathbf{I}_2 = \frac{20\angle 0^\circ}{80+j60} = 0.2\angle -36.86^\circ \text{ A}$$

$$\begin{aligned}\mathbf{V}_{Th} &= (12+j24)\,\mathbf{I}_1 - (30+j60)\,\mathbf{I}_2 \\ &= (26.83\angle 63.43^\circ)(0.49\angle -36.02^\circ) - (67.08\angle 63.43^\circ)(0.2\angle -36.86^\circ) \\ &= 0.33\angle 171.12^\circ \text{ V}\end{aligned}$$

Step II Calculation of $\mathbf{Z}_{Th}$ (Fig. 6.76)

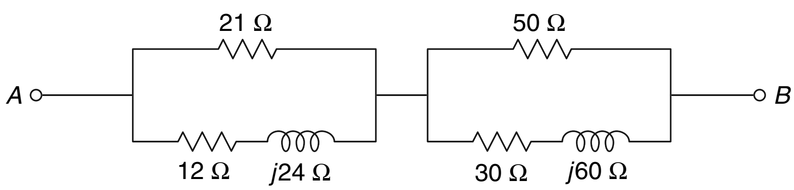


Fig. 6.76

$$\mathbf{Z}_{Th} = \frac{21(12+j24)}{33+j24} + \frac{50(30+j60)}{80+j60} = 47.4\angle 26.8^\circ\ \Omega$$

Step III Thevenin's Equivalent Network

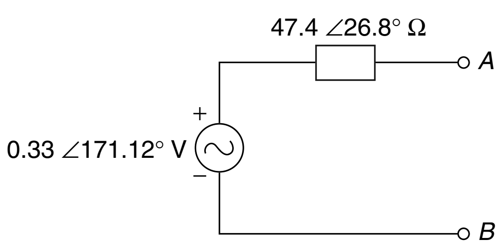


Fig. 6.77

Example 6.33 *Find Thevenin's equivalent network across terminals A and B for Fig. 6.78.*

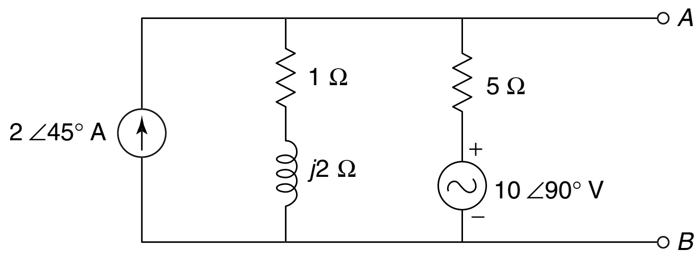


Fig. 6.78

Solution

Step I Calculation of $\mathbf{V}_{\mathrm{Th}}$ (Fig. 6.79)

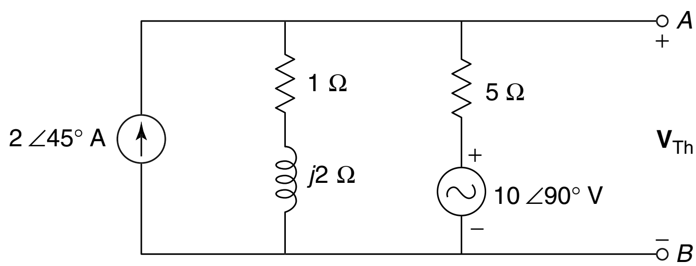


Fig. 6.79

Applying KCL at the node,

$$\frac{\mathbf{V}_{\mathrm{Th}}}{1+j2}+\frac{\mathbf{V}_{\mathrm{Th}}-10\angle 90^\circ}{5}=2\angle 45^\circ$$

$$\left(\frac{1}{1+j2}+\frac{1}{5}\right)\mathbf{V}_{\mathrm{Th}}=2\angle 45^\circ+2\angle 90^\circ$$

$$(0.57\angle -45^\circ)\mathbf{V}_{\mathrm{Th}}=3.7\angle 67.5^\circ$$

$$\mathbf{V}_{\mathrm{Th}}=6.49\angle 112.5^\circ \text{ V}$$

Step II Calculation of $\mathbf{Z}_{\mathrm{Th}}$ (Fig. 6.80)

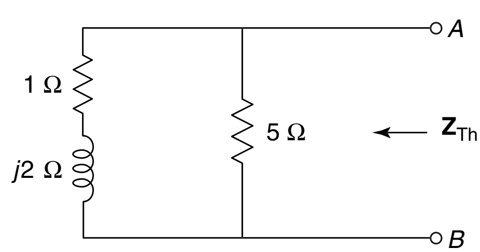


Fig. 6.80

$$\mathbf{Z}_{\mathrm{Th}}=\frac{5(1+j2)}{5+1+j2}=1.77\angle 45^\circ\ \Omega$$

Step III Thevenin's Equivalent Network (Fig. 6.81)

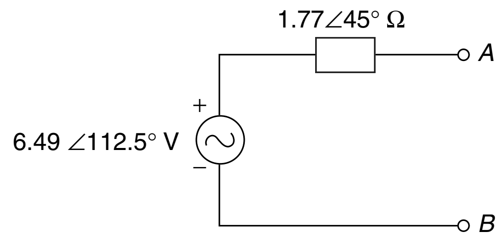


Fig. 6.81

Example 6.34 *Find the current through the* $(5 + j2)$ *Ω impedance in the network of Fig. 6.82.*

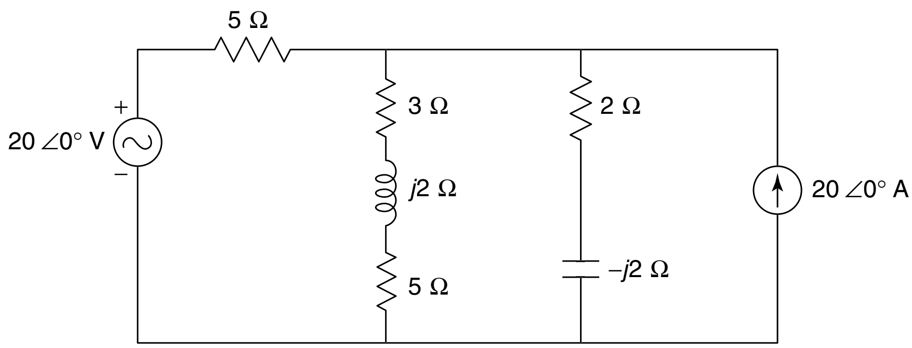


Fig. 6.82

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 6.83)

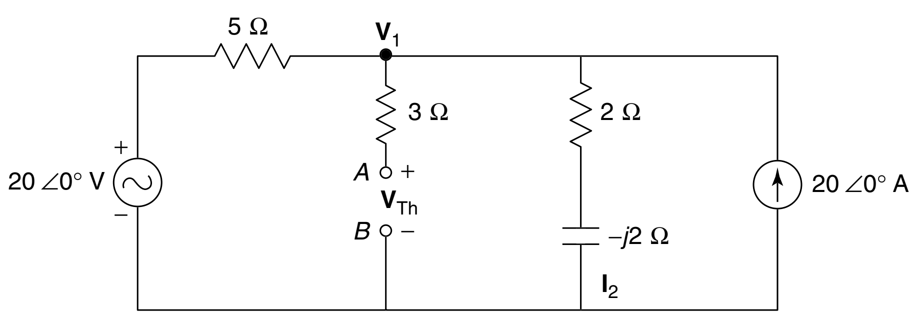


Fig. 6.83

Applying KCL at the node,

$$\frac{\mathbf{V}_1 - 20\angle 0^\circ}{5} + \frac{\mathbf{V}_1}{2 - j2} = 20\angle 0^\circ$$

$$\left(\frac{1}{5} + \frac{1}{2 - j2}\right)\mathbf{V}_1 = 20\angle 0^\circ + 4\angle 0^\circ$$

$$0.51\angle 29.05^\circ \; \mathbf{V}_1 = 24\angle 0^\circ$$

$$\mathbf{V}_1 = 47.06\angle -29.05^\circ \text{ V}$$

$$\mathbf{V}_{\text{Th}} = \mathbf{V}_1 = 47.06\angle -29.05^\circ \text{ V}$$

Step II Calculation of $\mathbf{Z}_{\text{Th}}$ (Fig. 6.84)

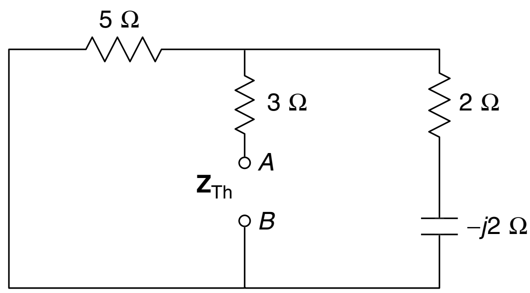


Fig. 6.84

$$\mathbf{Z}_{\text{Th}} = 3 + \frac{5(2-j2)}{5+2-j2} = 4.79\angle{-11.35°}\ \Omega$$

Step III Calculation of $\mathbf{I}_L$ (Fig. 6.85)

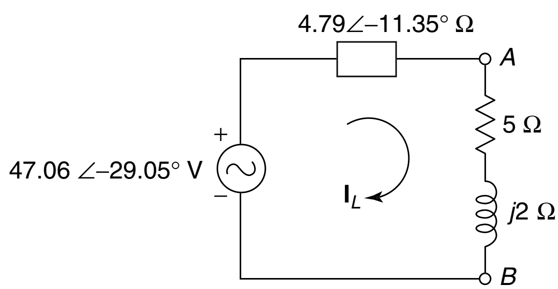


Fig. 6.85

$$\mathbf{I}_L = \frac{47.06\angle-29.05°}{4.79\angle{-11.35°}+5+j2} = 4.73\angle-39.96°\text{ A}$$

Example 6.35 *Find the current through the 5 Ω resistor in the network of Fig. 6.86.*

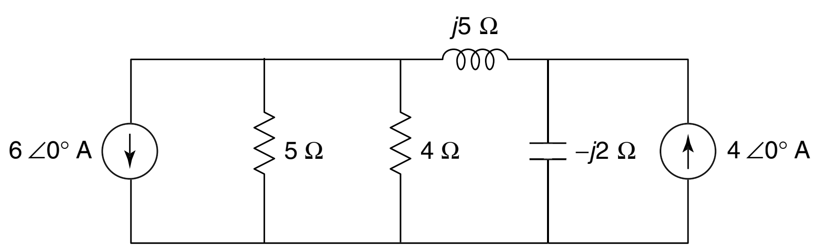


Fig. 6.86

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 6.87)

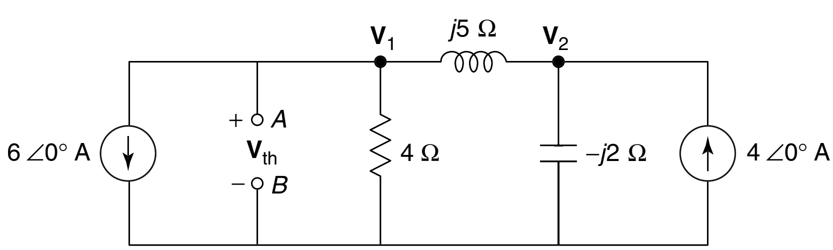


Fig. 6.87

Applying KCL at Node 1,

$$\frac{\mathbf{V}_1}{4}+\frac{\mathbf{V}_1-\mathbf{V}_2}{j5}+6\angle 0^\circ = 0$$

$$\left(\frac{1}{4}+\frac{1}{j5}\right)\mathbf{V}_1-\frac{1}{j5}\mathbf{V}_2 = -6\angle 0^\circ$$

$$(0.25-j0.2)\mathbf{V}_1+j0.2\mathbf{V}_2 = -6\angle 0^\circ \qquad \ldots\text{(i)}$$

Applying KCL at Node 2,

$$\frac{\mathbf{V}_2-\mathbf{V}_1}{j5}+\frac{\mathbf{V}_2}{-j2} = 4\angle 0^\circ$$

$$\left(-\frac{1}{j5}\right)\mathbf{V}_1+\left(\frac{1}{j5}-\frac{1}{j2}\right)\mathbf{V}_2 = 4\angle 0^\circ$$

$$j0.2\,\mathbf{V}_1+j0.3\,\mathbf{V}_2 = 4\angle 0^\circ \qquad \ldots\text{(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 0.25-j0.2 & j0.2 \\ j0.2 & j0.3 \end{bmatrix}\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}=\begin{bmatrix} -6\angle 0^\circ \\ 4\angle 0^\circ \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{V}_1 = \frac{\begin{vmatrix} -6\angle 0^\circ & j0.2 \\ 4\angle 0^\circ & j0.3 \end{vmatrix}}{\begin{vmatrix} 0.25-j0.2 & j0.2 \\ j0.2 & j0.3 \end{vmatrix}} = 20.8\angle -126.87^\circ \text{ V}$$

$$\mathbf{V}_{\text{Th}} = \mathbf{V}_1 = 20.8\angle -126.87^\circ \text{ V}$$

Step II Calculation of $\mathbf{Z}_{\text{Th}}$ (Fig. 6.88)

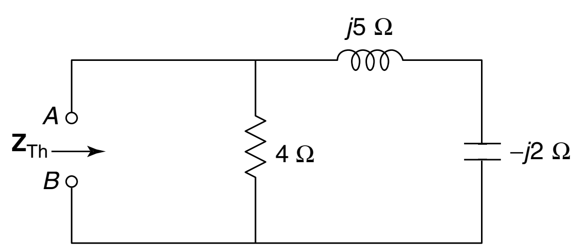


Fig. 6.88

$$\mathbf{Z}_{\text{Th}} = \frac{4(-j2+j5)}{4-j2+j5)} = 2.4\angle 53.13^\circ\ \Omega$$

Step III Calculation of $\mathbf{I}_L$ (Fig. 6.89)

$$\mathbf{I}_L = \frac{20.8\angle -126.87^\circ}{2.4\angle 53.13^\circ + 5} = 3.1\angle -143.47^\circ \text{ A}$$

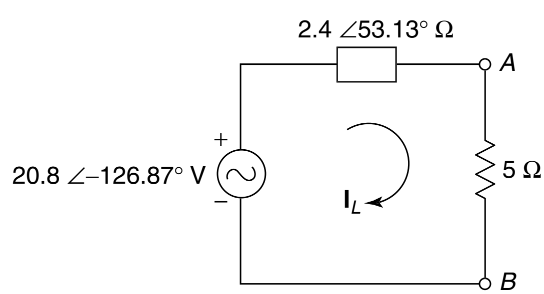


Fig. 6.89

Example 6.36 *In the network of Fig. 6.90, find the current through the 10 Ω resistor.*

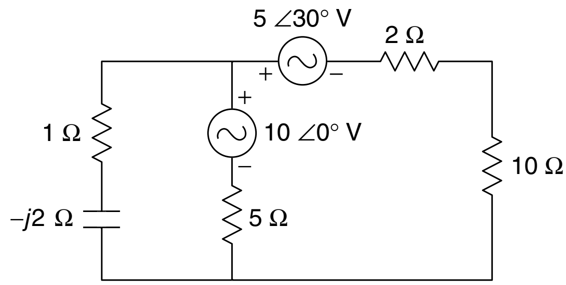


Fig. 6.90

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 6.91)

Applying KVL to the mesh,

$$j2\mathbf{I} - 1\mathbf{I} - 10\angle 0° - 5\mathbf{I} = 0$$

$$(j2 - 6)\mathbf{I} = 10\angle 0°$$

$$\mathbf{I} = 1.58\angle -161.57° \text{ A}$$

Writing $\mathbf{V}_{\text{Th}}$ equation,

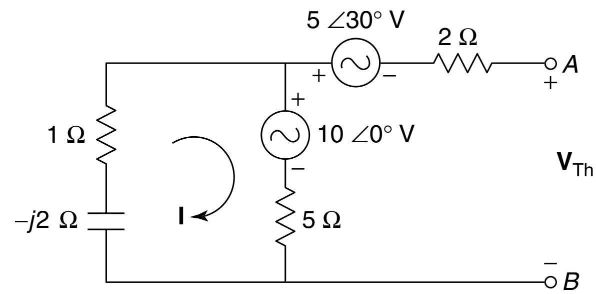


Fig. 6.91

$$5\mathbf{I} + 10\angle 0° - 5\angle 30° - 0 - \mathbf{V}_{\text{Th}} = 0$$

$$5(1.58\angle -161.57°) - 10\angle 0° - 5\angle 30° - \mathbf{V}_{\text{Th}} = 0$$

$$\mathbf{V}_{\text{Th}} = 5.32\angle -110.06° \text{ V}$$

Step II Calculation of $\mathbf{Z}_{\text{Th}}$ (Fig. 6.92)

$$\mathbf{Z}_{\text{Th}} = 2 + \frac{5(1 - j2)}{5 + 1 - j2} = 3.48\angle -21.04° \ \Omega$$

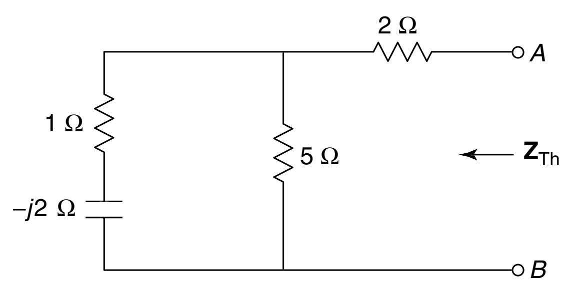


Fig. 6.92

Step III Calculation of $\mathbf{I}_L$ (Fig. 6.93)

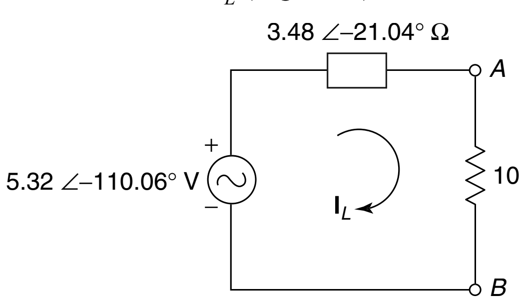


Fig. 6.93

$$\mathbf{I}_L = \frac{5.32\angle -110.06°}{3.48\angle -21.04° + 10} = 0.4\angle -104.67° \text{ A}$$

Example 6.37 *Find the current through $(4 + j6)\ \Omega$ impedance in the network of Fig. 6.94.*

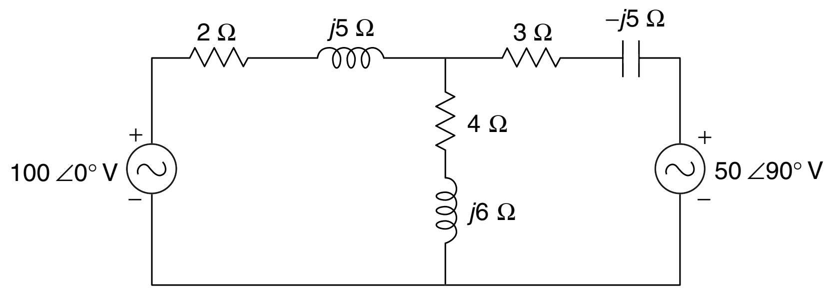


Fig. 6.94

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 6.95)

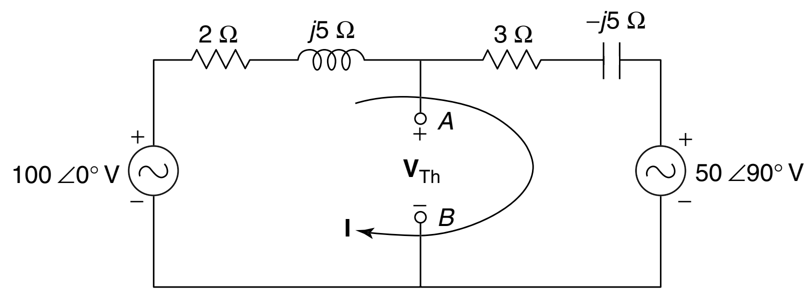


Fig. 6.95

Applying KVL to the mesh,

$$100\angle 0^\circ - 2\mathbf{I} - j5\mathbf{I} - 3\mathbf{I} + j5\mathbf{I} - 50\angle 90^\circ = 0$$

$$\mathbf{I} = 22.36\angle -26.57^\circ \text{ A}$$

Writing $\mathbf{V}_{\text{Th}}$ equation,

$$\mathbf{V}_{\text{Th}} - 3\mathbf{I} + j5\mathbf{I} - 50\angle 90^\circ = 0$$

$$\mathbf{V}_{\text{Th}} - (3 - j5)(22.36\angle -26.57^\circ) - 50\angle 90^\circ = 0$$

$$\mathbf{V}_{\text{Th}} = 80.61\angle -82.88^\circ \text{ V}$$

Step II Calculation of $\mathbf{Z}_{\text{Th}}$ (Fig. 6.96)

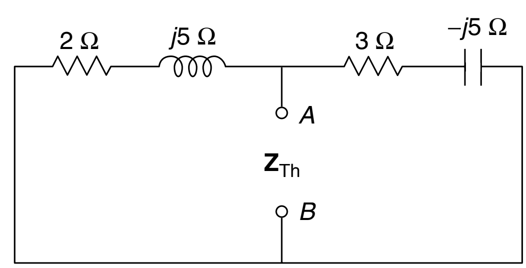


Fig. 6.96

$$\mathbf{Z}_{\text{Th}} = \frac{(2 + j5)(3 - j5)}{2 + j5 + 3 - j5} = 6.28\angle 9.16^\circ\ \Omega$$

Step III Calculation of $\mathbf{I}_L$ (Fig. 6.97)

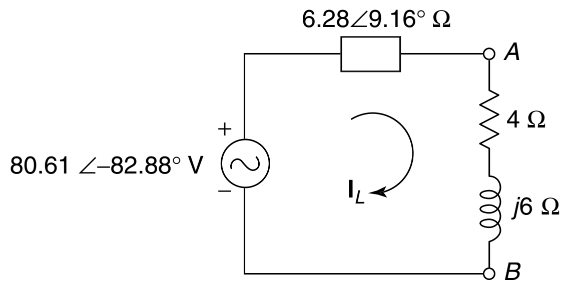


Fig. 6.97

$$\mathbf{I}_L = \frac{80.61\angle -82.88^\circ}{6.28\angle 9.16^\circ + 4 + j6} = 6.52\angle -117.34^\circ \text{ A}$$

Example 6.38 *Obtain Thevenin's equivalent network across terminals A and B in Fig. 6.98.*

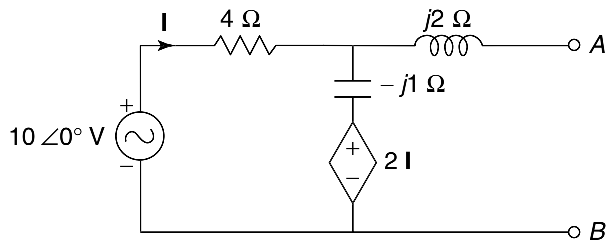


Fig. 6.98

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 6.99)

Applying KVL to the mesh,

$$10\angle 0^\circ - 4\mathbf{I} + j1\mathbf{I} - 2\mathbf{I} = 0$$

$$\mathbf{I} = 1.64\angle 9.46^\circ \text{ A}$$

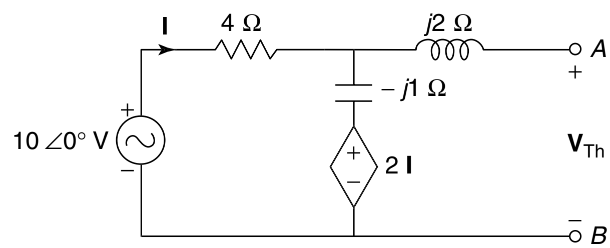


Fig. 6.99

Writing $\mathbf{V}_{\text{Th}}$ equation,

$$10\angle 0^\circ - 4\mathbf{I} - 0 - \mathbf{V}_{\text{Th}} = 0$$

$$10\angle 0^\circ - 4(1.64\angle 9.46^\circ) - \mathbf{V}_{\text{Th}} = 0$$

$$\mathbf{V}_{\text{Th}} = 3.69\angle -17^\circ \text{ V}$$

Step II Calculation of $\mathbf{I}_N$ (Fig. 6.100)

From Fig. 6.100,

$$\mathbf{I} = \mathbf{I}_1$$

Applying KVL to Mesh 1,

$$10\angle 0^\circ - 4\mathbf{I}_1 + j1(\mathbf{I}_1 - \mathbf{I}_2) - 2\mathbf{I} = 0$$

$$10\angle 0^\circ - 4\mathbf{I}_1 + j1\mathbf{I}_1 - j1\mathbf{I}_2 - 2\mathbf{I}_1 = 0$$

$$(6 - j1)\,\mathbf{I}_1 + j1\mathbf{I}_2 = 10\angle 0^\circ \quad \ldots\text{(i)}$$

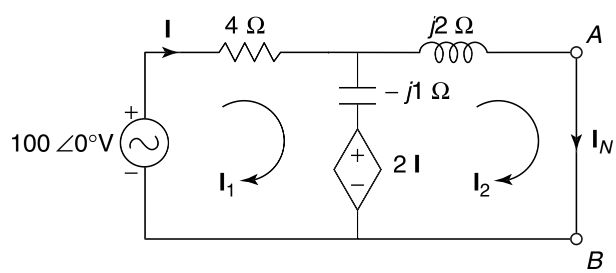


Fig. 6.100

Applying KVL to Mesh 2,

$$2\mathbf{I} + j1(\mathbf{I}_2 - \mathbf{I}_1) - j2\mathbf{I}_2 = 0$$

$$2\mathbf{I}_1 + j1\mathbf{I}_2 - j1\mathbf{I}_1 - j_2\mathbf{I}_2 = 0$$

$$(2-j1)\mathbf{I}_1 - j1\mathbf{I}_2 = 0 \qquad \text{...(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 6-j1 & j1 \\ 2-j1 & -j1 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 10\angle 0° \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 6-j1 & 10\angle 0° \\ 2-j1 & 0 \end{vmatrix}}{\begin{vmatrix} 6-j1 & j1 \\ 2-j1 & -j1 \end{vmatrix}} = 2.71\angle -102.53° \text{ A}$$

$$\mathbf{I}_N = \mathbf{I}_2 = 2.71\angle -102.53° \text{ A}$$

Step III Calculation of $\mathbf{Z}_{\text{Th}}$

$$\mathbf{Z}_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_N} = \frac{3.69\angle -17°}{2.71\angle -102.53°} = 1.36\angle 85.53° \ \Omega$$

Step IV Thevenin's Equivalent Network (Fig. 6.101)

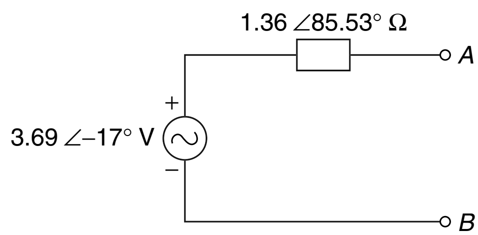


Fig. 6.101

Example 6.39 *Find Thevenin's equivalent network across terminals A and B for Fig. 6.102.*

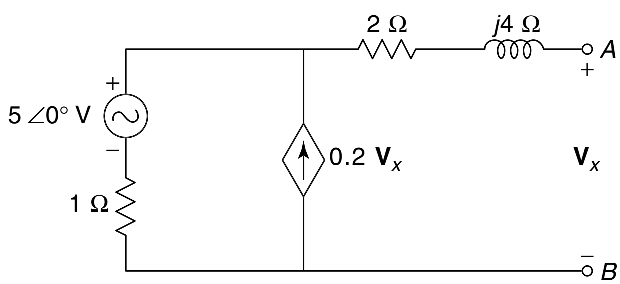


Fig. 6.102

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 6.103)

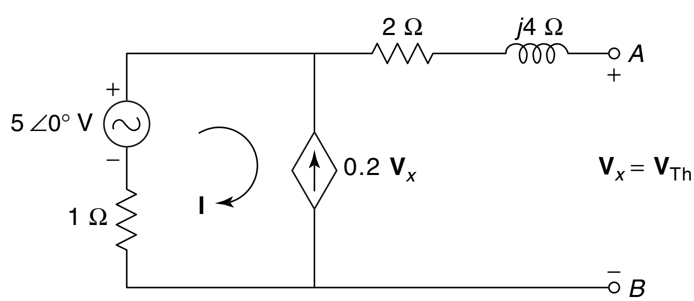


Fig. 6.103

From Fig. 6.103,

$$\mathbf{I} = -0.2\,\mathbf{V}_x \qquad \text{... (i)}$$

Writing $\mathbf{V}_{\text{Th}}$ equation,

$$-\mathbf{I} + 5\angle 0° - 0 - \mathbf{V}_x = 0$$

$$0.2\mathbf{V}_x + 5\angle 0° - \mathbf{V}_x = 0$$
$$\mathbf{V}_x = 6.25\angle 0° \text{ V}$$
$$\mathbf{V}_{\text{Th}} = \mathbf{V}_x = 6.25\angle 0° \text{ V}$$

Step II Calculation of $\mathbf{I}_N$ (Fig. 6.104)

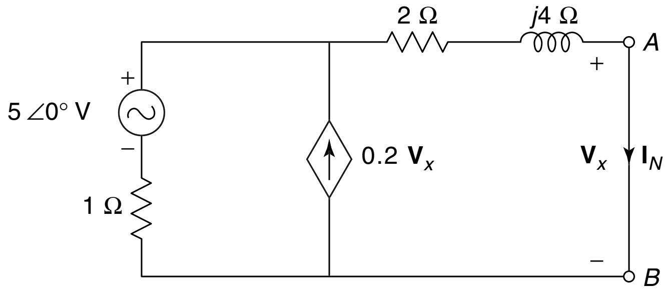


Fig. 6.104

From Fig. 6.104,

$$\mathbf{V}_x = 0$$

The dependent source depends on the controlling variable $\mathbf{V}_x$. When $\mathbf{V}_x = 0$, the dependent source vanishes, i.e. $0.2\mathbf{V}_x = 0$ as shown in Fig. 6.105.

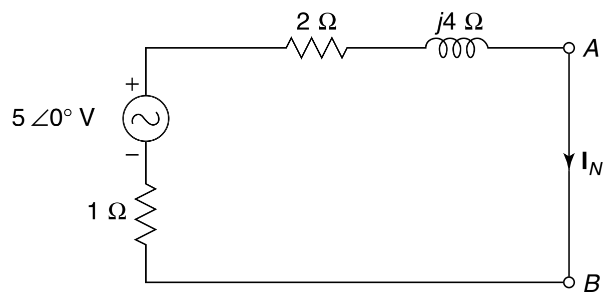


Fig. 6.105

$$\mathbf{I}_N = \frac{5\angle 0°}{1+2+j4} = 1\angle -53.13° \text{ A}$$

Step III Calculation of $\mathbf{Z}_{\text{Th}}$

$$\mathbf{Z}_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_N} = \frac{6.25\angle 0°}{1\angle -53.13°} = 6.25\angle 53.13° \ \Omega$$

Step IV Thevenin's Equivalent Network (Fig. 6.106)

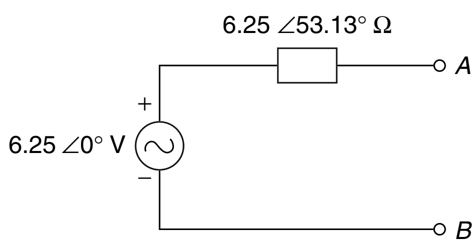


Fig. 6.106

6.6 NORTON'S THEOREM

Norton's theorem states that *any linear network can be replaced by a current source* $\mathbf{I}_N$ *parallel with an impedance* $\mathbf{Z}_N$ *where* $\mathbf{I}_N$ *is the current flowing through the short-circuited path placed across the terminals.*

Example 6.40 *Obtain Norton's equivalent network between terminals A and B as shown in Fig. 6.107.*

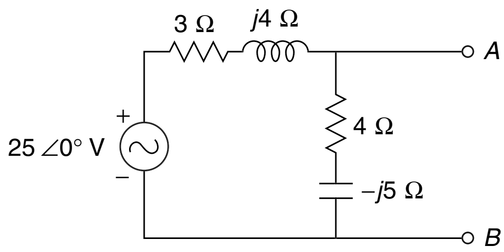


Fig. 6.107

Solution

Step I Calculation of $\mathbf{I}_N$ (Fig. 6.108)

When a short circuit is placed across $(4 - j4)\ \Omega$ impedance, it gets shorted as shown in Fig. 6.109.

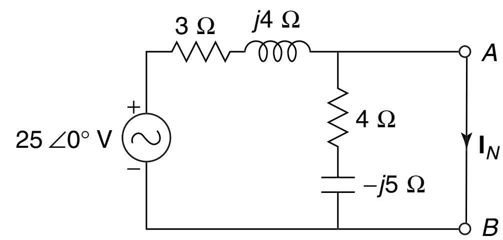


Fig. 6.108

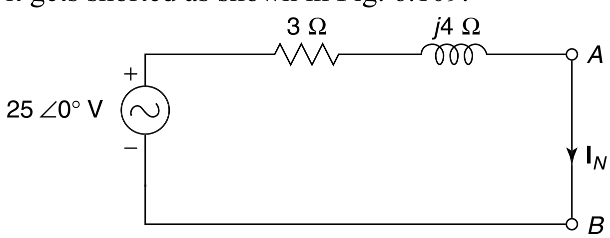


Fig. 6.109

$$\mathbf{I}_N = \frac{25\angle 0^\circ}{3 + j4} = 5\angle -53.13^\circ \text{ A}$$

Step II Calculation of $\mathbf{Z}_N$ (Fig. 6.110)

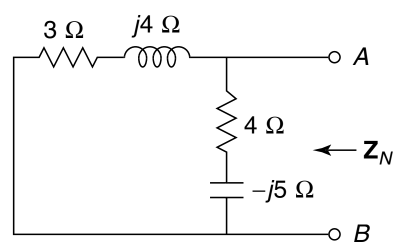


Fig. 6.110

$$\mathbf{Z}_N = \frac{(3 + j4)(4 - j5)}{3 + j4 + 4 - j5} = 4.53\angle 9.92^\circ\ \Omega$$

Step III Norton's Equivalent Network

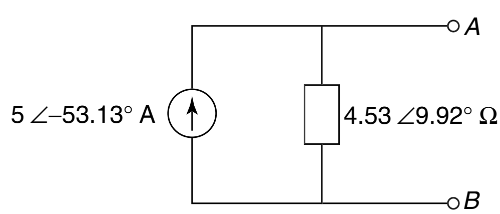


Fig. 6.111

Example 6.41 *Obtain Norton's equivalent network at the terminals A and B in Fig. 6.112.*

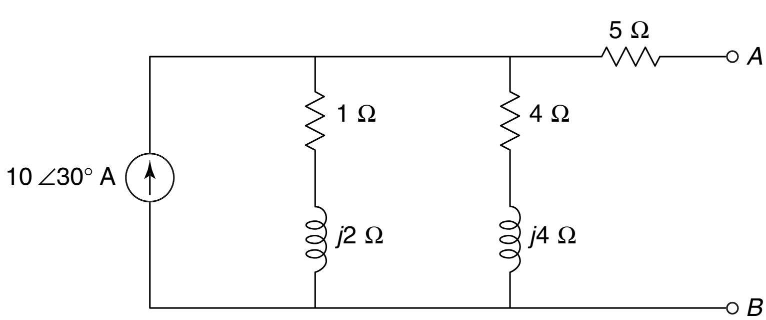


Fig. 6.112

Solution

Step I Calculation of $\mathbf{I}_N$ (Fig. 6.113)

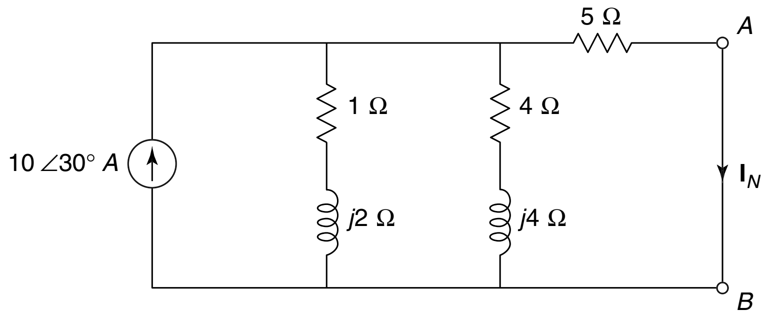


Fig. 6.113

By series-parallel reduction technique (Fig. 6.114)

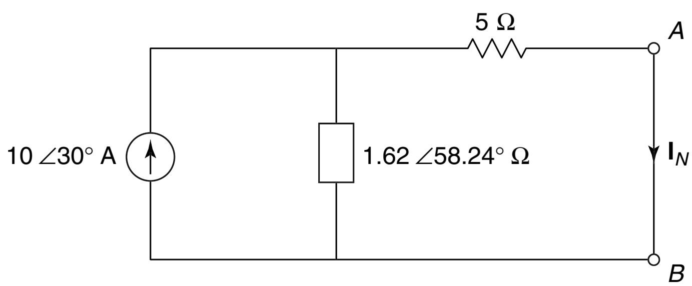


Fig. 6.114

$$\mathbf{I}_N = (10\angle 30°)\left(\frac{1.62\angle 58.24°}{1.62\angle 58.24° + 5}\right) = 2.69\angle 75° \text{ A}$$

Step II Calculation of $\mathbf{Z}_N$ (Fig. 6.115)

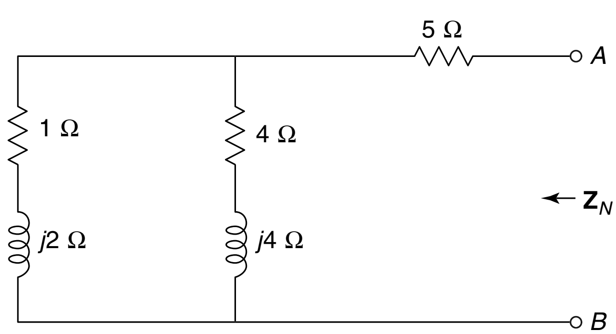


Fig. 6.115

$$\mathbf{Z}_N = 5 + \frac{(1+j2)(4+j4)}{1+j2+4+j4} = 6.01\angle 13.24° \ \Omega$$

Step III Norton's Equivalent Network (Fig. 6.116)

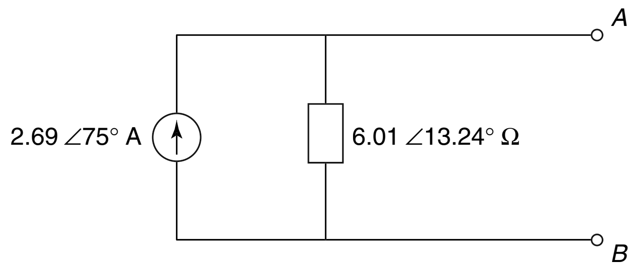


Fig. 6.116

Example 6.42 *Find Norton's equivalent network across terminals A and B in Fig. 6.117.*

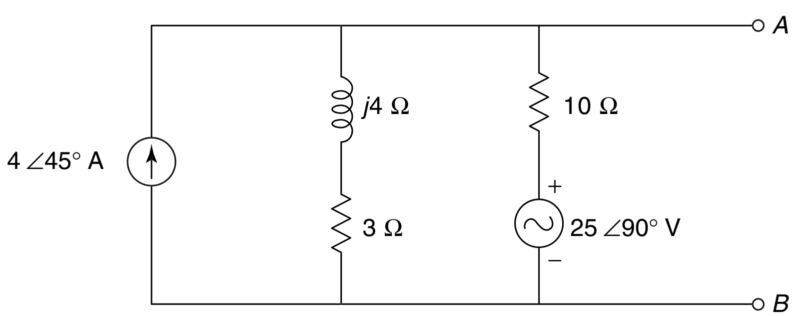


Fig. 6.117

Solution

Step I Calculation of $\mathbf{I}_N$ (Fig. 6.118)

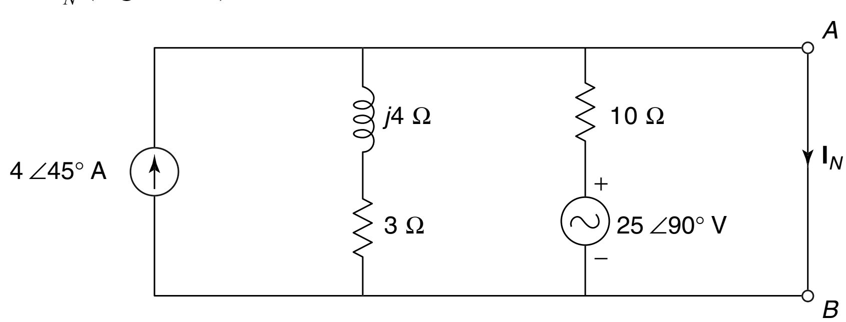


Fig. 6.118

When a short circuit is placed across the $(3+j4)\ \Omega$ impedance, it gets shorted as shown in Fig. 6.119.

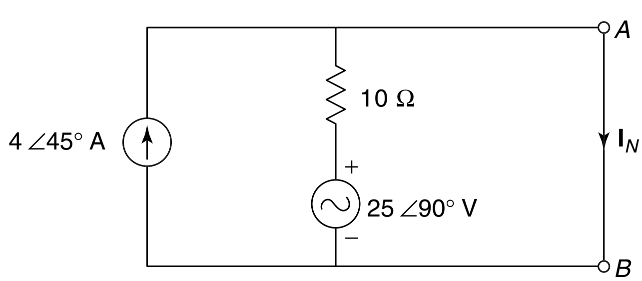


Fig. 6.119

By source transformation, the network is redrawn as shown in Fig. 6.120.

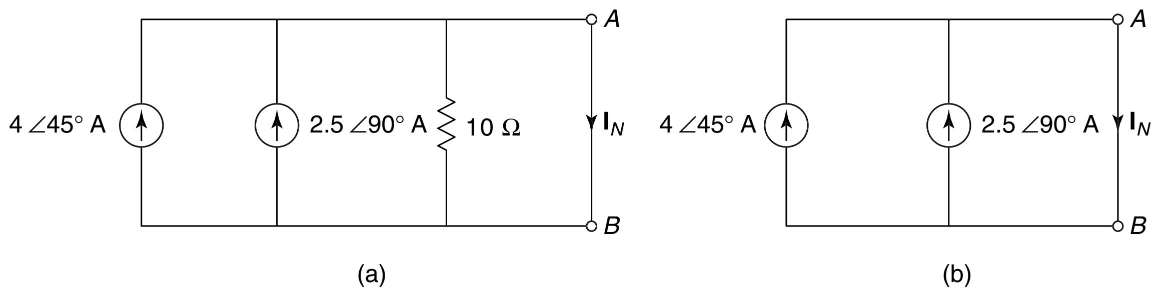


Fig. 6.120

$$I_N = 4\angle 45° + 2.5\angle 90° = 6.03\angle 62.04° \text{ A}$$

Step II Calculation of $\mathbf{Z}_N$ (Fig. 6.121)

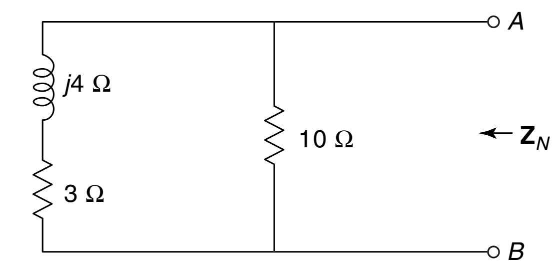


Fig. 6.121

$$\mathbf{Z}_N = \frac{10(3+j4)}{10+3+j4} = 3.68\angle 36.03° \ \Omega$$

Step III Norton's Equivalent Network (Fig. 6.122)

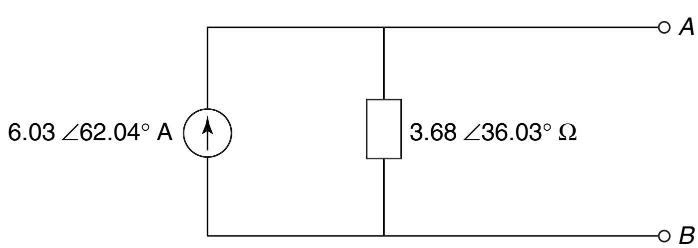


Fig. 6.122

Example 6.43 *Obtain the Norton's equivalent network for Fig. 6.123.*

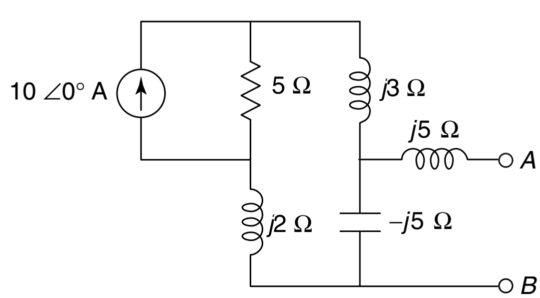


Fig. 6.123

Solution

Step I Calculation of $\mathbf{I}_N$ (Fig. 6.124)

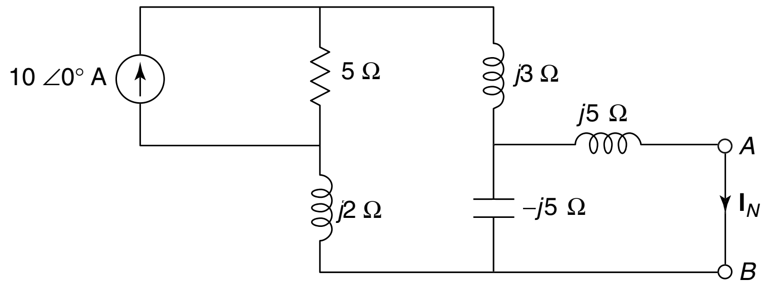


Fig. 6.124

By source transformation, the network can be redrawn as shown in Fig. 6.125.

Writing KVL equations in matrix form,

$$\begin{bmatrix} 5 & j5 \\ j5 & 0 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 50\angle 0^\circ \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 5 & 50\angle 0^\circ \\ j5 & 0 \end{vmatrix}}{\begin{vmatrix} 5 & j5 \\ j5 & 0 \end{vmatrix}} = 10\angle -90^\circ \text{ A}$$

$$\mathbf{I}_N = \mathbf{I}_2 = 10\angle -90^\circ \text{ A}$$

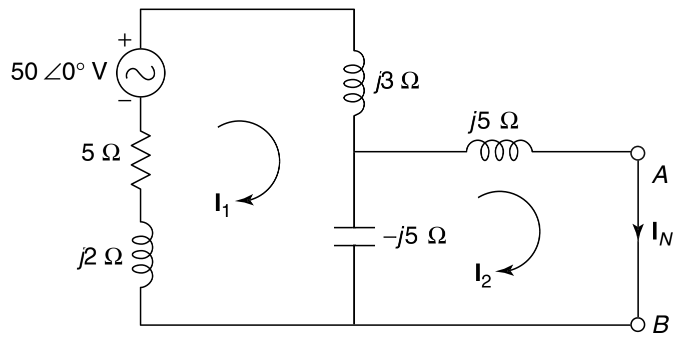


Fig. 6.125

Step II Calculation of $\mathbf{Z}_N$ (Fig. 6.126)

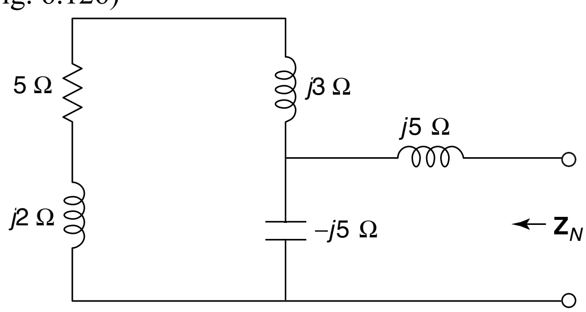


Fig. 6.126

$$\mathbf{Z}_N = j5 + \frac{(5+j5)(-j5)}{5+j5-j5} = 5\ \Omega$$

Step III Norton's Equivalent Network (Fig. 6.127)

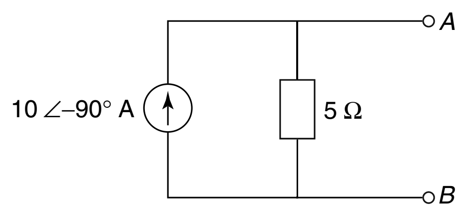


Fig. 6.127

Example 6.44 *Obtain the Norton's equivalent network for Fig. 6.128.*

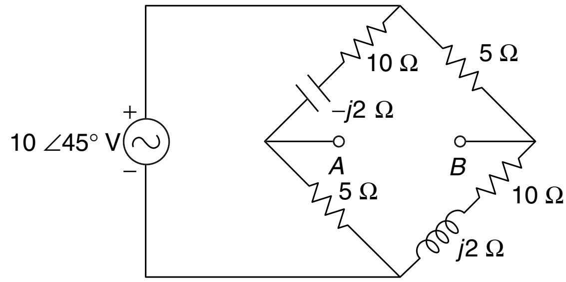


Fig. 6.128

Solution

Step I Calculation of $\mathbf{I}_N$ (Fig. 6.129)

Writing KVL equations in matrix form,

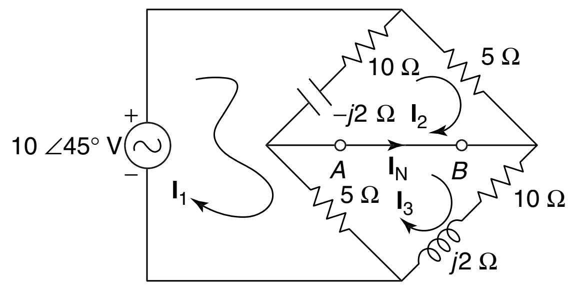


Fig. 6.129

$$\begin{bmatrix} 15-j2 & -10+j2 & -5 \\ -10+j2 & 15-j2 & 0 \\ -5 & 0 & 15+j2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 10\angle 45^\circ \\ 0 \\ 0 \end{bmatrix}$$

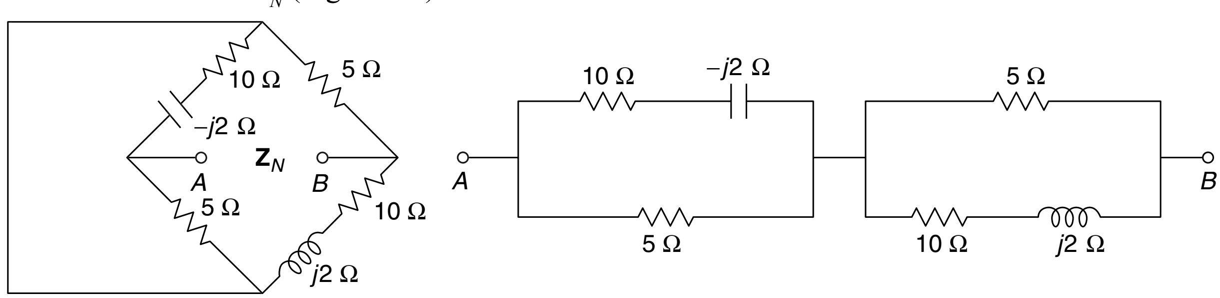
By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 15-j2 & 10\angle 45^\circ & -5 \\ -10+j2 & 0 & 0 \\ -5 & 0 & 15+j2 \end{vmatrix}}{\begin{vmatrix} 15-j2 & -10+j2 & -5 \\ -10+j2 & 15-j2 & 0 \\ -5 & 0 & 15+j2 \end{vmatrix}} = 1\angle 41.28^\circ \text{ A}$$

$$\mathbf{I}_3 = \frac{\begin{vmatrix} 15-j2 & -10+j2 & 10\angle 45^\circ \\ -10+j2 & 15-j2 & 0 \\ -5 & 0 & 0 \end{vmatrix}}{\begin{vmatrix} 15-j2 & -10+j2 & -5 \\ -10+j2 & 15-j2 & 0 \\ -5 & 0 & 15+j2 \end{vmatrix}} = 0.49\angle 37.41^\circ \text{ A}$$

$$\mathbf{I}_N = \mathbf{I}_3 - \mathbf{I}_2 = 0.49\angle 37.41 - 1\angle 41.28^\circ = 0.51\angle -135^\circ \text{ A}$$

Step II Calculation of $\mathbf{Z}_N$ (Fig. 6.130)



(a) (b)

Fig. 6.130

$$\mathbf{Z}_N = \frac{5(10-j2)}{5+10-j2} + \frac{5(10+j2)}{5+10+j2} = 6.72\ \Omega$$

Step III Norton's Equivalent Network (Fig. 6.131)

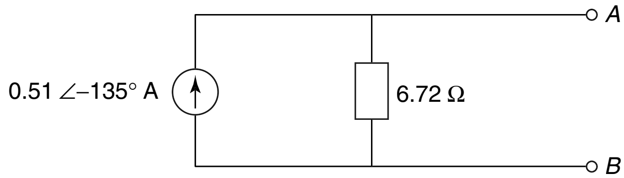


Fig. 6.131

Example 6.45 *Find the current through the 8 Ω resistor in the Network of Fig. 6.132.*

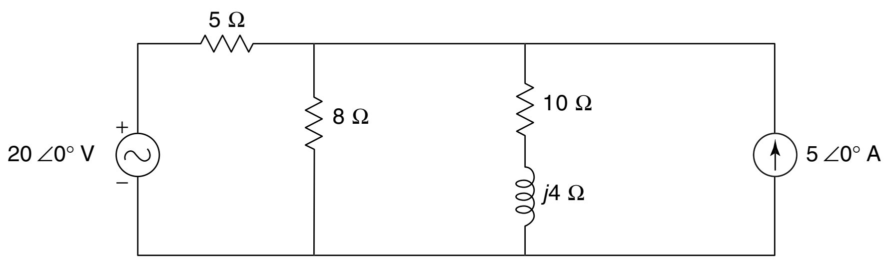


Fig. 6.132

Solution

Step I Calculation of $\mathbf{I}_N$ (Fig. 6.133)

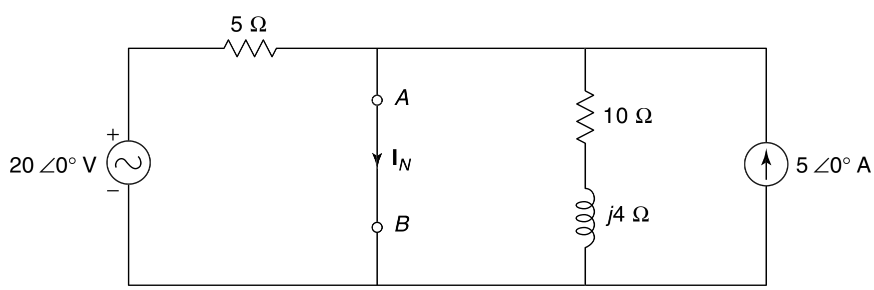


Fig. 6.133

When a short circuit is placed across the $(10+j4)\ \Omega$ impedance, it gets shorted as shown in fig. 6.134.

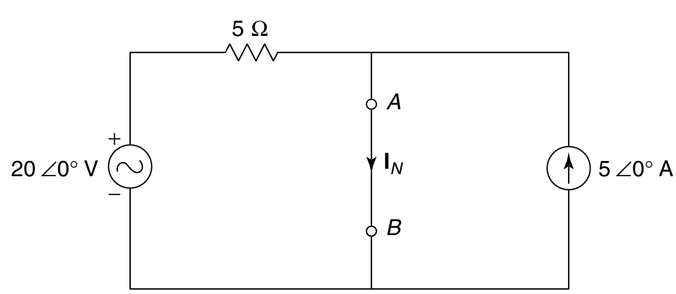


Fig. 6.134

By source transformation, the network is redrawn as shown in Fig. 6.135.

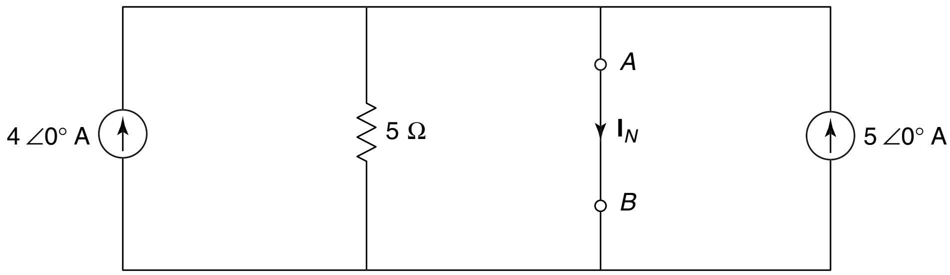


Fig. 6.135

$$\mathbf{I}_N = 4\angle 0° + 5\angle 0° = 9\angle 0° \text{ A}$$

Step II Calculation of $\mathbf{Z}_N$ (Fig. 6.136)

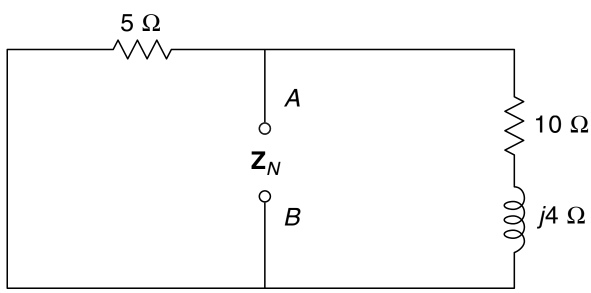


Fig. 6.136

$$\mathbf{Z}_N = \frac{5(10+j4)}{5+10+j4} = 3.47\angle 6.87° \ \Omega$$

Step III Calculation of $\mathbf{I}_L$ (Fig. 6.137)

$$\mathbf{I}_L = \frac{9\angle 0°}{3.47\angle 6.87° + 8} = 0.79\angle -2.08° \text{ A}$$

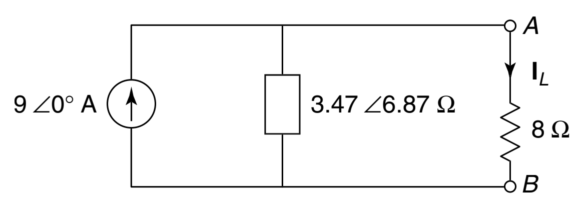


Fig. 6.137

Example 6.46 *Obtain Norton's equivalent network across the terminals A and B in Fig. 6.138.*

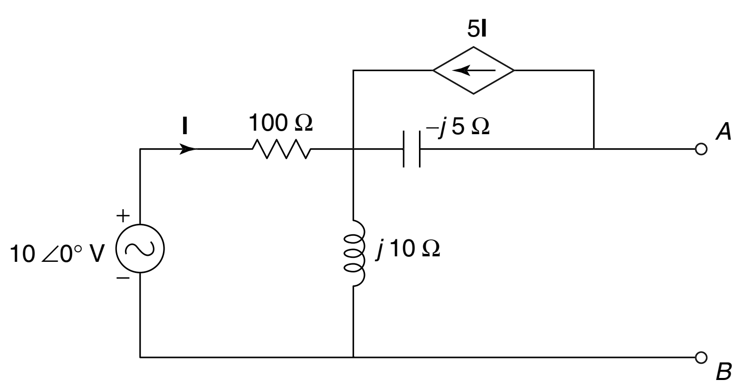


Fig. 6.138

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 6.139)

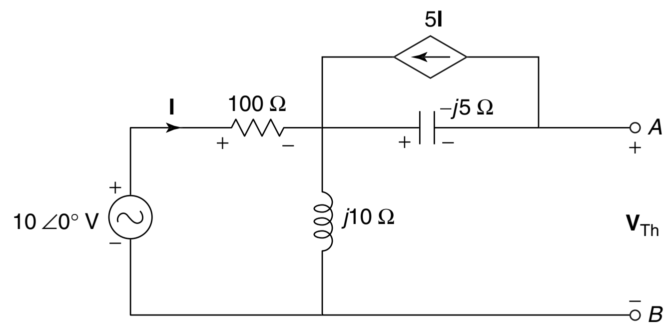


Fig. 6.139

$$\mathbf{I} = \frac{10\angle 0^\circ}{100 + j10} = 0.1\angle -5.71^\circ \text{ A}$$

Writing $\mathbf{V}_{\text{Th}}$ equation,

$$10\angle 0^\circ - 100\,\mathbf{I} - (-j5)(5\mathbf{I}) - \mathbf{V}_{\text{Th}} = 0$$

$$10\angle 0^\circ - 100(0.1\angle -5.71^\circ) + (j5)(5)(0.1\angle -5.71^\circ) - \mathbf{V}_{\text{Th}} = 0$$

$$\mathbf{V}_{\text{Th}} = 3.5\angle 85.1^\circ \text{ V}$$

Step II Calculation of $\mathbf{I}_N$ (Fig. 6.140)

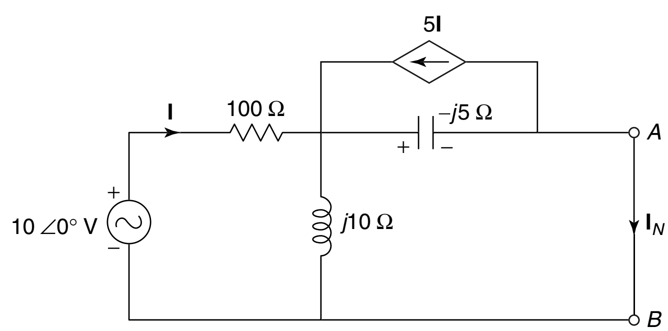


Fig. 6.140

By source transformation, the network is redrawn as shown in Fig. 6.141.

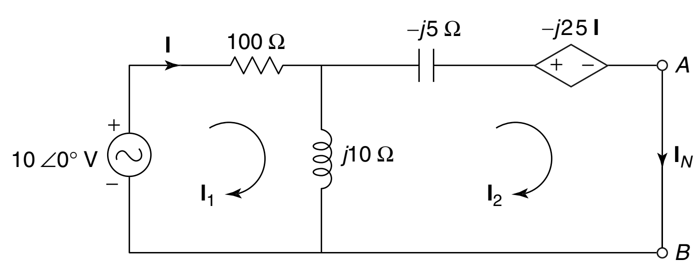


Fig. 6.141

From Fig. 6.141,

$$\mathbf{I} = \mathbf{I}_1 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 1,

$$10\angle 0^\circ - 100\,\mathbf{I}_1 - j10(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(100 + j10)\,\mathbf{I}_1 - j10\mathbf{I}_2 = 10\angle 0^\circ \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 2,

$$-j10(\mathbf{I}_2 - \mathbf{I}_1) + j5\mathbf{I}_2 + j25\mathbf{I} = 0$$

$$-j10\mathbf{I}_2 + j10\mathbf{I}_1 + j5\mathbf{I}_2 + j25\mathbf{I}_1 = 0$$

$$j35\mathbf{I}_1 - j5\mathbf{I}_2 = 0 \qquad \ldots\text{(iii)}$$

Writing Eqs (ii) and (iii) in matrix form,

$$\begin{bmatrix} 100 + j10 & -j10 \\ j35 & -j5 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 10\angle 0^\circ \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 100 + j10 & 10\angle 0^\circ \\ j35 & 0 \end{vmatrix}}{\begin{vmatrix} 100 + j10 & -j10 \\ j35 & -j5 \end{vmatrix}} = 0.6\angle 30.96^\circ \text{ A}$$

$$\mathbf{I}_N = \mathbf{I}_2 = 0.6\angle 30.96^\circ \text{ A}$$

Step III Calculation of $\mathbf{Z}_N$

$$\mathbf{Z}_N = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_N} = \frac{3.5\angle 85.1^\circ}{0.6\angle 30.96^\circ} = 5.83\angle 54.14^\circ\ \Omega$$

Step IV Norton's Equivalent Network (Fig. 6.142)

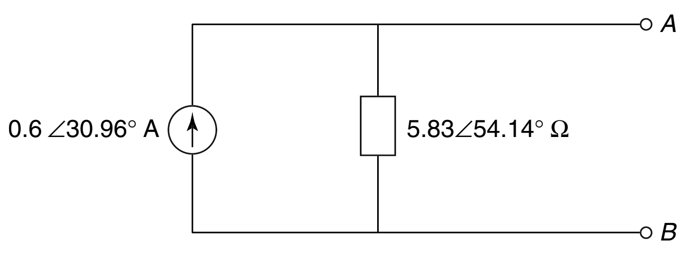


Fig. 6.142

6.7 MAXIMUM POWER TRANSFER THEOREM

This theorem is used to determine the value of load impedance for which the source will transfer maximum power.

Consider a simple network as shown in Fig. 6.143. There are three possible cases for load impedance $\mathbf{Z}_L$.

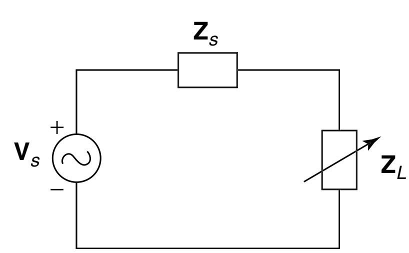


Fig. 6.143

Case (i) When the load impedance is variable resistance (Fig. 6.144)

$$\mathbf{I}_L = \frac{\mathbf{V}_s}{\mathbf{Z}_s + \mathbf{Z}_L} = \frac{\mathbf{V}_s}{R_s + jX_s + R_L}$$

$$|\mathbf{I}_L| = \frac{|\mathbf{V}_s|}{\sqrt{(R_s + R_L)^2 + X_s^2}}$$

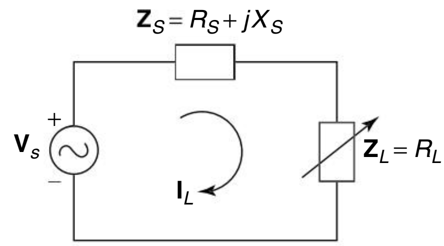


Fig. 6.144 *Purely resistive load*

The power delivered to the load is

$$P_L = |\mathbf{I}_L|^2 R_L = \frac{|\mathbf{V}_s|^2 R_L}{\sqrt{(R_s + R_L)^2 + X_s{}^2}}$$

For power to be maximum,

$$\frac{dP_L}{dR_L} = 0$$

$$|\mathbf{V}_s|^2 \left[\frac{\{(R_s + R_L)^2 + X_s^2\} - 2R_L(R_s + R_L)}{[(R_s + R_L)^2 + X_s^2]^2} \right] = 0$$

$$(R_s + R_L)^2 + X_s^2 - 2R_L(R_s + R_L) = 0$$

$$R_s^2 + 2R_sR_L + R_L^2 + X_s^2 - 2R_LR_s - 2R_L^2 = 0$$

$$R_s^2 + X_s^2 - R_L^2 = 0$$

$$R_L^2 = R_s^2 + X_s^2$$

$$R_L = \sqrt{R_s^2 + X_s^2} = |Z_s|$$

Hence, load resistance R_L should be equal to the magnitude of the source impedance for maximum power transfer.

Case (ii) When the load impedance is a complex impedance with variable resistance and variable reactance (Fig. 6.145)

$$\mathbf{I}_L = \frac{\mathbf{V}_s}{\mathbf{Z}_s + \mathbf{Z}_L}$$

$$|\mathbf{I}_L| = \frac{|\mathbf{V}_s|}{\sqrt{(R_s + R_L)^2 + (X_s + X_L)^2}}$$

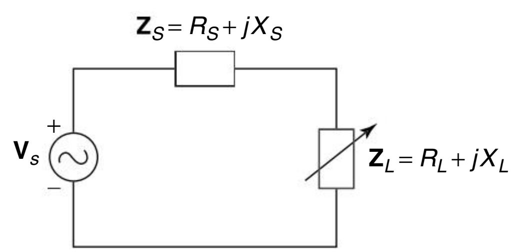


Fig. 6.145 *Complex impedance load*

The power delivered to the load is

$$P_L = |\mathbf{I}_L|^2 R_L = \frac{|\mathbf{V}_S|^2 R_L}{(R_s + R_L)^2 + (X_s + X_L)^2}$$

For maximum value of P_L, denominator of the equation should be small, ie. $X_L = -X_s$.

$$P_L = \frac{|\mathbf{V}_S|^2 R_L}{(R_s + R_L)^2}$$

Differentiating the above equation w.r.t. R_L and equating to zero,

$$\frac{dP_L}{dR_L} = |\mathbf{V}_s|^2 \left[\frac{(R_s + R_L)^2 - 2\,R_L\,(R_s + R_L)}{(R_s + R_L)^2} \right] = 0$$

$$(R_s + R_L)^2 - 2\,R_L\,(R_s + R_L) = 0$$

$$R_s^2 + 2\,R_s R_L + R_L^2 - 2\,R_L R_s - 2R_L^2 = 0$$

$$R_s^2 - R_L^2 = 0$$

$$R_L^2 = R_s^2$$

$$R_L = R_s$$

Hence, load resistance R_L should be equal to source resistance R_L and load reactance X_L should be equal to negative value of source reactance for maximum power transfer.

$$\mathbf{Z}_L = \mathbf{Z}_s^* = R_s - jX_s$$

i.e. load impedance should be a complex conjugate of the source impedance.

Case (iii) When the load impedance is a complex impedance with variable resistance and fixed reactance (Fig. 6.146)

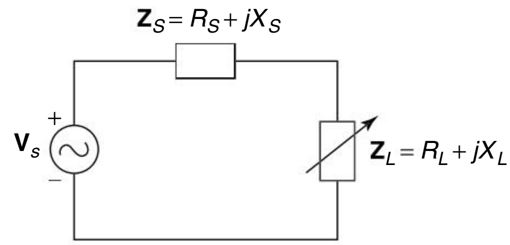


Fig. 6.146 *Complex impedance load*

$$\mathbf{I}_L = \frac{\mathbf{V}_s}{\mathbf{Z}_s + \mathbf{Z}_L}$$

$$|\mathbf{I}_L| = \frac{|\mathbf{V}_s|}{\sqrt{(R_s + R_L)^2 + (X_s + X_L)^2}}$$

The power delivered to the load is

$$P_L = |\mathbf{I}_L|^2\,R_L = \frac{|\mathbf{V}_s|^2\,R_L}{\sqrt{(R_s + R_L)^2 + (X_s + X_L)^2}}$$

For maximum power,

$$\frac{dP_L}{dR_L} = 0$$

$$|\mathbf{V}_s|^2 \left[\frac{(R_s + R_L)^2 + (X_s + X_L)^2 - 2\,R_L(R_s + R_L)}{\{(R_s + R_L)^2 + (X_s + X_L)^2\}^2} \right] = 0$$

$$(R_s + R_L)^2 + (X_s + X_L)^2 - 2\,R_L(R_s + R_L) = 0$$

$$R_s^2 + 2\,R_s\,R_L + R_L^2 + (X_s + X_L)^2 - 2\,R_L R_s - 2\,R_L^2 = 0$$

$$R_s^2 + (X_s + X_L)^2 - R_L^2 = 0$$

$$R_L^2 = R_s^2 + (X_s + X_L)^2$$

$$R_L = \sqrt{R_s^2 + (X_s + X_L)^2}$$

$$= |R_s + j(X_s + X_L)|$$

$$= |R_s + jX_s + jX_L|$$

$$= |Z_s + jX_L|$$

Hence, load resistance R_L should be equal to the magnitude of the impedance $\mathbf{Z}_s + jX_L$, i.e. $|\mathbf{Z}_s + jX_L|$ for maximum power transfer.

Example 6.47 *For maximum power transfer, find the value of $\mathbf{Z}_L$ in the network of Fig. 6.147 if (i) $\mathbf{Z}_L$ is an impedance, and (ii) $\mathbf{Z}_L$ is pure resistance.*

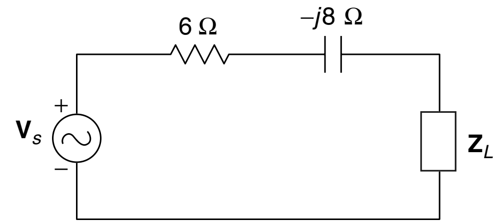


Fig. 6.147

Solution

$$\mathbf{Z}_s = (6 - j8)\ \Omega$$

(i) If $\mathbf{Z}_L$ is an impedance

For maximum power transfer, $\mathbf{Z}_L = \mathbf{Z}_s^* = (6 + j8)\ \Omega$

(ii) If $\mathbf{Z}_L$ is a resistance

For maximum power transfer, $\mathbf{Z}_L = |\mathbf{Z}_S| = |6 + j8| = 10\ \Omega$

Example 6.48 *For the maximum power transfer, find the value of $\mathbf{Z}_L$ in the network of Fig. 6.148 for the following cases:*

(i) $\mathbf{Z}_L$ is variable resistance, (ii) $\mathbf{Z}_L$ is complex impedance, with variable resistance and variable reactance, and (iii) $\mathbf{Z}_L$ is complex impedance with variable resistance and fixed reactance of j5 Ω.

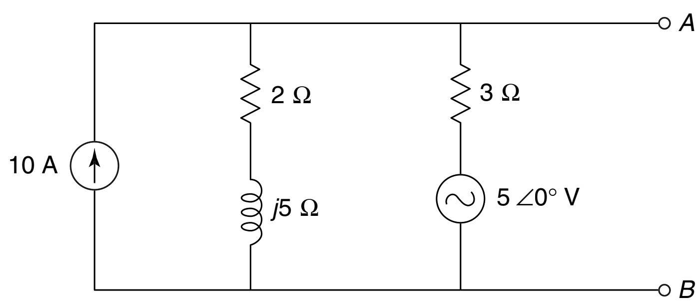


Fig. 6.148

Solution Thevenin's impedance can be calculated by replacing voltage source by a short circuit and current source by an open circuit.

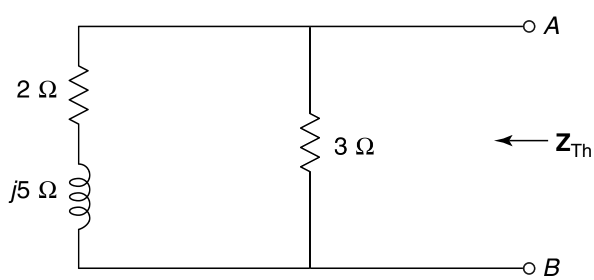


Fig. 6.149

$$\mathbf{Z}_{\text{Th}} = \frac{3(2+j5)}{3+2+j5} = (2.1+j0.9)\ \Omega$$

For maximum power transfer, value of $\mathbf{Z}_L$ will be,

(i) $\mathbf{Z}_L$ is variable resistance

$$\mathbf{Z}_L = |\mathbf{Z}_{\text{Th}}| = |2.1+j0.9| = 2.28\ \Omega$$

(ii) $\mathbf{Z}_L$ is complex impedance with variable resistance and variable reactance

$$\mathbf{Z}_L = \mathbf{Z}_{\text{Th}}^* = (2.1-j0.9)\ \Omega$$

(iii) $\mathbf{Z}_L$ is complex impedance with variable resistance and fixed reactance of $j5\ \Omega$

$$\mathbf{Z}_L = |\mathbf{Z}_{\text{Th}} + j5| = |2.1+j\,0.9+j5| = 6.26\ \Omega$$

Example 6.49 *Find the impedance $\mathbf{Z}_L$ so that maximum power can be transferred to it in the network of Fig. 6.150. Find maximum power.*

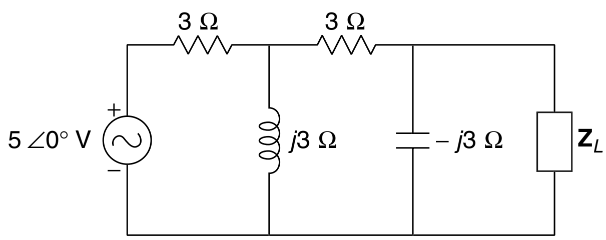


Fig. 6.150

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 6.151)

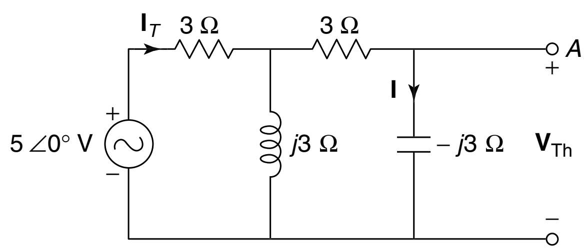


Fig. 6.151

$$\mathbf{Z}_T = 3 + \frac{j3(3-j3)}{3+j3-j3} = 6.71\angle 26.57^\circ\ \Omega$$

$$\mathbf{I}_T = \frac{5\angle 0^\circ}{6.71\angle 26.57^\circ} = 0.75\angle -26.57^\circ\ \text{A}$$

By current division rule,

$$\mathbf{I} = 0.75\angle -26.57^\circ \times \frac{j3}{3+j3-j3} = 0.75\angle 63.43^\circ\ \text{A}$$

$$\mathbf{V}_{\text{Th}} = (-j3)(0.75\angle 63.43^\circ) = 2.24\angle -26.57^\circ\ \text{V}$$

Step II Calculation of $\mathbf{Z}_{\text{Th}}$ (Fig. 6.152)

$$\mathbf{Z}_{\text{Th}} = [(3 \parallel j3) + 3] \parallel (-j3)$$
$$= 3 \angle -53.12° \ \Omega$$
$$= (1.8 - j2.4) \ \Omega$$

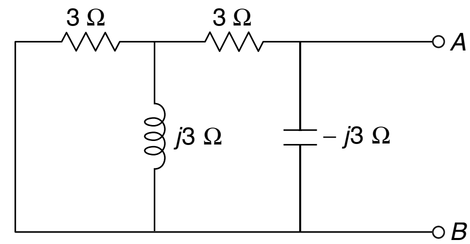


Fig. 6.152

Step III Calculation of $\mathbf{Z}_L$

For maximum power transfer, the load impedance should be a complex conjugate of the source impedance.

$$\mathbf{Z}_L = (1.8 + j2.4) \ \Omega$$

Step IV Calculation of P_{max} (Fig. 6.153)

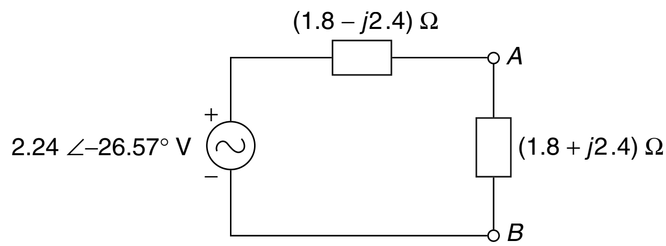


Fig. 6.153

$$P_{\text{max}} = \frac{|\mathbf{V}_{\text{Th}}|^2}{4R_L} = \frac{|2.24|^2}{4 \times 1.8} = 0.7 \text{ W}$$

Example 6.50 *Find the value of $\mathbf{Z}_L$ for maximum power transfer in the network shown and find maximum power.*

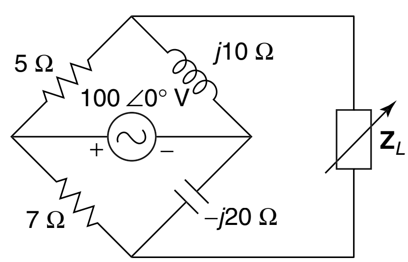


Fig. 6.154

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 6.155)

$$\mathbf{I}_1 = \frac{100\angle 0°}{5 + j10} = 8.94\angle -63.43° \text{ A}$$

$$\mathbf{I}_2 = \frac{100\angle 0°}{7 - j20} = 4.72\angle 70.7° \text{ A}$$

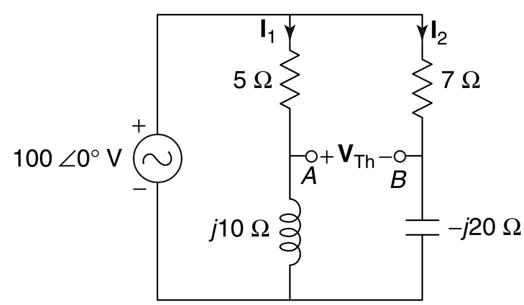


Fig. 6.155

$$\mathbf{V}_{\text{Th}} = \mathbf{V}_A - \mathbf{V}_B = (8.94\angle -63.43°)(j10) - (4.72\angle 70.7°)(-j20) = 71.76\angle 97.3° \text{ V}$$

Step II Calculation of $\mathbf{Z}_{\text{Th}}$ (Fig. 6.156)

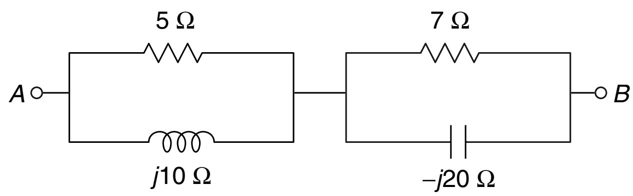


Fig. 6.156

$$\mathbf{Z}_{\text{Th}} = \frac{5(j10)}{5+j10} + \frac{7(-j20)}{7-j20} = \frac{50\angle 90°}{11.18\angle 63.43°} + \frac{140\angle -90°}{21.19\angle -70.7°} = (10.23 - j0.18)\ \Omega$$

Step III For maximum power transfer, the load impedance should be complex conjugate of the source impedance.

$$\mathbf{Z}_L = (10.23 + j0.18)\ \Omega$$

Step IV Calculation of P_{max} (Fig. 6.157)

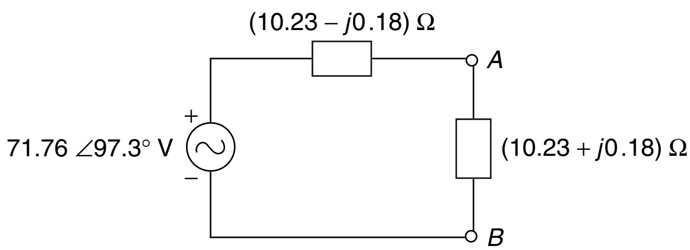


Fig. 6.157

$$P_{\text{max}} = \frac{|\mathbf{V}_{\text{Th}}|^2}{4R_L} = \frac{|71.76|^2}{4 \times 10.23} = 125.84 \text{ W}$$

Example 6.51 *Find the value of load impedance $\mathbf{Z}_L$ so that maximum power can be transferred to it in the network of Fig. 6.158. Find maximum power.*

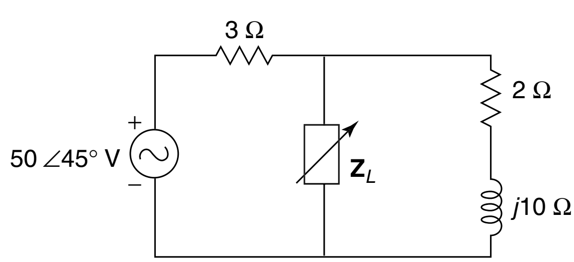


Fig. 6.158

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 6.159)

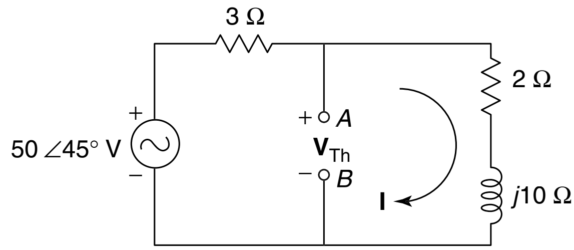


Fig. 6.159

$$\mathbf{I} = \frac{50\angle 45^\circ}{3+2+j10} = 4.47\angle -18.43^\circ \text{ A}$$

$$\mathbf{V}_{\text{Th}} = (2+j10)\,\mathbf{I} = (2+j10)(4.47\angle -18.43^\circ) = 45.6\angle 60.26^\circ \text{ V}$$

Step II Calculation of $\mathbf{Z}_{\text{Th}}$ (Fig. 6.160)

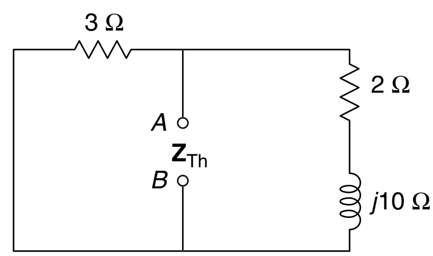


Fig. 6.160

$$\mathbf{Z}_{\text{Th}} = \frac{3(2+j10)}{3+2+j10} = (2.64 + j0.72)\ \Omega$$

Step III Calculation of $\mathbf{Z}_L$

For maximum power transfer, the load impedance should be complex conjugate of the source impedance.

$$\mathbf{Z}_L = (2.64 - j0.72)\ \Omega$$

Step IV Calculation of $P_{\max}$ (Fig. 6.161)

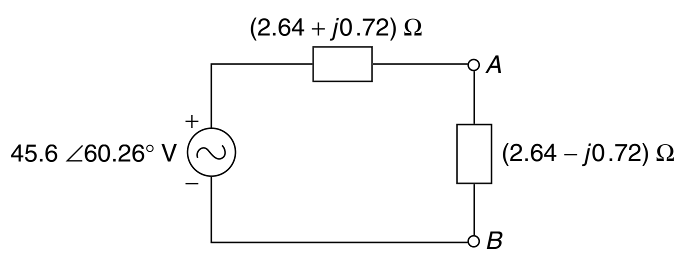


Fig. 6.161

$$P_{\max} = \frac{|\mathbf{V}_{\text{Th}}|^2}{4R_L} = \frac{|45.6|^2}{4\times 2.64} = 196.91 \text{ W}$$

Example 6.52 *Determine the load $\mathbf{Z}_L$ required to be connected in the network of Fig. 6.162 for maximum power transfer. Determine the maximum power drawn.*

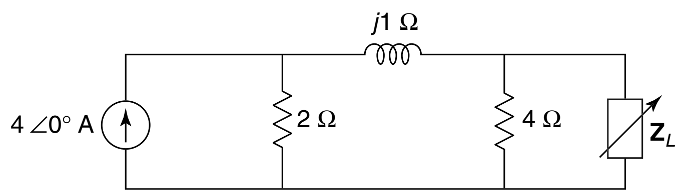


Fig. 6.162

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 6.163)

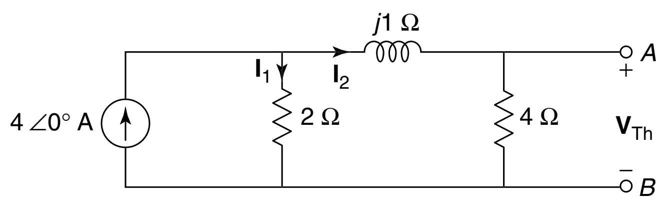


Fig. 6.163

$$\mathbf{I}_2 = 4\angle 0° \times \frac{2}{6+j1} = 1.315\angle -9.46° \text{ A}$$

$$\mathbf{V}_{\text{Th}} = 4\mathbf{I}_2 = 4(1.315\angle -9.46°) = 5.26\angle -9.46° \text{ V}$$

Step II Calculation of $\mathbf{Z}_{\text{Th}}$ (Fig. 6.164)

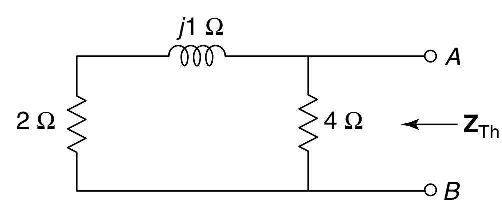


Fig. 6.164

$$\mathbf{Z}_{\text{Th}} = \frac{4(2+j1)}{4+2+j1} = 1.47\angle 17.1° = (1.41 + j0.43)\ \Omega$$

Step III Calculation of $\mathbf{Z}_L$

For maximum power transfer, the load impedance should be the complex conjugate of the source impedance.

$$\mathbf{Z}_L = (1.41 - j0.43)\ \Omega$$

Step IV Calculation of $P_{\max}$ (Fig. 6.165)

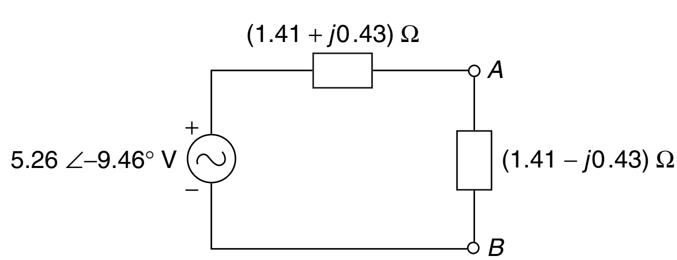


Fig. 6.165

$$P_{\max} = \frac{|\mathbf{V}_{\text{Th}}|^2}{4R_L} = \frac{|5.26|^2}{4 \times 1.41} = 4.91 \text{ W}$$

Example 6.53 *In the network shown in Fig. 6.166, find the value of $\mathbf{Z}_L$ for which the power transferred will be maximum. Also find maximum power.*

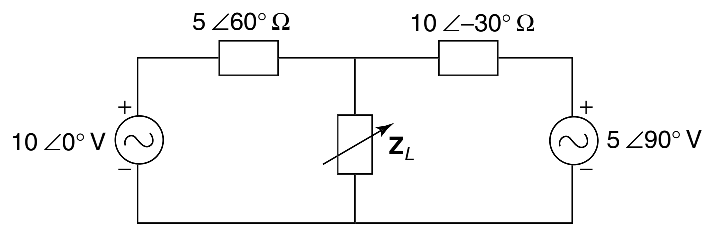


Fig. 6.166

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 6.167)

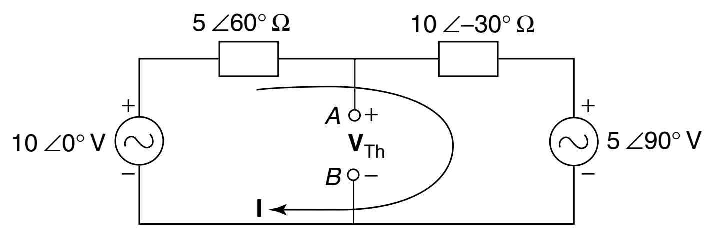


Fig. 6.167

Applying KVL to the mesh,

$$10\angle 0° - (5\angle 60°)\mathbf{I} - (10\angle -30°)\mathbf{I} - 5\angle 90° = 0$$
$$11.18\angle -26.57° - (11.18\angle -3.43°)\mathbf{I} = 0$$
$$\mathbf{I} = 1\angle -23.14° \text{ A}$$

Writing $\mathbf{V}_{\text{Th}}$ equation,

$$10\angle 0° - (5\angle 60°)\mathbf{I} - \mathbf{V}_{\text{Th}} = 0$$
$$10\angle 0° - (5\angle 60°)(1\angle -23.14°) - \mathbf{V}_{\text{Th}} = 0$$
$$\mathbf{V}_{\text{Th}} = 6.71\angle -26.56° \text{ V}$$

Step II Calculation of $\mathbf{Z}_{\text{Th}}$ (Fig. 6.168)

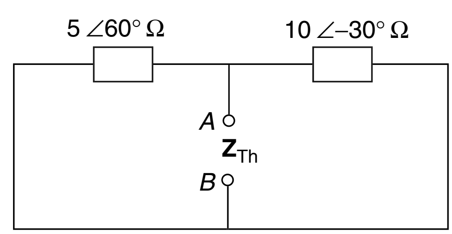


Fig. 6.168

$$\mathbf{Z}_{\text{Th}} = \frac{(5\angle 60°)(10\angle -30°)}{5\angle 60° + 10\angle -30°} = 4.47\angle 33.43° \ \Omega = (3.73 + j2.46)\ \Omega$$

Step III Calculation of $\mathbf{Z}_L$

For maximum power transfer, the load impedance should be the complex conjugate of the source impedance.

$$Z_L = Z_{\text{Th}}^* = (3.73 - j2.46)\ \Omega$$

Step IV Calculation of P_{max} (Fig. 6.169)

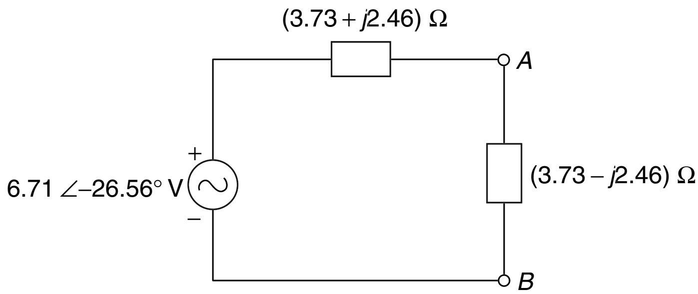


Fig. 6.169

$$P_{\text{max}} = \frac{|\mathbf{V}_{\text{Th}}|^2}{4R_L} = \frac{(6.71)^2}{4 \times 3.73} = 3.02\ \text{W}$$

Example 6.54 *In the network shown in Fig. 6.170, find the value of* $\mathbf{Z}_L$ *so that power transfer from the source is maximum. Also find maximum power.*

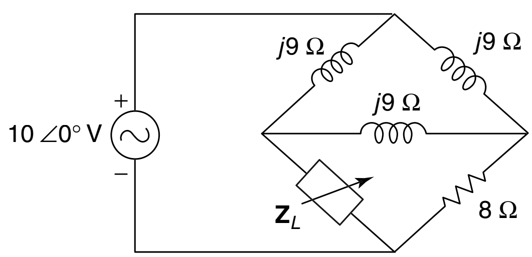


Fig. 6.170

Solution

Step I Calculation of $\mathbf{V}_{\text{Th}}$ (Fig. 6.171)

Applying Star-delta transformation (Fig. 6.172)

$$\mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z}_3 = \frac{(j9)(j9)}{j9 + j9 + j9} = j3\ \Omega$$

$\mathbf{V}_{\text{Th}}$ = Voltage drop across $(8 + j3)\Omega$ impedence

$$= (8 + j3)\frac{10\angle 0^\circ}{8 + j3 + j3} = 8.54\angle -16.31^\circ\ \text{V}$$

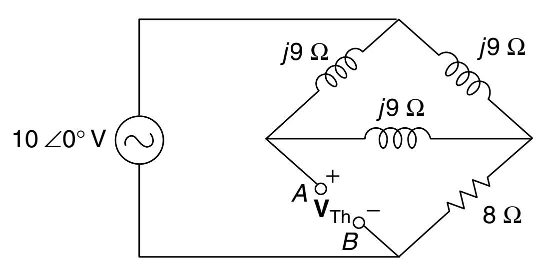


Fig. 6.171

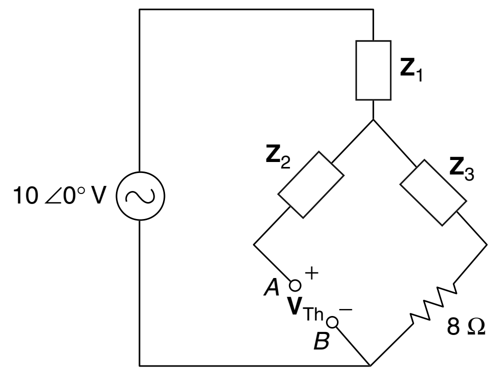


Fig. 6.172

Step II Calculation of $\mathbf{Z}_{\text{Th}}$ (Fig. 6.173)

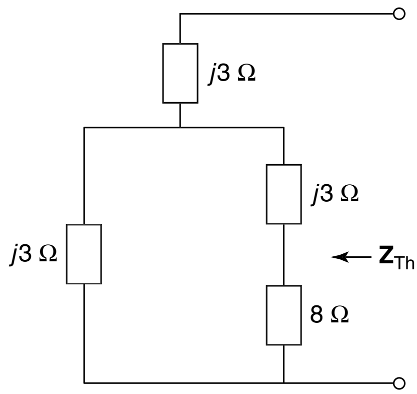


Fig. 6.173

$$\mathbf{Z}_{\text{Th}} = j3 + \frac{j3(8+j3)}{j3+8+j3)} = 5.51\angle 82.49°\ \Omega = (0.72 + j5.46)\ \Omega$$

Step III Calculation of $\mathbf{Z}_L$

For maximum power transfer, the load impedance should be the complex conjugate of the source impedance.

$$\mathbf{Z}_L = \mathbf{Z}^*_{\text{Th}} = (0.72 - j5.46)\ \Omega$$

Step IV Calculation of P_{max} (Fig. 6.174)

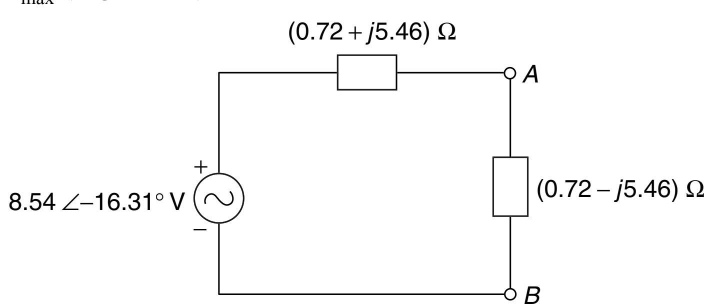


Fig. 6.174

$$P_{\text{max}} = \frac{|\mathbf{V}_{\text{Th}}|^2}{4R_L} = \frac{(8.54)^2}{4 \times 0.72} = 25.32\ \text{W}$$

Example 6.55 *For the network shown in Fig. 6.175, find the value of $\mathbf{Z}_L$ that will transfer maximum power from the source. Also find maximum power.*

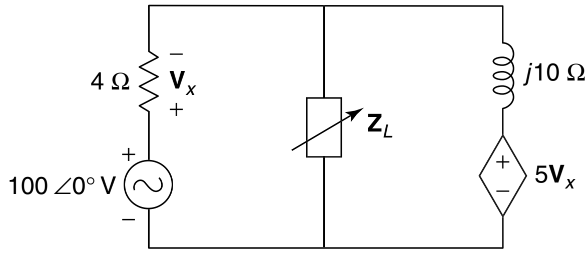


Fig. 6.175

Solution

Step I Calculation of $\mathbf{V}_{Th}$ (Fig. 6.176)

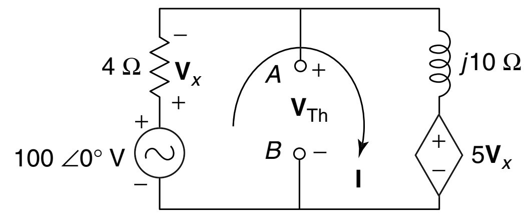


Fig. 6.176

From Fig 6.176,

$$\mathbf{V}_x = 4\mathbf{I}$$

Applying KVL to the mesh,

$$100\angle 0° - 4\mathbf{I} - j\,10\,\mathbf{I} - 5\mathbf{V}_x = 0$$

$$100\angle 0° - (4 + j10)\,\mathbf{I} - 5(4\mathbf{I}) = 0$$

$$\mathbf{I} = \frac{100\ \angle 0°}{24 + j10} = 3.85\ \angle -22.62°\ \text{A}$$

Writing $\mathbf{V}_{Th}$ equation,

$$100\ \angle\ 0° - 4\mathbf{I} - \mathbf{V}_{Th} = 0$$

$$100\angle 0° - 4(3.85\ \angle -22.62°) - \mathbf{V}_{Th} = 0$$

$$\mathbf{V}_{Th} = 86\angle 3.95°\ \text{V}$$

Step II Calculation of $\mathbf{I}_N$ (Fig. 6.177)

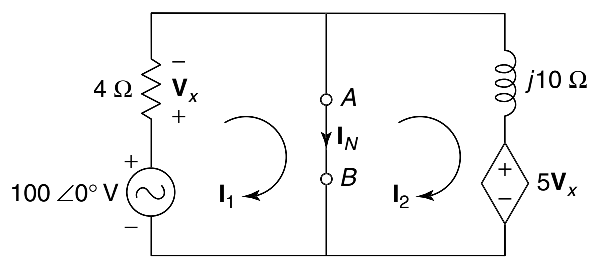


Fig. 6.177

From Fig. 6.177,

$$\mathbf{V}_x = 4\mathbf{I}_1$$

Applying KVL to Mesh 1,

$$100\ \angle 0° - 4\mathbf{I}_1 = 0$$

$$\mathbf{I}_1 = 25\ \text{A}$$

Applying KVL to Mesh 2,

$$-j10\mathbf{I}_2 - 5\mathbf{V}_x = 0$$

$$-j10\mathbf{I}_2 - 5(4\mathbf{I}_1) = 0$$

$$-j10\mathbf{I}_2 - 5(100) = 0$$

$$\mathbf{I}_2 = 50\angle 90°\text{A}$$

$$\mathbf{I}_N = \mathbf{I}_1 - \mathbf{I}_2 = 25 - 50\angle 90° = 55.9\angle -63.43°\text{A}$$

Step III Calculation of $\mathbf{Z}_{Th}$

$$\mathbf{Z}_{Th} = \frac{\mathbf{V}_{Th}}{\mathbf{I}_N} = \frac{86\angle 3.95°}{55.9\angle -63.43°} = 1.54\angle 67.38°\ \Omega = (0.59 + j1.42)\ \Omega$$

Step IV Calculation of $\mathbf{Z}_L$

For maximum power transfer, $\mathbf{Z}_L = \mathbf{Z}^*_{\text{Th}} = (0.59 - j1.42)\ \Omega$

Step V Calculation of P_{max} (Fig. 6.178)

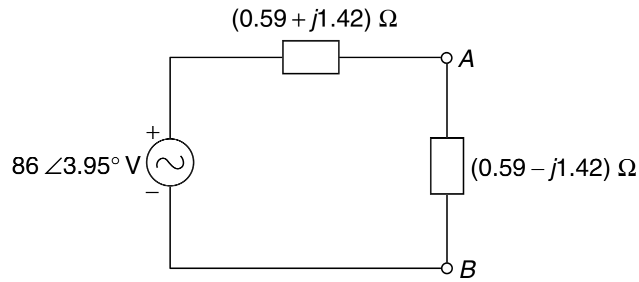


Fig. 6.178

$$P_{\text{max}} = \frac{|\mathbf{V}_{\text{Th}}|^2}{4R_L} = \frac{(86)^2}{4 \times 0.59} = 3133.9\ \text{W}$$

6.8 RECIPROCITY THEOREM

The Reciprocity theorem states that '*In a linear, bilateral, active, single-source network, the ratio of excitation to response remains same when the positions of excitation and response are interchanged.*'

Example 6.56 *Find the current through the 6 Ω resistor and verify the reciprocity theorem.*

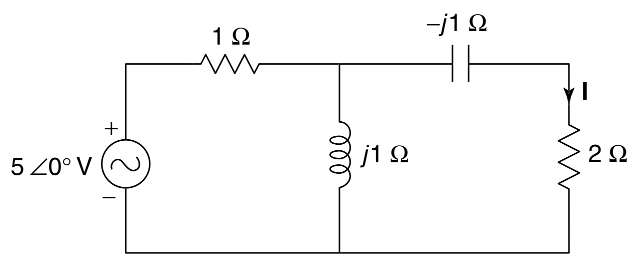


Fig. 6.179

Solution

Case I Calculation of current **I** when excitation and response are not interchanged (Fig. 6.180)

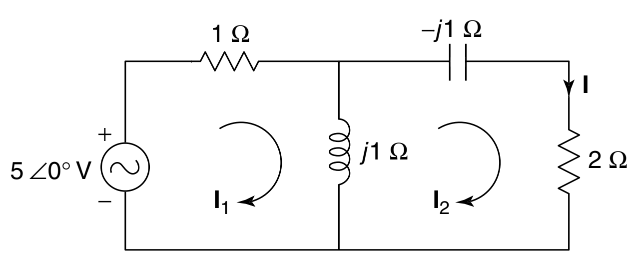


Fig. 6.180

Applying KVL to Mesh 1,

$$5\angle 0° - 1\mathbf{I}_1 - j1(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(1 + j1)\,\mathbf{I}_1 - j1\mathbf{I}_2 = 5\angle 0° \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-j1(\mathbf{I}_2-\mathbf{I}_1)+j1\mathbf{I}_2-2\mathbf{I}_2=0$$

$$-j1\mathbf{I}_1+2\mathbf{I}_2=0 \qquad \ldots\text{(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix}1+j1 & -j1\\ -j1 & 2\end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix}=\begin{bmatrix}5\angle 0^\circ\\ 0\end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2=\frac{\begin{vmatrix}1+j1 & 5\angle 0^\circ\\ -j1 & 0\end{vmatrix}}{\begin{vmatrix}1+j1 & -j1\\ -j1 & 2\end{vmatrix}}=1.39\angle 56.31^\circ \text{ A}$$

$$\mathbf{I}=\mathbf{I}_2=1.39\angle 56.31^\circ \text{ A}$$

Case II Calculation of current **I** when excitation and response are interchanged (Fig. 6.181)

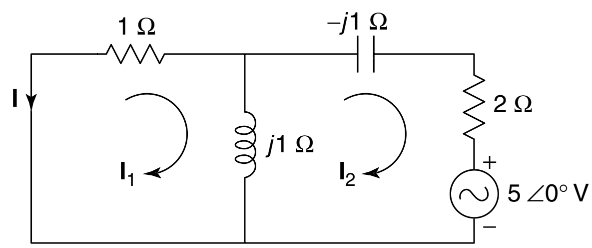


Fig. 6.181

Applying KVL to Mesh 1,

$$-1\mathbf{I}_1-j1(\mathbf{I}_1-\mathbf{I}_2)=0$$

$$(1+j1)\mathbf{I}_1-j1\mathbf{I}_2=0 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-j1(\mathbf{I}_2-\mathbf{I}_1)+j1\mathbf{I}_2-2\mathbf{I}_2-5\angle 0^\circ=0$$

$$-j1\mathbf{I}_1+2\mathbf{I}_2=-5\angle 0^\circ \qquad \ldots\text{(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix}1+j1 & -j1\\ -j1 & 2\end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix}=\begin{bmatrix}0\\ -5\angle 0^\circ\end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_1=\frac{\begin{vmatrix}0 & -j1\\ -5\angle 0^\circ & 2\end{vmatrix}}{\begin{vmatrix}1+j1 & -j1\\ -j1 & 2\end{vmatrix}}=1.38\angle -123.69^\circ \text{ A}$$

$$\mathbf{I}=-\mathbf{I}_1=1.39\angle 56.31^\circ \text{ A}$$

Since the current **I** is same in both the cases, the reciprocity theorem is verified.

Example 6.57 *In the network of Fig. 6.182, find the voltage $\mathbf{V}_x$ and verify the reciprocity theorem.*

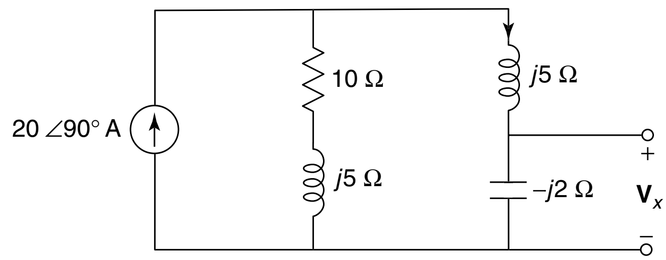


Fig. 6.182

Solution

Case I Calculation of voltage $\mathbf{V}_x$ when excitation and response are interchanged. (Fig. 6.183)

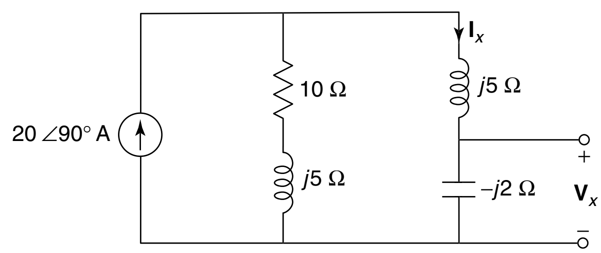


Fig. 6.183

By current division rule,

$$\mathbf{I}_x = (20\angle 90°)\frac{(10+j5)}{(10+j5)+(j5-j2)} = 17.46\angle 77.91°\ \text{A}$$

$$\mathbf{V}_x = (-j2)\mathbf{I}_x = (-j2)(17.46\angle 77.91°) = 34.92\ \angle -12.09°\ \text{V}$$

Case II Calculation of voltage $\mathbf{V}_x$ when excitation and response are interchanged (Fig. 6.184)

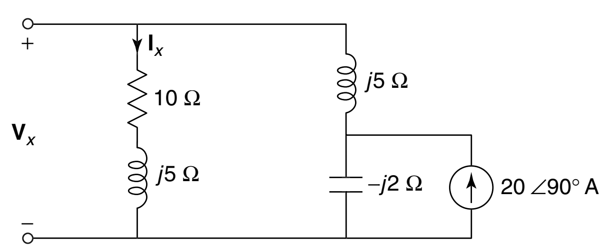


Fig. 6.184

$$\mathbf{I}_x = (20\angle 90°)\frac{(-j2)}{(-j2)+(10+j5+j5)} = 3.12\angle -38.66°\text{A}$$

$$\mathbf{V}_x = (10+j5)\mathbf{I}_x = (10+j5)(3.12\angle -38.66°) = 34.88\angle -12.09°\text{V}$$

Since the voltage $\mathbf{V}_x$ is same in both the cases, the reciprocity theorem is verified.

Example 6.58 *Find **I** and verify the reciprocity theorem for the network shown in Fig. 6.185.*

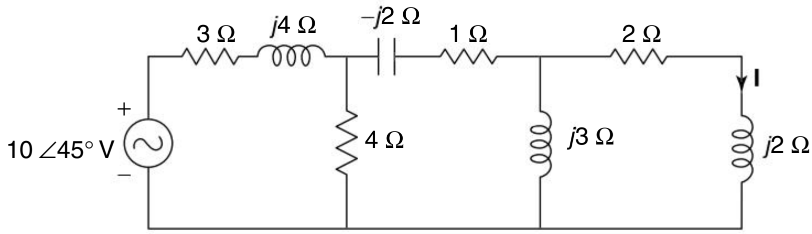


Fig. 6.185

Solution

Case I Calculation of **I** when excitation and response are not interchanged (Fig. 6.186)

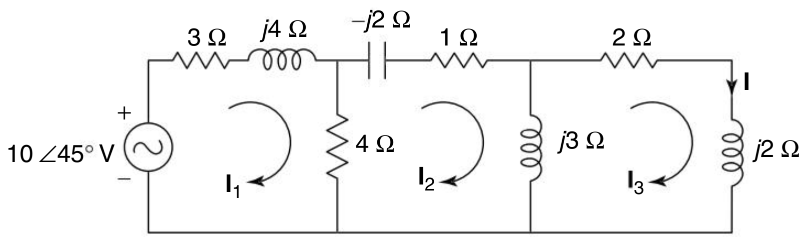


Fig. 6.186

Applying KVL to Mesh 1,

$$10\angle 45^\circ - (3+j4)\mathbf{I}_1 - 4(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(7+j4)\mathbf{I}_1 - 4\mathbf{I}_2 = 10\angle 45^\circ \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-4(\mathbf{I}_2 - \mathbf{I}_1) - (1-j2)\,\mathbf{I}_2 - j3(\mathbf{I}_2 - \mathbf{I}_3) = 0$$

$$-4\mathbf{I}_1 + (5+j1)\mathbf{I}_2 - j3\mathbf{I}_3 = 0 \qquad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-j3(\mathbf{I}_3 - \mathbf{I}_2) - 2\mathbf{I}_3 - j2\mathbf{I}_3 = 0$$

$$-j3\mathbf{I}_2 + (2+j5)\mathbf{I}_3 = 0 \qquad \text{...(iii)}$$

Writing Eqs (i), (ii) and (iii) in matrix form,

$$\begin{bmatrix} 7+j4 & -4 & 0 \\ -4 & 5+j1 & -j3 \\ 0 & -j3 & 2+j5 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 10\angle 45^\circ \\ 0 \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_3 = \frac{\begin{vmatrix} 7+j4 & -4 & 10\angle 45^\circ \\ -4 & 5+j1 & 0 \\ 0 & -j3 & 0 \end{vmatrix}}{\begin{vmatrix} 7+j4 & -4 & 0 \\ -4 & 5+j1 & -j3 \\ 0 & -j3 & 2+j5 \end{vmatrix}} = 0.704\angle 30.72^\circ \text{ A}$$

$$\mathbf{I} = \mathbf{I}_3 = 0.704\angle 30.72^\circ \text{A}$$

Case II Calculation of **I** when excitation and response are interchanged (Fig. 6.187)

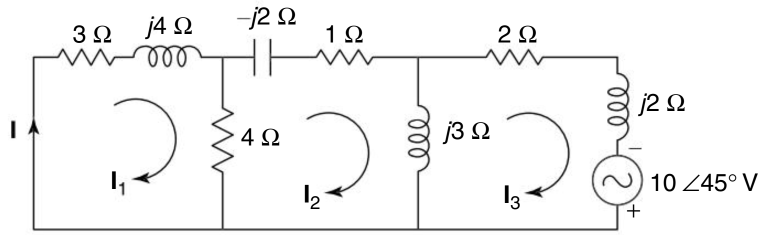


Fig. 6.187

Applying KVL to Mesh 1,

$$-(3+j4)\mathbf{I}_1-4(\mathbf{I}_1-\mathbf{I}_2)=0$$

$$(7+j4)\mathbf{I}_1-4\mathbf{I}_2=0 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-4(\mathbf{I}_2-\mathbf{I}_1)-(1-j2)\,\mathbf{I}_2-j3(\mathbf{I}_2-\mathbf{I}_3)=0$$

$$-4\mathbf{I}_1+(5+j1)\mathbf{I}_2-j3\mathbf{I}_3=0 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-j3(\mathbf{I}_3-\mathbf{I}_2)-2\mathbf{I}_3-j2\mathbf{I}_3+10\angle 45^\circ=0$$

$$-j3\mathbf{I}_2+(2+j5)\mathbf{I}_3=10\angle 45^\circ \qquad \ldots\text{(iii)}$$

Writing Eqs (i), (ii) and (iii) in matrix form,

$$\begin{bmatrix} 7+j4 & -4 & 0 \\ -4 & 5+j1 & -j3 \\ 0 & -j3 & 2+j5 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 10\angle 45^\circ \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_1=\frac{\begin{vmatrix} 0 & -4 & 0 \\ 0 & 5+j1 & -j3 \\ 10\angle 45^\circ & -j3 & 2+j5 \end{vmatrix}}{\begin{vmatrix} 7+j4 & -4 & 0 \\ -4 & 5+j1 & -j3 \\ 0 & -j3 & 2+j5 \end{vmatrix}}=0.704\angle 30.72^\circ\text{A}$$

$$\mathbf{I}=\mathbf{I}_1=0.704\angle 30.72^\circ\text{A}$$

Since the current **I** is same in both the cases, the reciprocity theorem, is verified.

6.9 MILLMAN'S THEOREM

Millman's theorem states that '*If there are n voltage sources $\boldsymbol{V}_1$, $\boldsymbol{V}_2$. … $\boldsymbol{V}_n$ with internal impedances $\boldsymbol{Z}_1$, $\boldsymbol{Z}_2$, … $\boldsymbol{Z}_n$ respectively connected in parallel then these voltage sources can be replaced by a single voltage source $\boldsymbol{V}_m$ and a single series impedance $\boldsymbol{Z}_m$*.

$$\mathbf{V}_m=\frac{\mathbf{V}_1\,\mathbf{Y}_1+\mathbf{V}_2\,\mathbf{Y}_2+\ldots+\mathbf{V}_n\,\mathbf{Y}_n}{\mathbf{Y}_1+\mathbf{Y}_2+\ldots+\mathbf{Y}_n}$$

$$\mathbf{Z}_m=\frac{1}{\mathbf{Y}_m}=\frac{1}{\mathbf{Y}_1+\mathbf{Y}_2+\ldots+\mathbf{Y}_n}$$

Example 6.59 *Find the current through the 40 Ω resistor for the network shown in Fig. 6.188.*

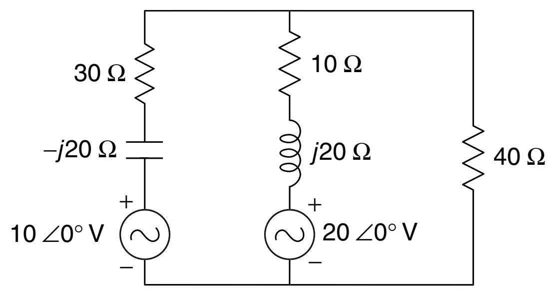


Fig. 6.188

Solution

Step I Calculation of $\mathbf{V}_m$

$$\mathbf{V}_m = \frac{\mathbf{V}_1\mathbf{Y}_1 + \mathbf{V}_2\mathbf{Y}_2}{\mathbf{Y}_1 + \mathbf{Y}_2} = \frac{(10\angle 0^\circ)\left(\dfrac{1}{30 - j20}\right) + (20\angle 0^\circ)\left(\dfrac{1}{10 + j20}\right)}{\dfrac{1}{30 - j20} + \dfrac{1}{10 + j20}} = 18.2\angle{-15.95^\circ}\text{V}$$

Step II Calculation of $\mathbf{Z}_m$

$$\mathbf{Z}_m = \frac{1}{\mathbf{Y}_m} = \frac{1}{\mathbf{Y}_1 + \mathbf{Y}_2} = \frac{1}{\dfrac{1}{30 - j20} + \dfrac{1}{10 + j20}} = 20.15\angle 29.74^\circ\ \Omega$$

Step III Calculation of $\mathbf{I}_L$ (Fig. 6.189)

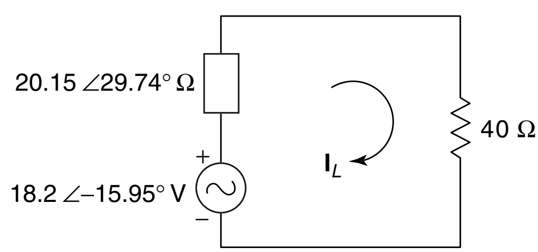


Fig. 6.189

$$\text{I}_L = \frac{18.2\angle{-15.95^\circ}}{20.15\angle 29.74^\circ + 40} = 0.31\angle{-25.81^\circ}\ \text{A}$$

Example 6.60 *Find the current **I** in the network shown in Fig. 6.190.*

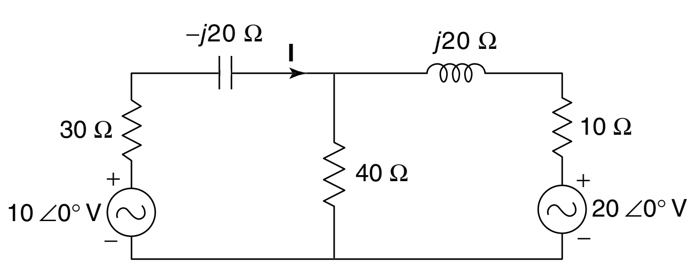


Fig. 6.190

Solution The network is redrawn as shown in Fig. 6.191.

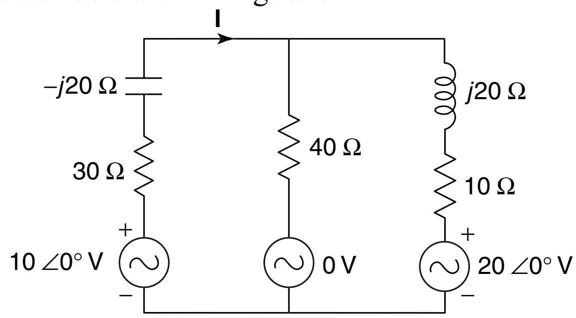


Fig. 6.191

Step I Calculation of $\mathbf{V}_m$

$$\mathbf{V}_m = \frac{\mathbf{V}_1\mathbf{Y}_1 + \mathbf{V}_2\mathbf{Y}_2}{\mathbf{Y}_1 + \mathbf{Y}_2} = \frac{0\left(\frac{1}{40}\right) + (20\angle 0^\circ)\left(\frac{1}{10+j20}\right)}{\frac{1}{40} + \frac{1}{10+j20}} = 14.86\angle -21.8^\circ \text{ V}$$

Step II Calculation of $\mathbf{Z}_m$

$$\mathbf{Z}_m = \frac{1}{\mathbf{Y}_m} = \frac{1}{\mathbf{Y}_1 + \mathbf{Y}_2} = \frac{1}{\frac{1}{40} + \frac{1}{10+j20}} = 16.61\angle 41.63^\circ \ \Omega$$

Step III Calculation of **I** (Fig. 6.192)

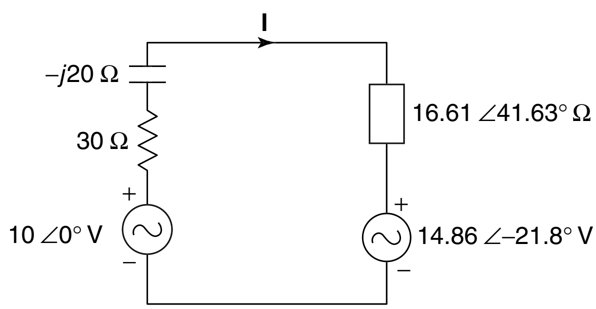


Fig. 6.192

$$\mathbf{I} = \frac{10 - 14.86\angle -21.8^\circ}{30 - j20 + 16.61\angle 41.63^\circ} = 0.15\angle 136.47^\circ \text{ A}$$

Example 6.61 *Apply the dual Millman theorem and find the power loss in the (2 + j2) Ω impedance.*

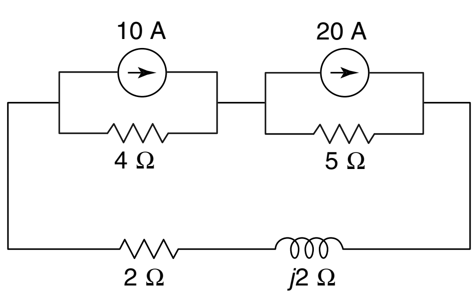


Fig. 6.193

Solution

Step I Calculation of $\mathbf{I}_m$

$$\mathbf{I}_m = \frac{\mathbf{I}_1\mathbf{Z}_1 + \mathbf{I}_2\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(10)(4)+(20)(5)}{4+5} = 15.56 \text{ A}$$

Step II Calculation of $\mathbf{Z}_m$

$$\mathbf{Z}_m = \mathbf{Z}_1 + \mathbf{Z}_2 = 4+5 = 9\ \Omega$$

Step III Calculation of P (Fig. 6.194)

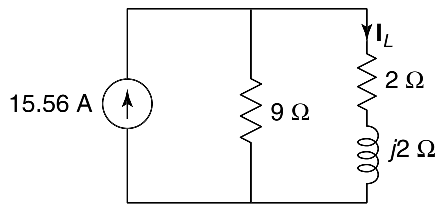


Fig. 6.194

By current division rule,

$$\mathbf{I}_L = 15.56 \times \frac{9}{9+2+j2} = 12.53\angle -10.3^\circ \text{ A}$$

$$P = \mathbf{I}_L^2 R_L = (12.53)^2 \times 2 = 314 \text{ W}$$

Example 6.62 *In the network shown in Fig. 6.195, what load Z_L will receive the maximum power. Also find maximum power.*

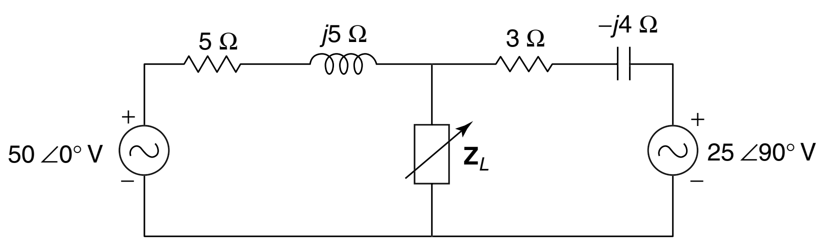


Fig. 6.195

Solution

Step I Calculation of $\mathbf{V}_m$

$$\mathbf{V}_m = \frac{\mathbf{V}_1\mathbf{Y}_1 + \mathbf{V}_2\mathbf{Y}_2}{\mathbf{Y}_1 + \mathbf{Y}_2} = \frac{(50\angle 0^\circ)\left(\dfrac{1}{5+j5}\right) + (25\angle 90^\circ)\left(\dfrac{1}{3-j4}\right)}{\dfrac{1}{5+j5} + \dfrac{1}{3-j4}} = 9.81\angle -78.69^\circ \text{V}$$

Step II Calculation of $\mathbf{Z}_m$

$$\mathbf{Z}_m = \frac{1}{\mathbf{Y}_m} = \frac{1}{\mathbf{Y}_1 + \mathbf{Y}_2} = \frac{1}{\dfrac{1}{5+j5} + \dfrac{1}{3-j4}} = 4.39\angle -15.26^\circ\ \Omega = (4.23 - j1.15)\ \Omega$$

Step II Calculation of change in current by compensation theorem

$$\delta \mathbf{Z} = (1 + j1) - (5 + j2) = (-4 - j1)\ \Omega$$
$$\mathbf{V}_c = \mathbf{I}_L \delta \mathbf{Z} = (1.44\angle -25^\circ)(-4 - j1) = 5.94\angle 169.04^\circ \text{ V}$$

The compensating network is shown in Fig. 6.208.

$$\delta \mathbf{I}_L = -\frac{5.94\angle 169.04^\circ}{2.94\angle -11.31^\circ + 5 + j2 - 4 - j1}$$
$$= 1.52\angle -17.18^\circ \text{ A}$$

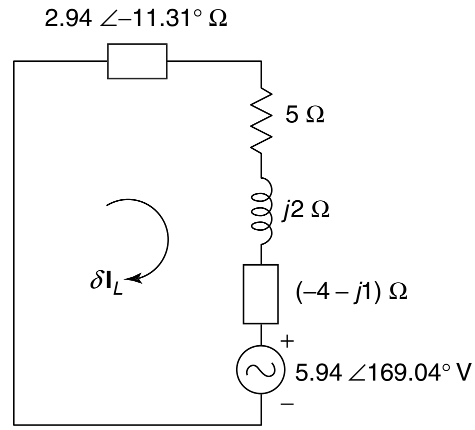


Fig. 6.208

Exercises

MESH ANALYSIS

6.1 Find the current through the 3 + j4 Ω impedance in Fig. 6.209.

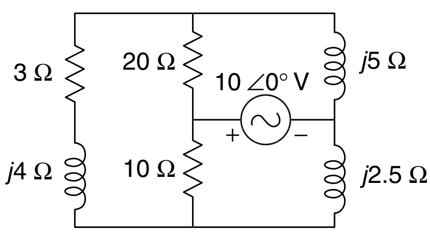


Fig. 6.209

[0]

6.2 In the network of Fig. 6.210, find $\mathbf{V}_0$.

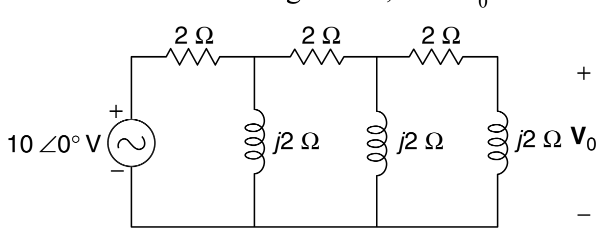


Fig. 6.210

[1.56 ∠128.7° V]

6.3 Find the current $\mathbf{I}_3$ in the network of Fig. 6.211.

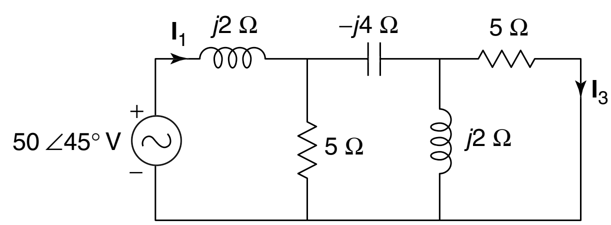


Fig. 6.211

[11.6 ∠113.2° A]

6.4 In the network of Fig.6.212, find $\mathbf{V}_2$ which results in zero current through the 4 Ω resistor.

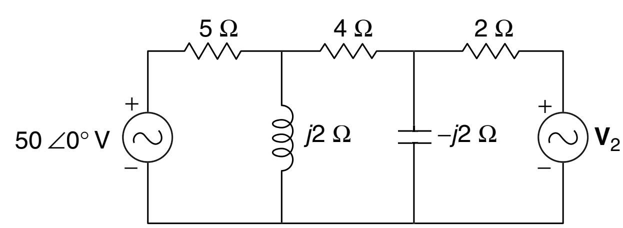


Fig. 6.212

[26.3 ∠113.2° V]

NODE ANALYSIS

6.5 For the network shown in Fig. 6.213, find the voltage $\mathbf{V}_{AB}$.

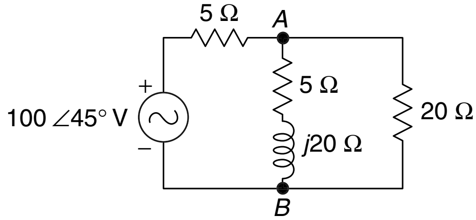


Fig. 6.213

[75.4 ∠55.2° V]

6.6 Find the voltages at nodes 1 and 2 in the network of Fig. 6.214.

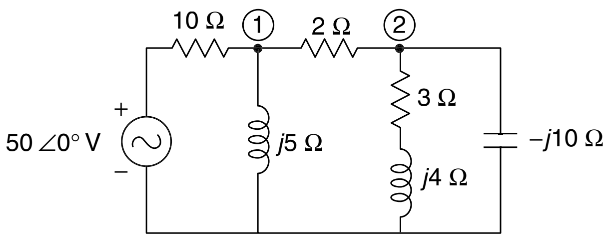


Fig. 6.214

[15.95 ∠49.94° V, 12.9 ∠55.5° V]

6.7 In the network of Fig. 6.215, find the current in the 10 ∠30° V source.

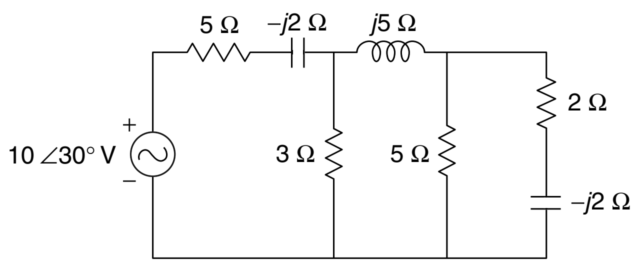


Fig. 6.215

[1.44 ∠38.8° A]

SUPERPOSITION THEOREM

6.8 For the network shown in Fig. 6.216, find the current in the 10 Ω resistor.

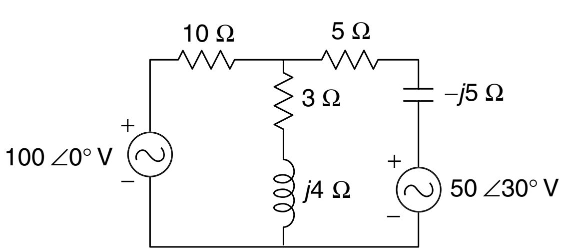


Fig. 6.216

[73.4 ∠−21.84° A]

6.9 In the network of Fig. 6.217, find the current through capacitance.

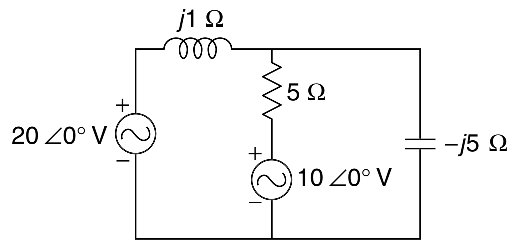


Fig. 6.217

[4.86 ∠80.8° A]

THEVENIN'S THEOREM

6.10 Obtain Thevenin's equivalent network for the network shown in Fig. 6.218.

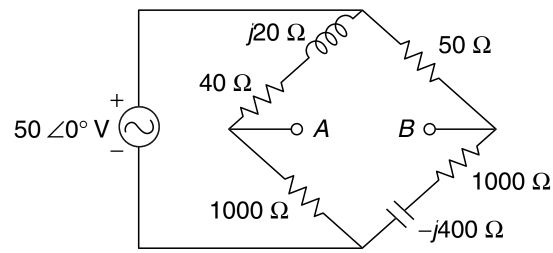


Fig. 6.218

[0.192 ∠−43.4° V, 88.7 ∠11.55° Ω]

6.11 Obtain Thevenin's equivalent network for Fig. 6.219.

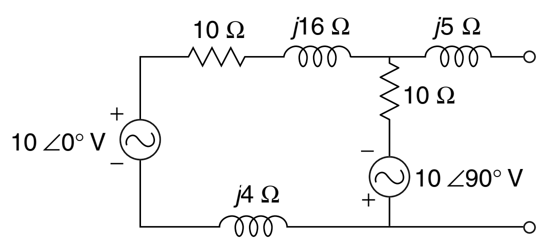


Fig. 6.219

[11.17 ∠−63.4° V, 10.6 ∠45° Ω]

NORTON'S THEOREM

6.12 Find Norton's equivalent network for Fig. 6.220.

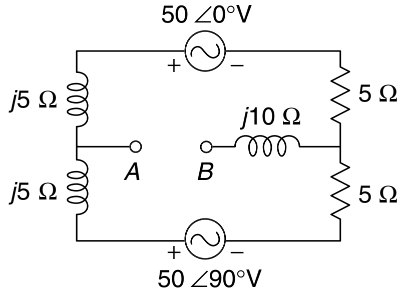


Fig. 6.220

[2.77 ∠–33.7° A, 2.5 + j12.5 Ω]

6.13 Find the current through the (3 + j4) Ω impedance in the network of Fig. 6.221.

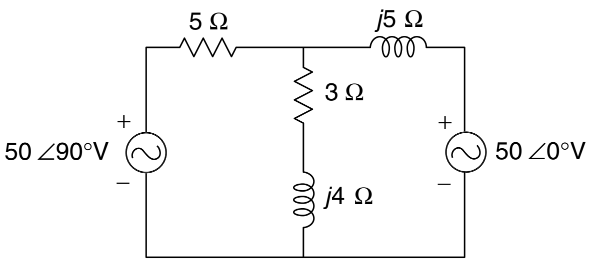


Fig. 6.221

[8.3 ∠85.2° A]

MAXIMUM POWER TRANSFER THEOREM

6.14 Determine the maximum power delivered to the load in the network shown in Fig. 6.222.

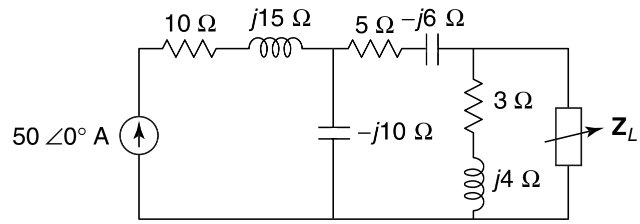


Fig. 6.222

[1032.35 W]

6.15 For the network shown in Fig. 6.223, find the value of $\mathbf{Z}_L$ that will receive the maximum power. Determine also this power.

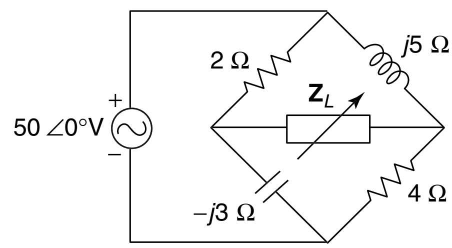


Fig. 6.223

[3.82 – j1.03 Ω, 54.5 W]

Objective-Type Questions

6.1 In Fig. 6.224, the equivalent impedance seen across terminals a, b, is

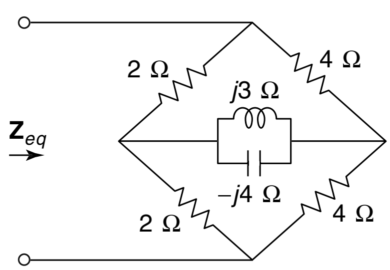


Fig. 6.224

(a) $\dfrac{16}{3}$ Ω (b) $\dfrac{8}{3}$ Ω

(C) $\left(\dfrac{8}{3}+j12\right)$ Ω (d) none of the above

6.2 The Thevenin equivalent voltage $\mathbf{V}_{Th}$ appearing between the terminals A and B of the network shown in Fig. 6.225 is given by

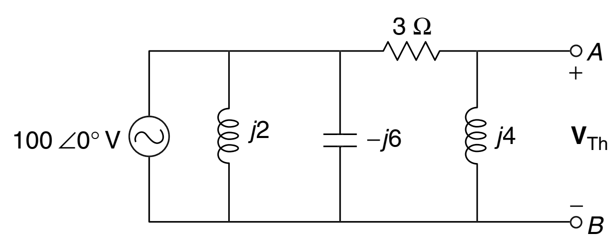


Fig. 6.225

(a) $j16(3 - j4)$ (b) $j16(3 + j4)$

(c) $16(3 + j4)$ (d) $16\,(3 - j4)$

6.3 A source of angular frequency of 1 rad/s has a source impedance consisting of a 1 Ω resistance